Bayesian O-mode profile inversion with dynamic initialisation

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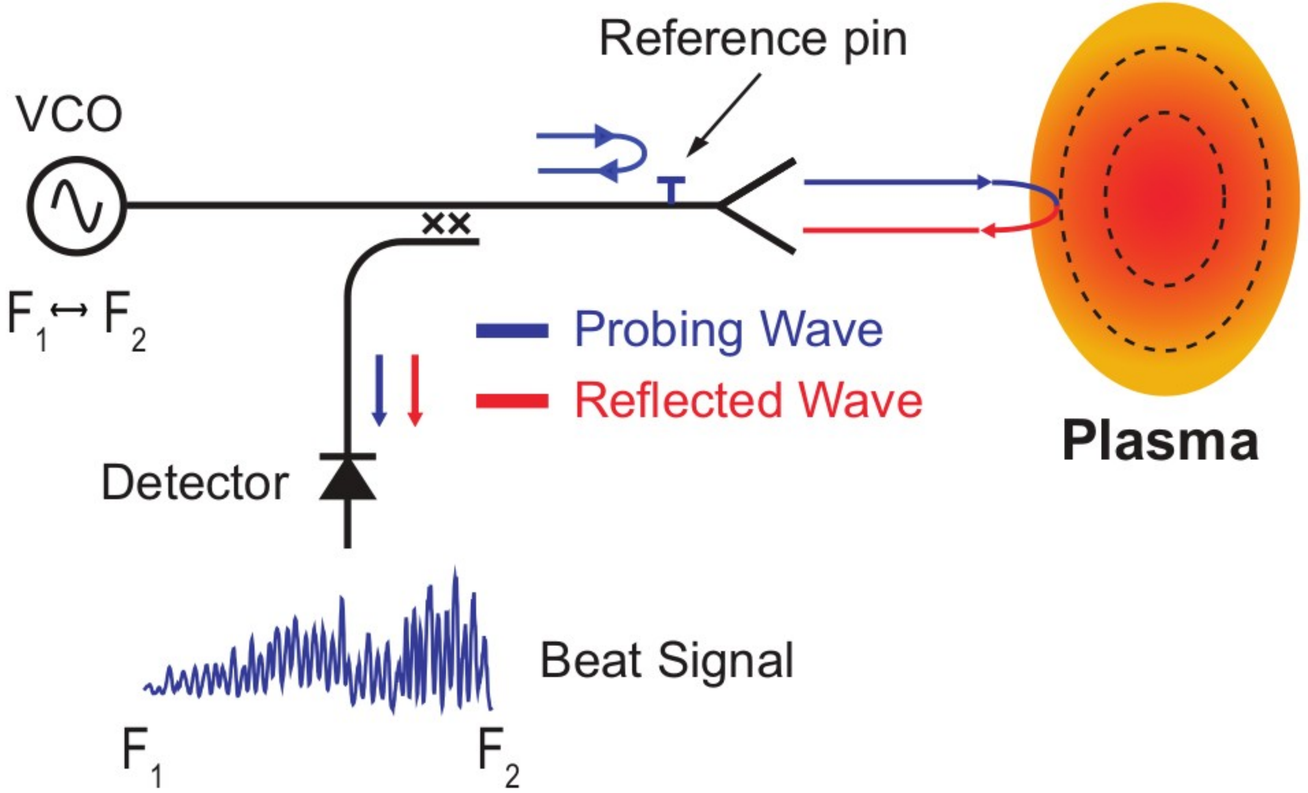
Diagnostics basics: O-Mode density reflectometry

At AUG

- Microwave chirp ~ 17-75 GHz
- Reflected at cut-off density
- Overlaid with reference signal
- Beating, due to changing phase delay
- Low-Pass Filter
- Sampling at 40 MHz
- Extract beat frequency
- Group delay ~ beat freq.

$$\tau_g = \frac{\omega_B}{\kappa}$$

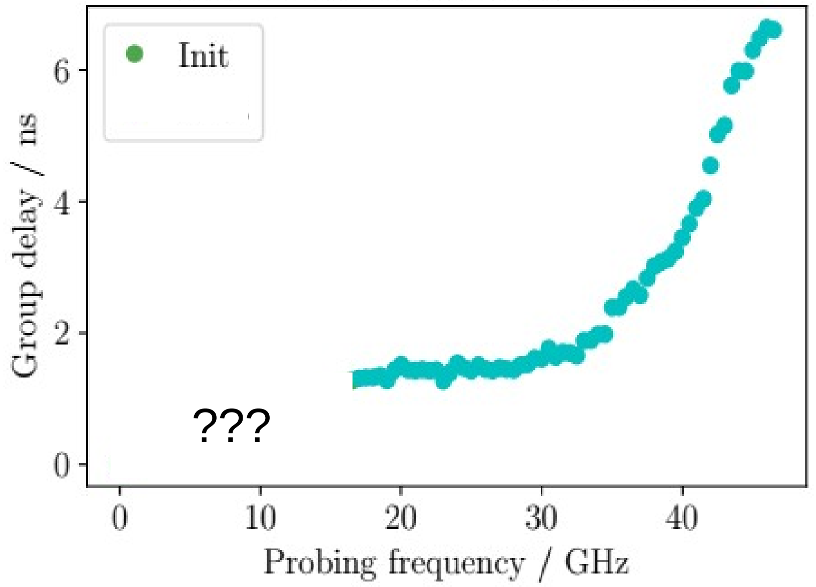
LFS Sector 5 @ z=0.14m, HFS Sector 4 @z=0.07m

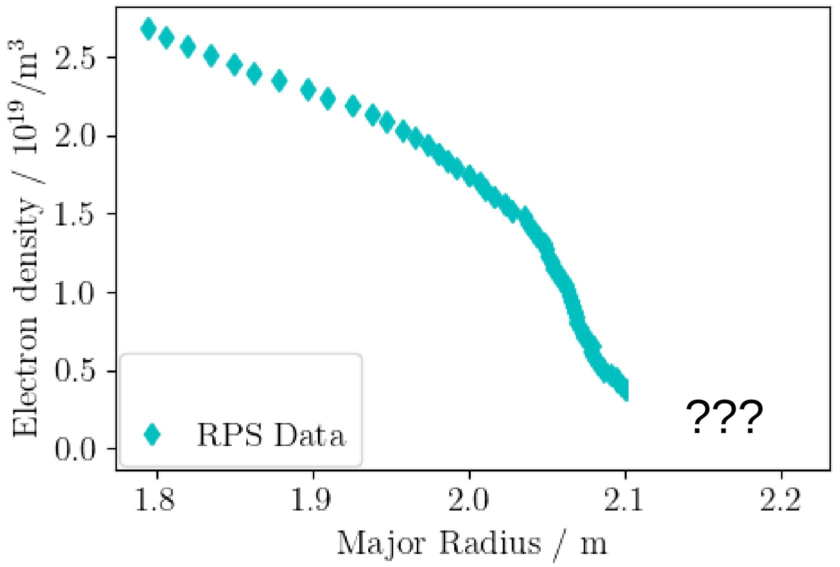


Why initialisation

Initialisation issue

- First frequency ~ 17Ghz
- First probed density 3.6e18 / m³





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Usual solution:

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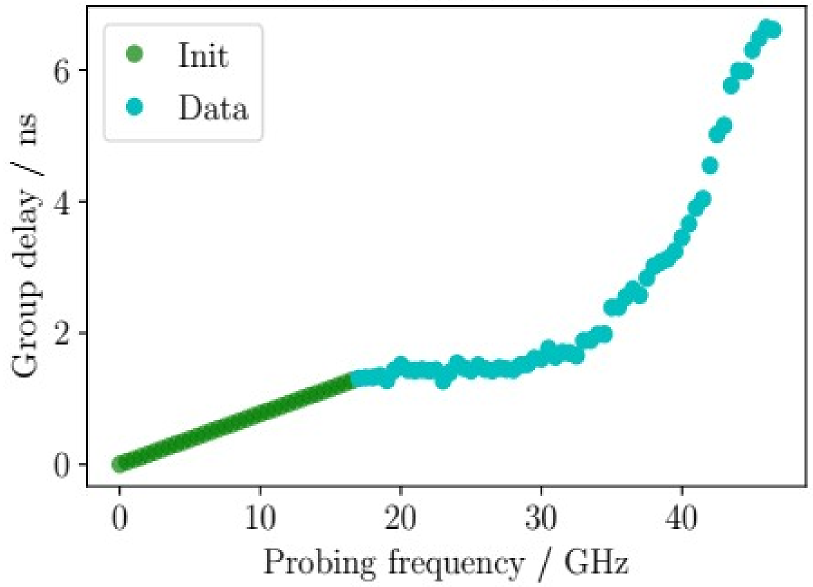
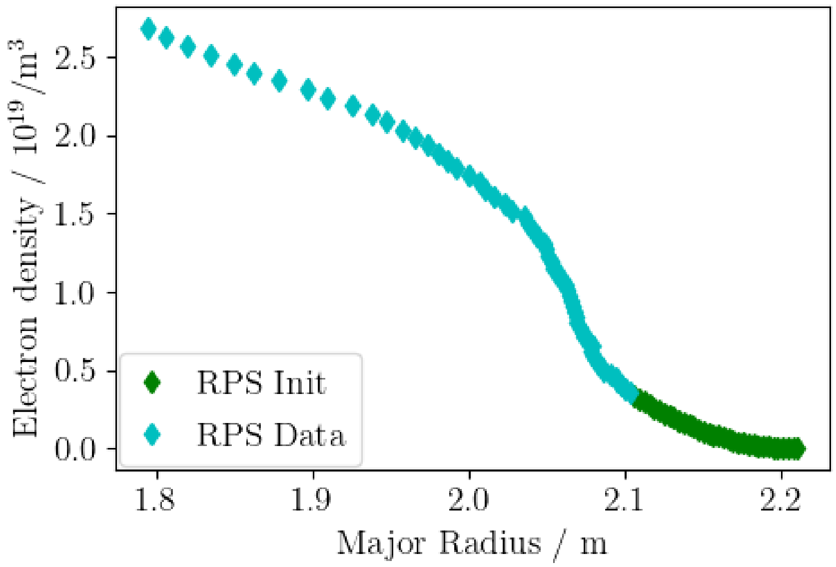
  Linear group delay interpolation
  → density along LOS i





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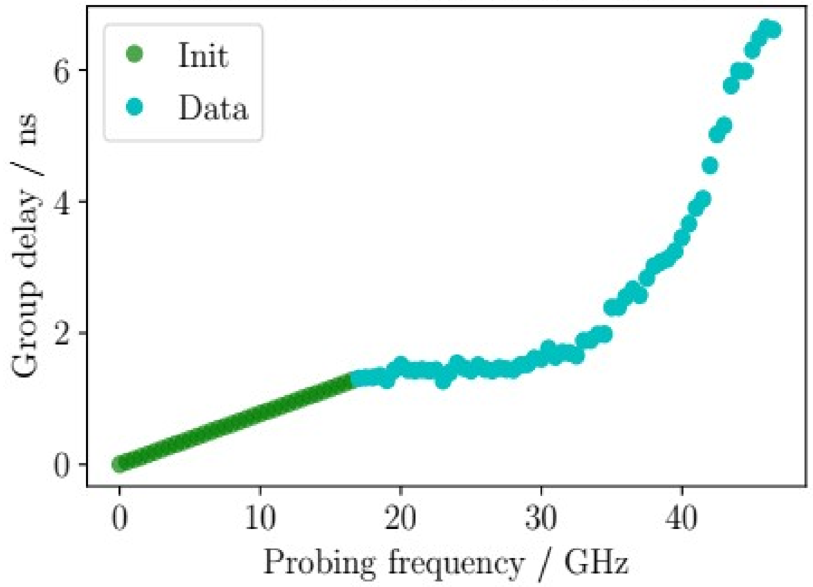
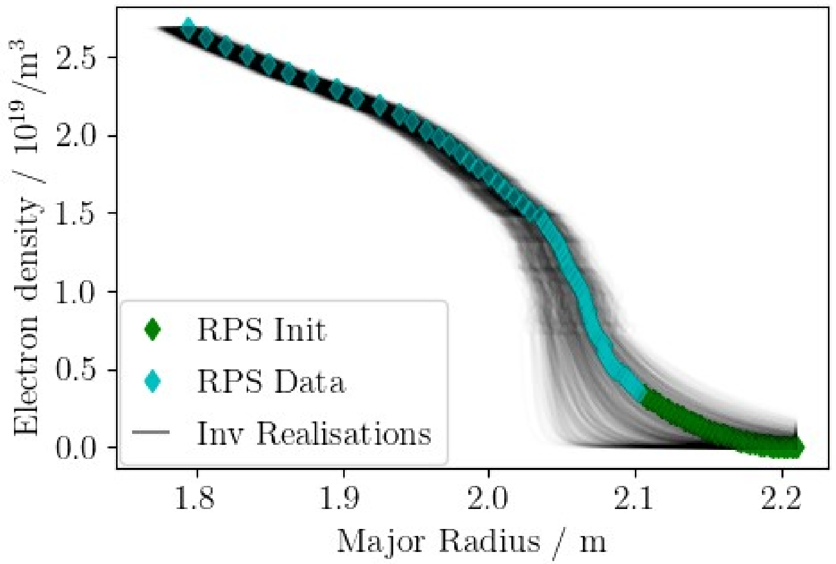
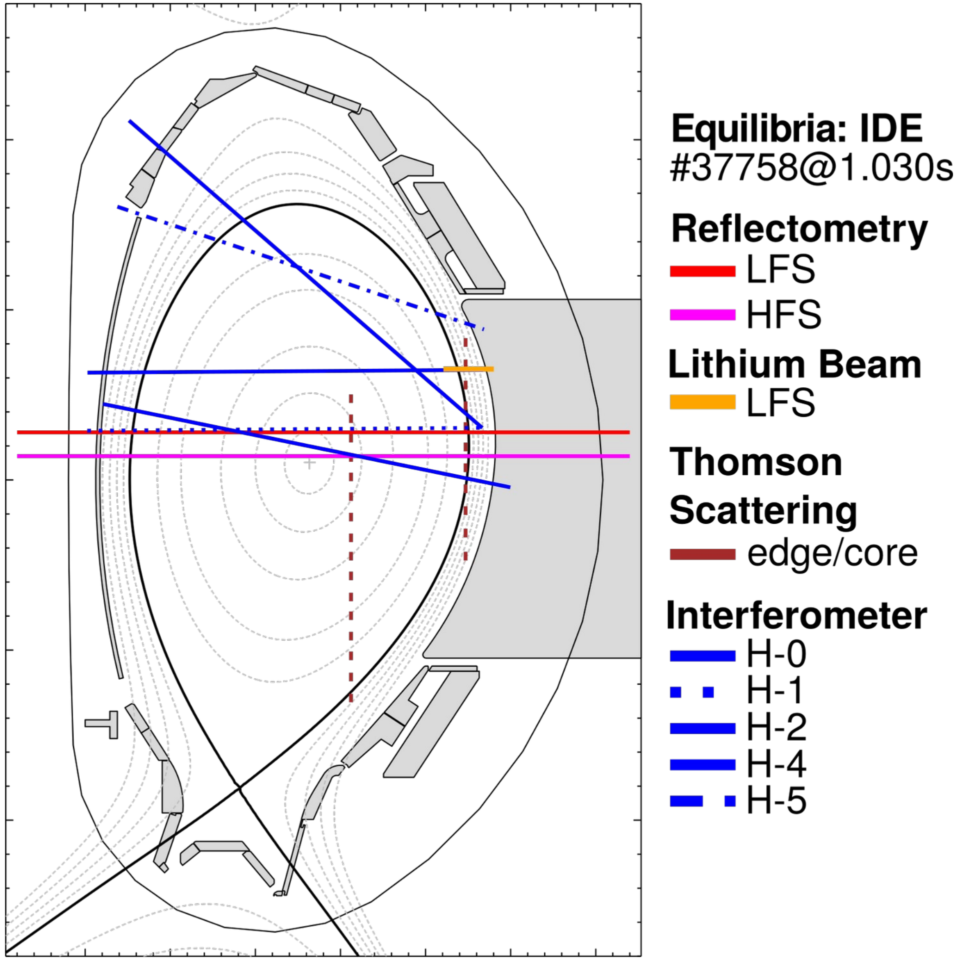




At AUG: Integrated Data Analysis

- Model for $n_e(\rho_{pol})$ an $T_e(\rho_{pol}) \rightarrow$ Spline
- Fit data from many diagnostics
  - TS → n_e, T_e
  - Interferometry → n_e
  - Lithium Beam → $n_e(T_e, Z_{eff})$
  - ECE → $T_e(n_e, B)$
  - BES → $n_e(Z_{eff})$
- Reflectometry forward Model $\tau(n_e)$ **[1]**
  Key: n_e linear between support points

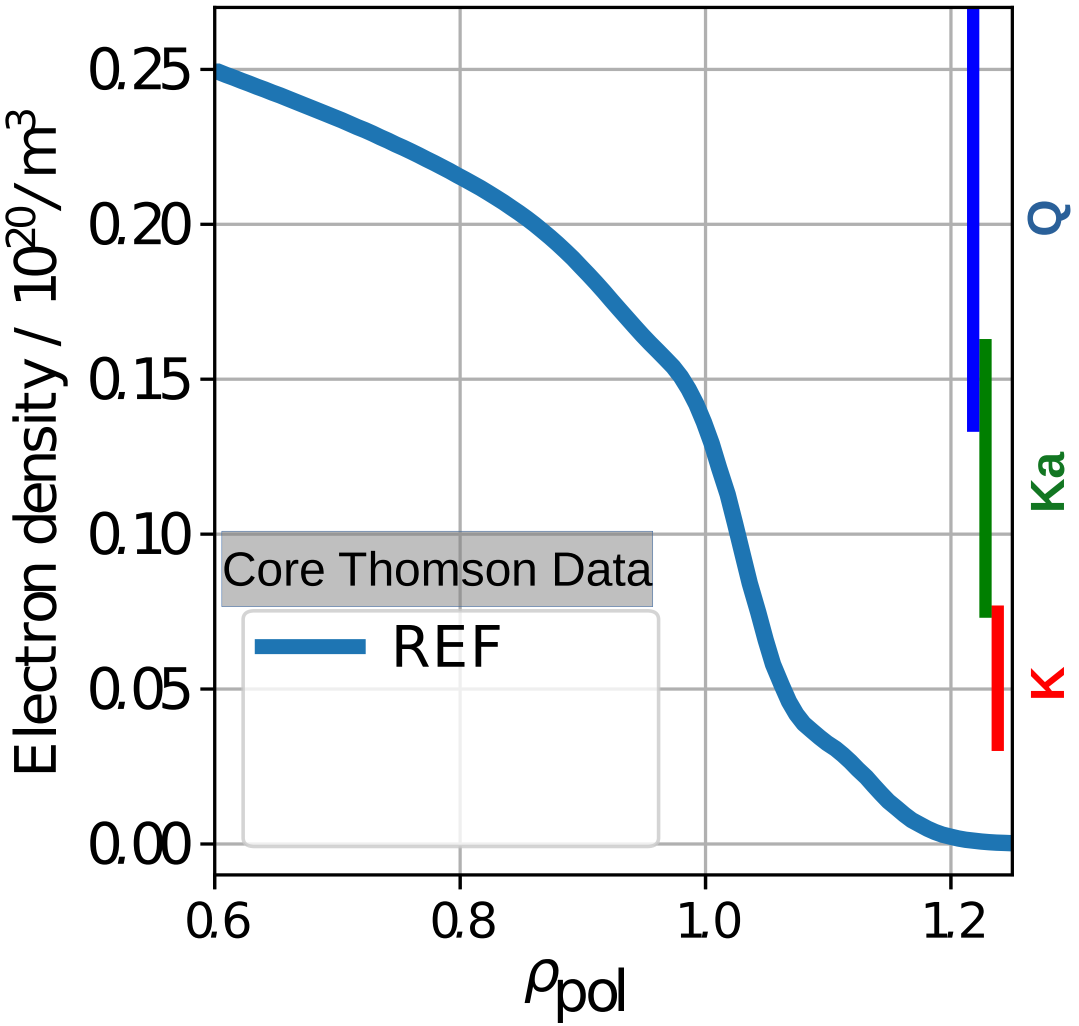


[1] D Stieglitz, J Santos, R Fischer, RSI 2023 “Implementation and validation of swept density reflectometry for integrated data analysis at ASDEX Upgrade“

Motivation for stand-alone analysis vs IDA

Stand alone analysis useful for

- Checking measurements quickly
- Basis for comparison to other diagnostics
- Construct databasis
- Position control of plasma
  → Presented results relevant



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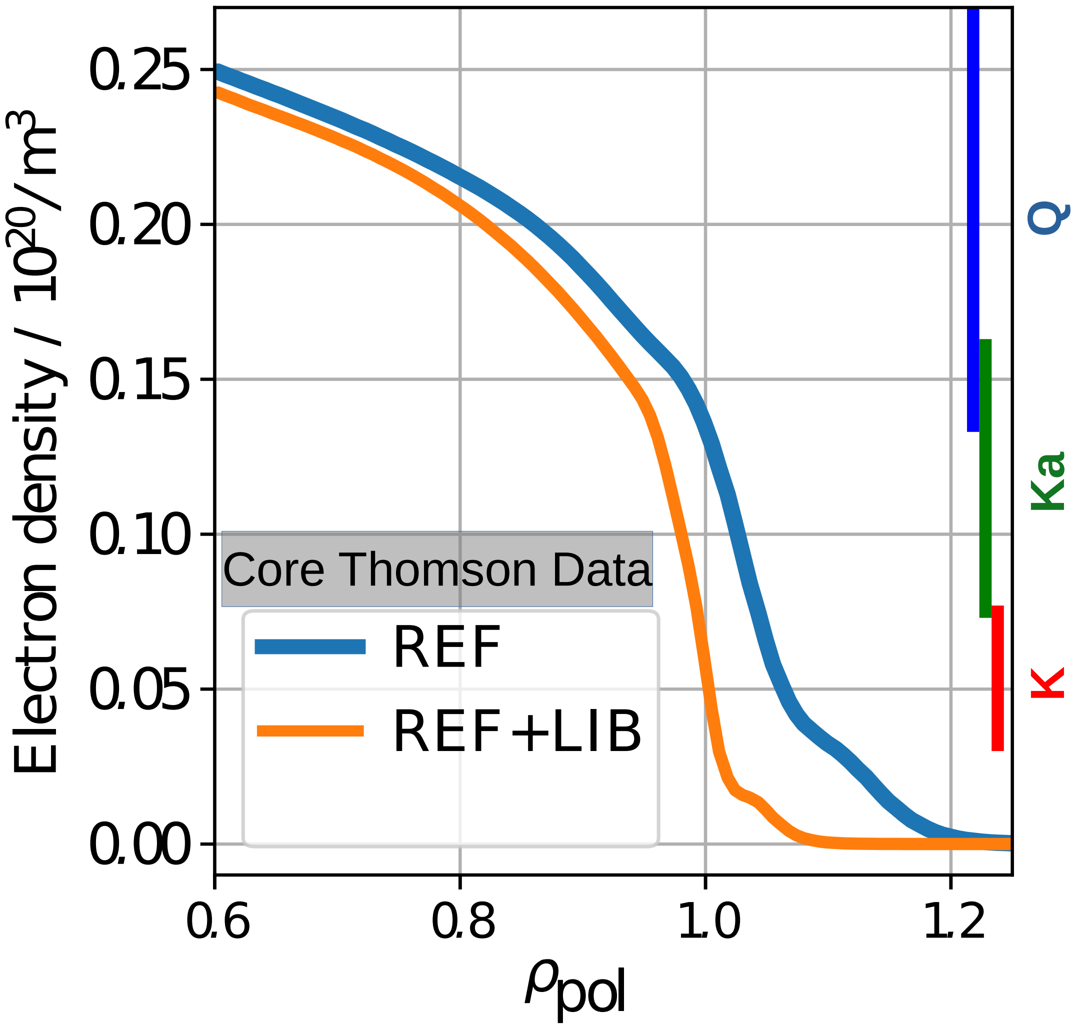
- Integrated Data Analysis (IDA)
  → combine with other diagnostics, e.g. Lithium Beam



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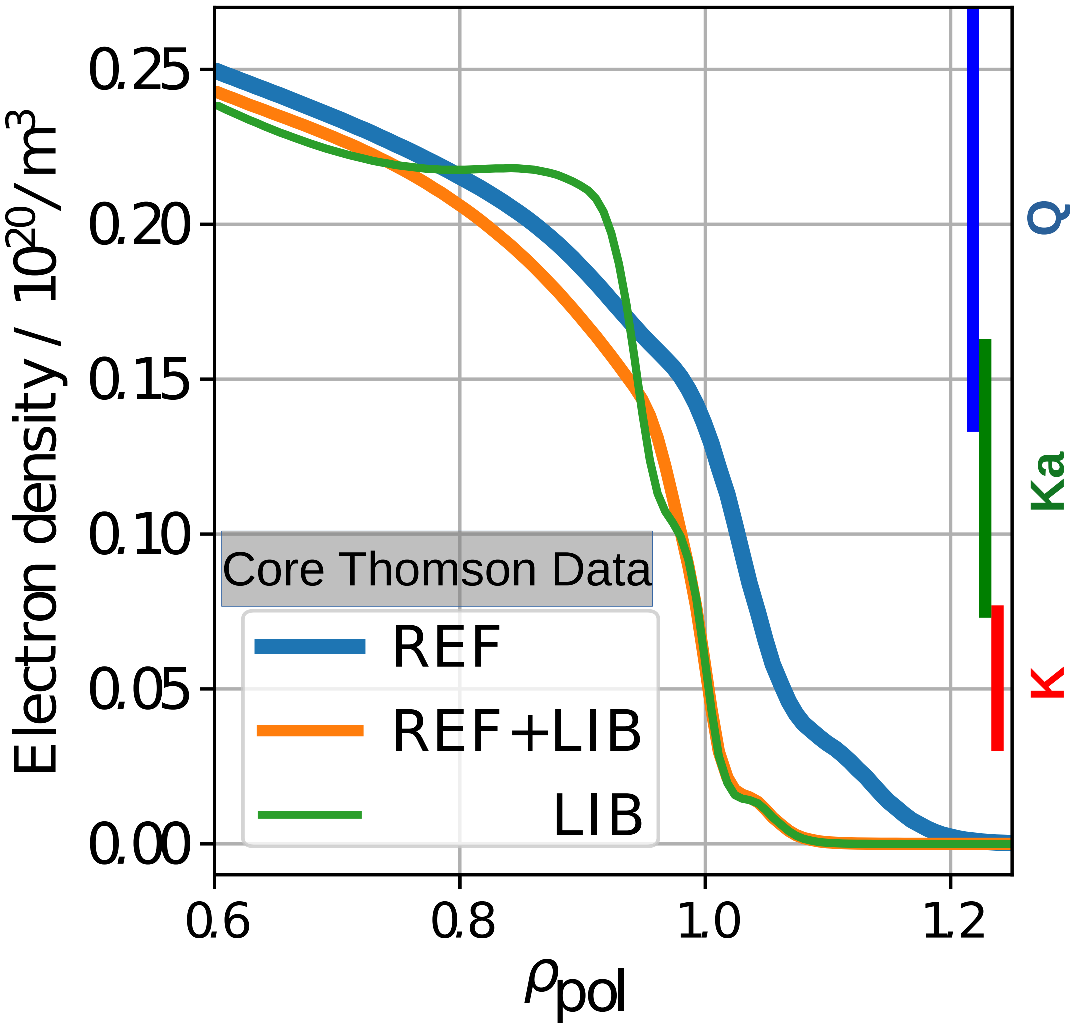


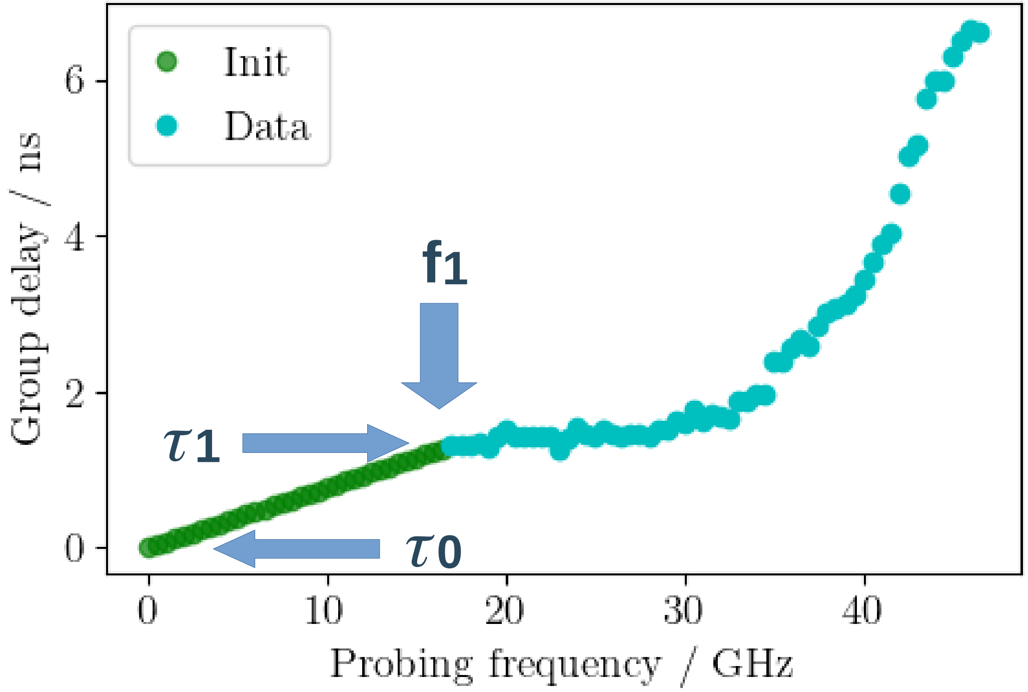
Overview: the individual parts

- How to initialise
  - Thinking “forward”: density to measurement
  - Linear group delay vs exponential density
  - Fitting first few data points & priors
- Examples and comparison of initialisation
- Discussion of benefits & drawbacks

Initialisation: defining parameters

1) Group Delays

- τ_1 & f_1 first measured values
- τ_0 sets initial Radius R_0



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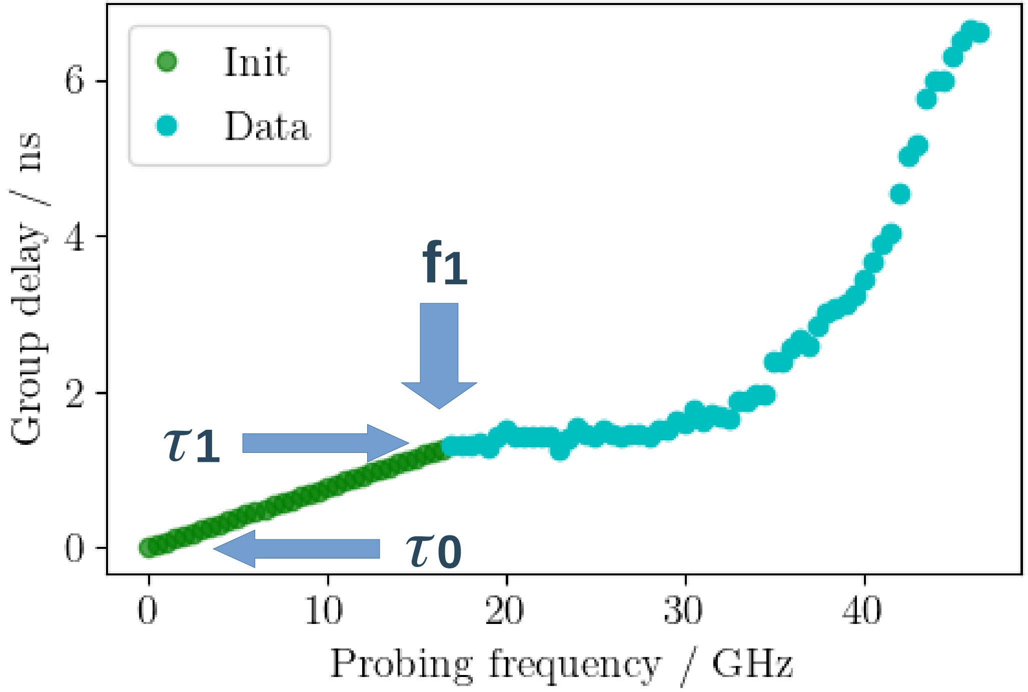
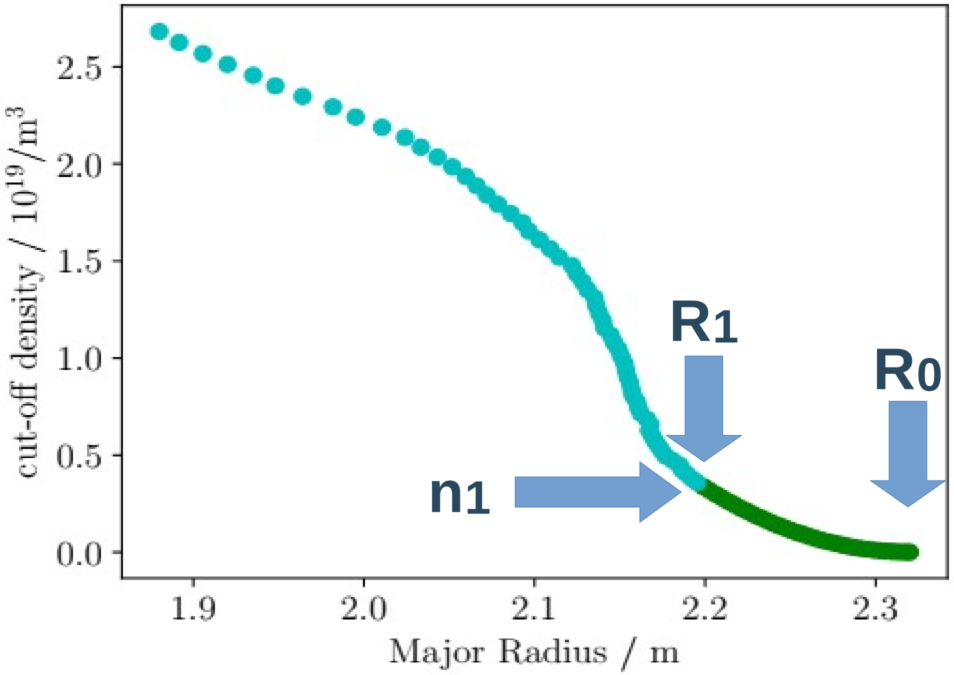


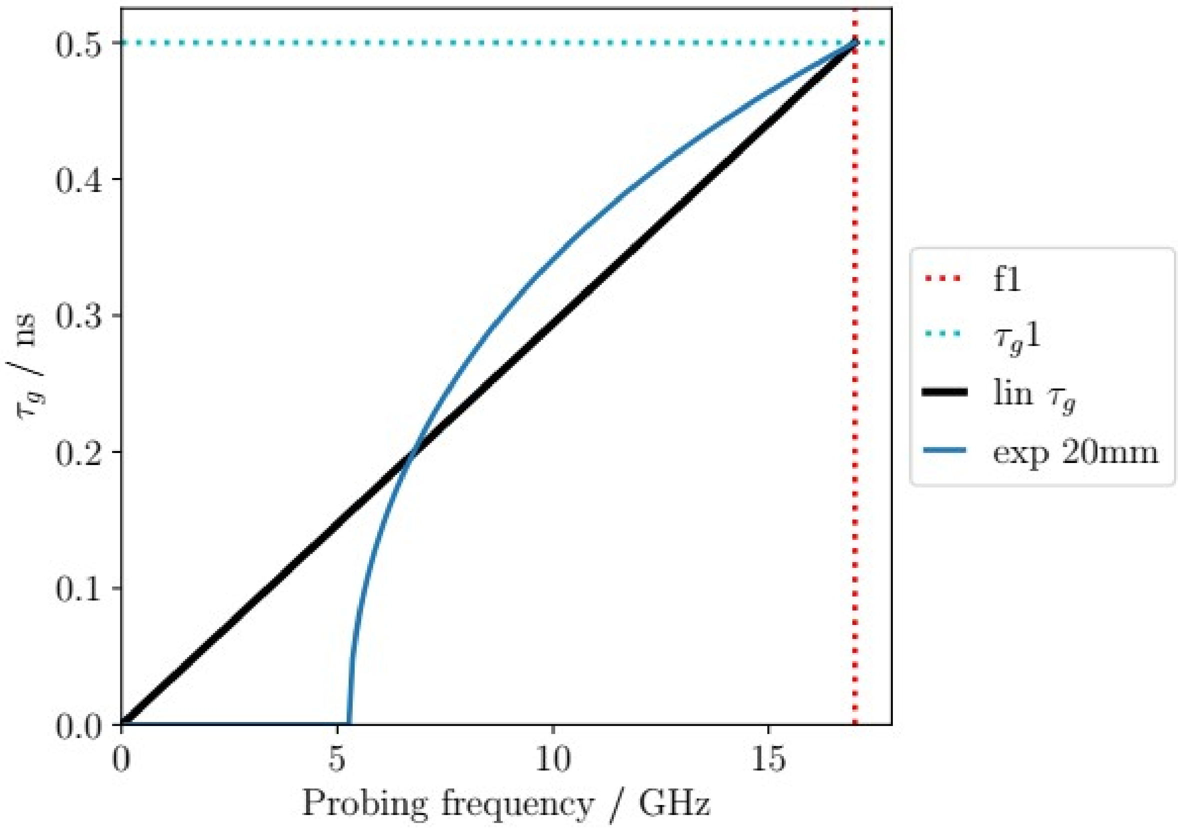
2) Density profile

- n_1 first observed cut-off
- $R_1(\tau_{init})$



Initialisation: Variability

Initialisation Guess



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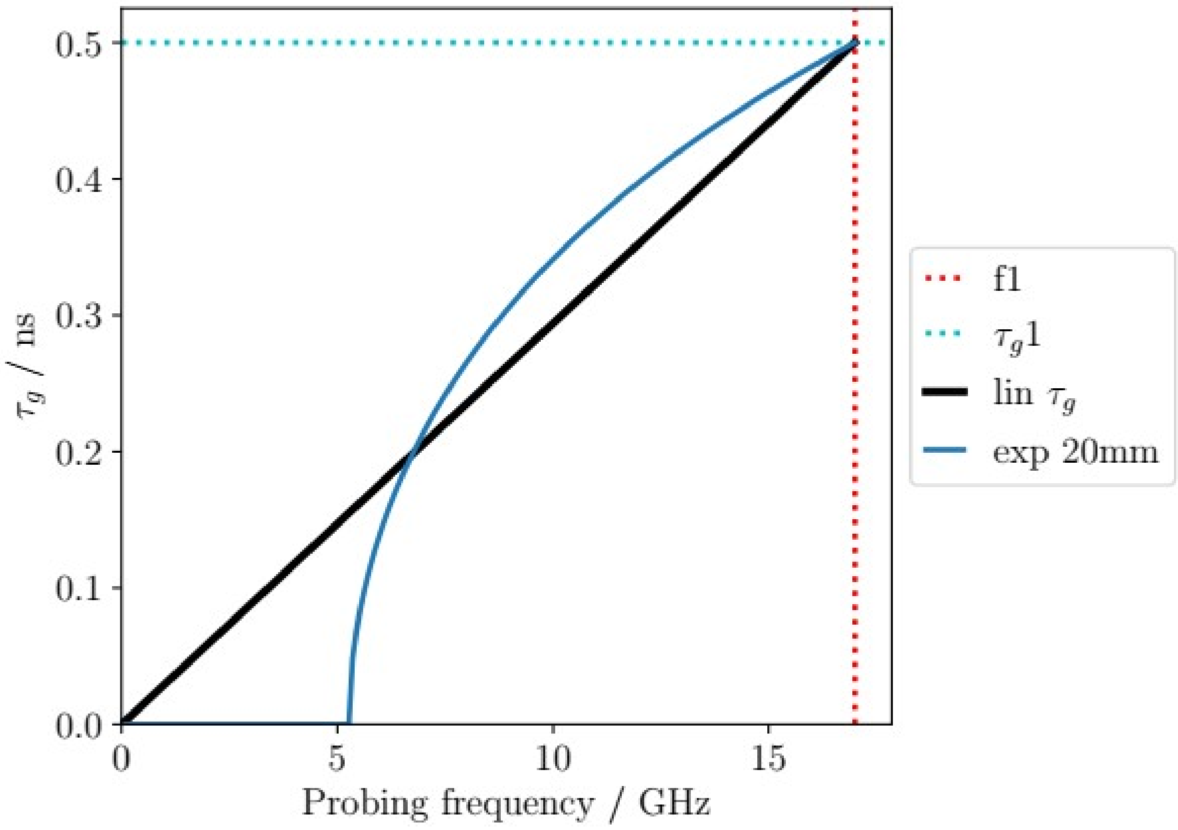
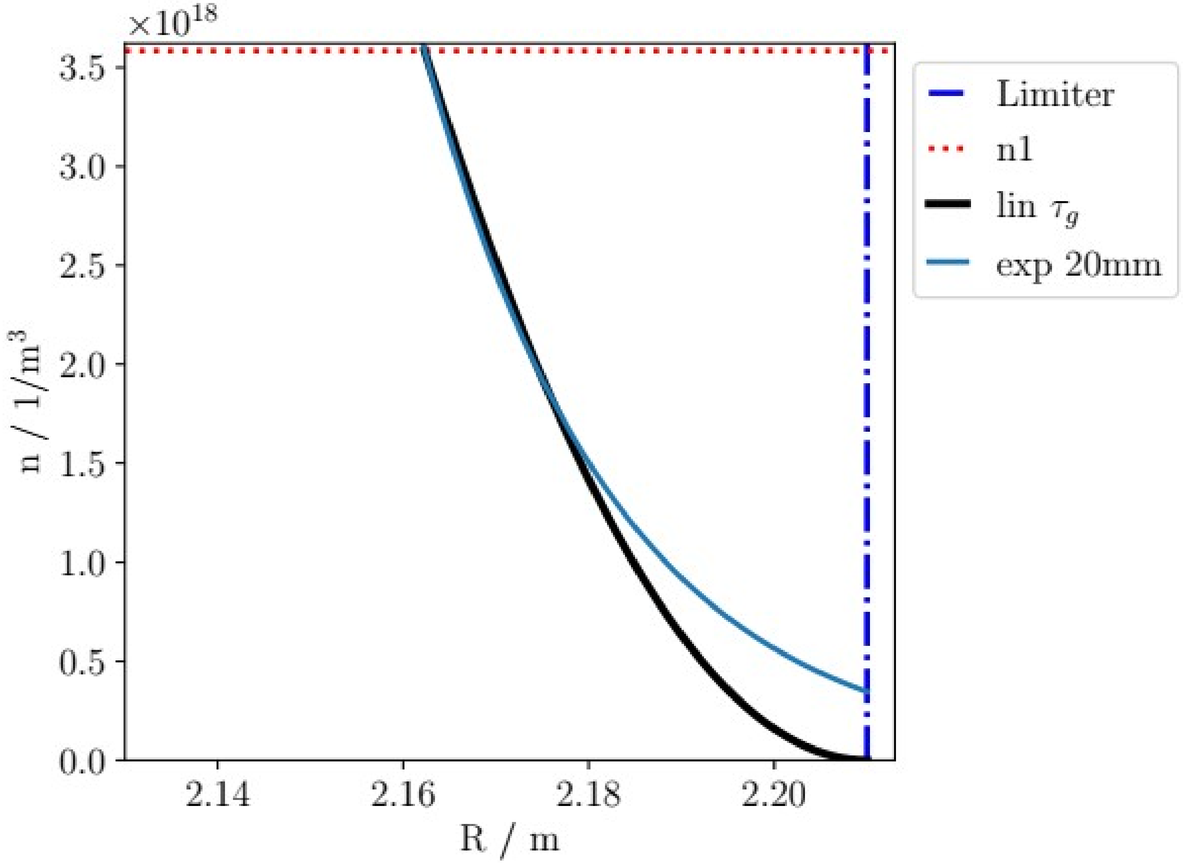


Density Profile



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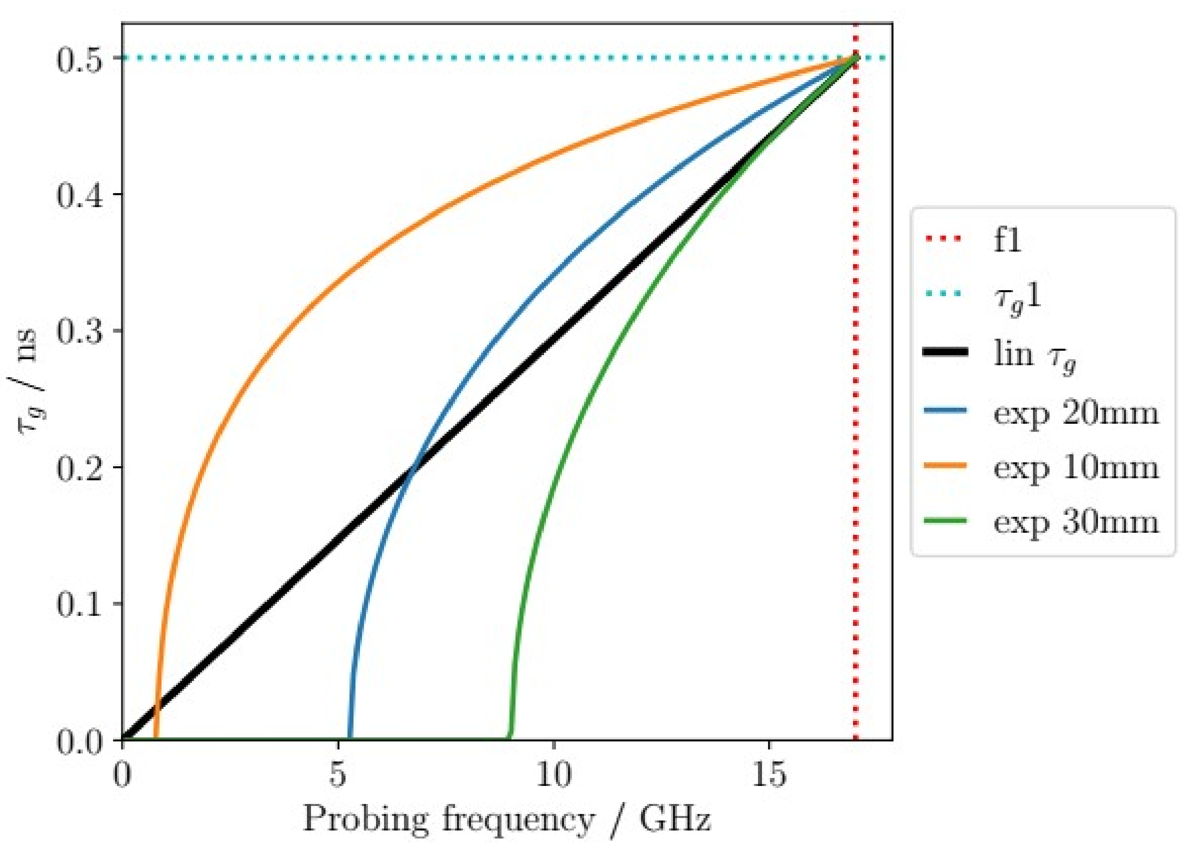

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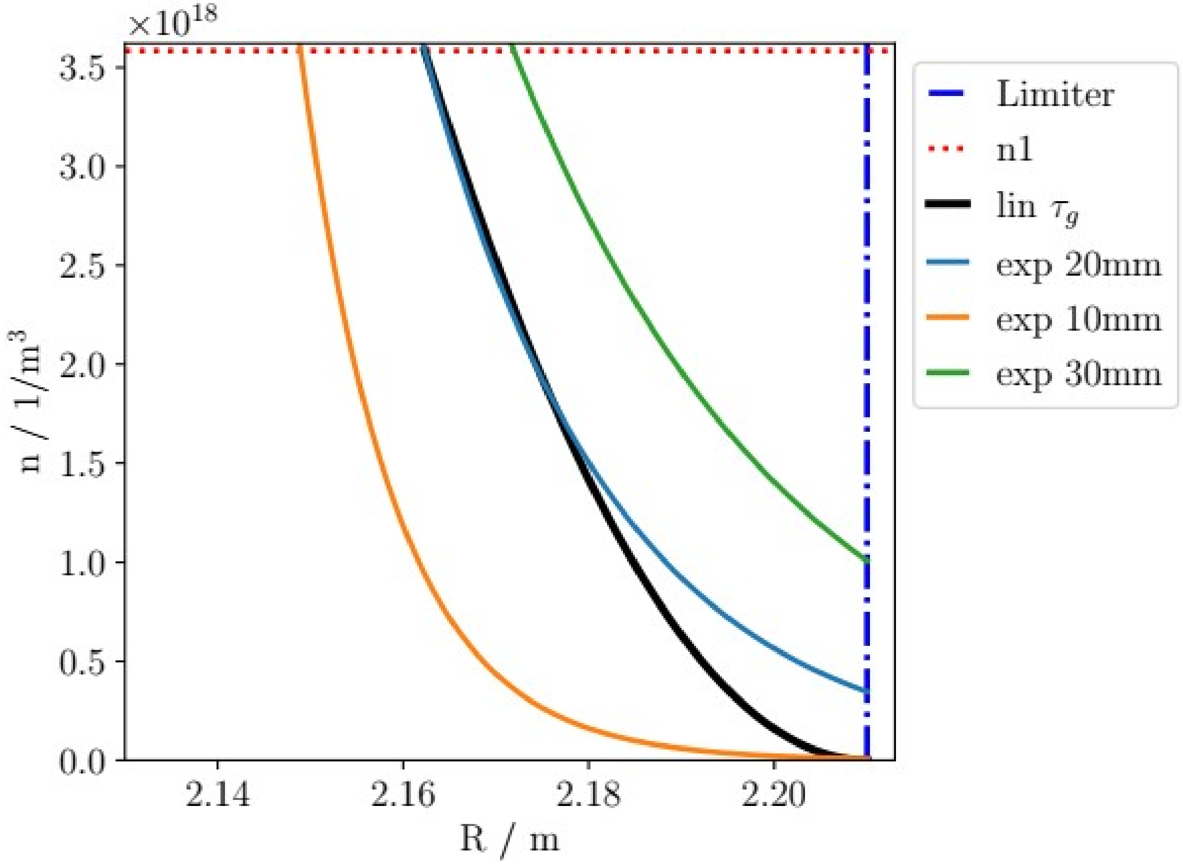

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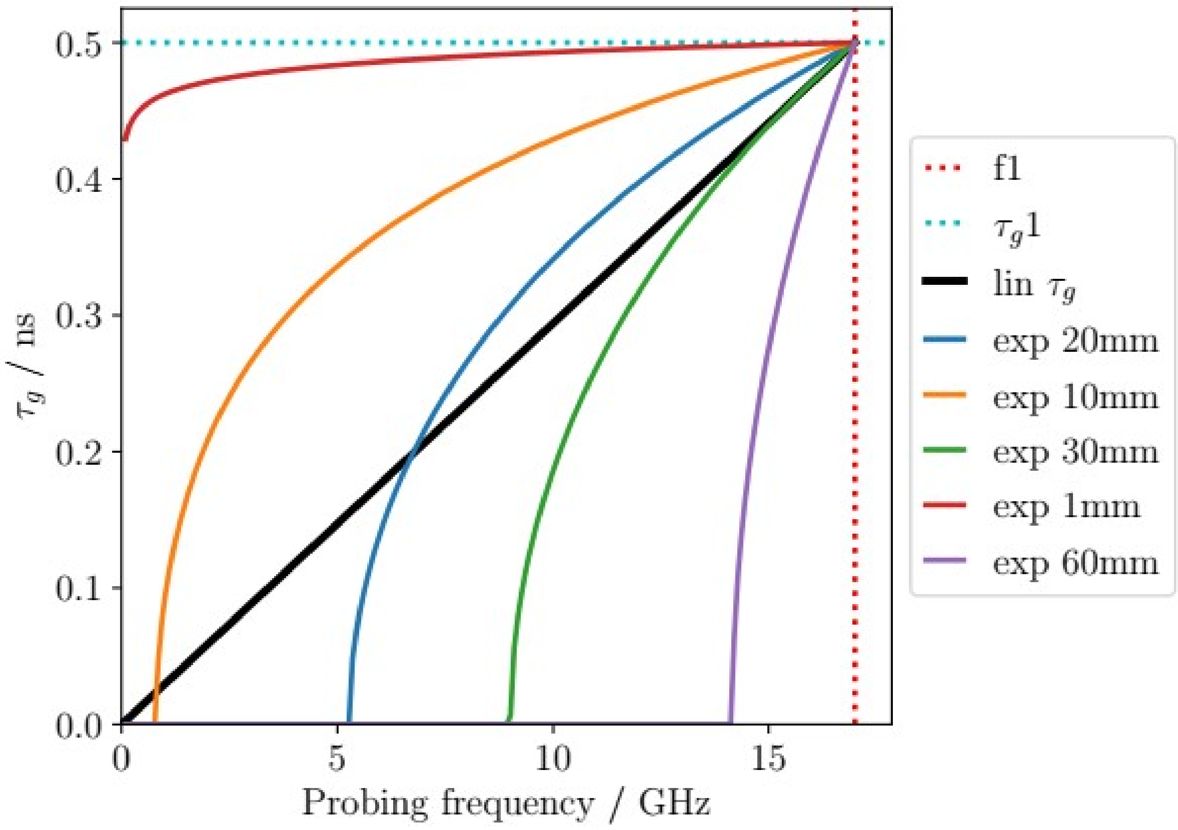

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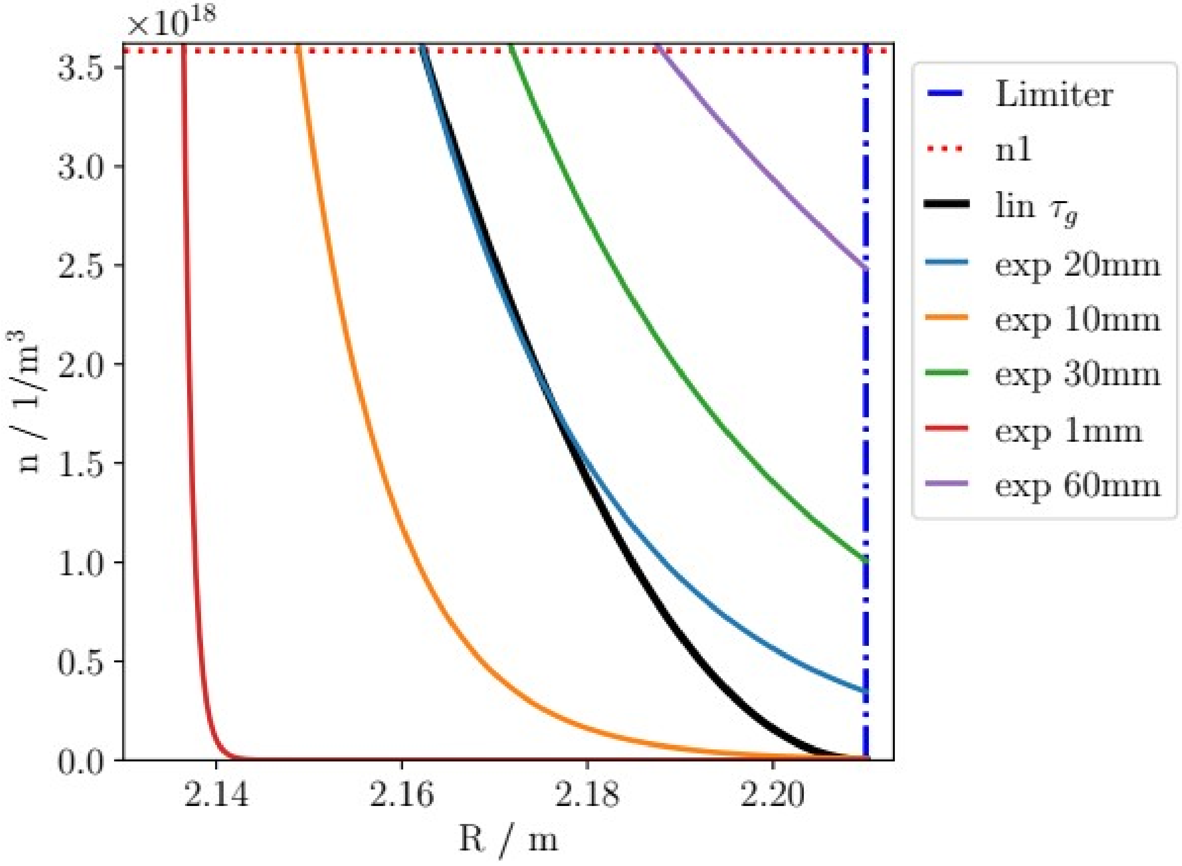
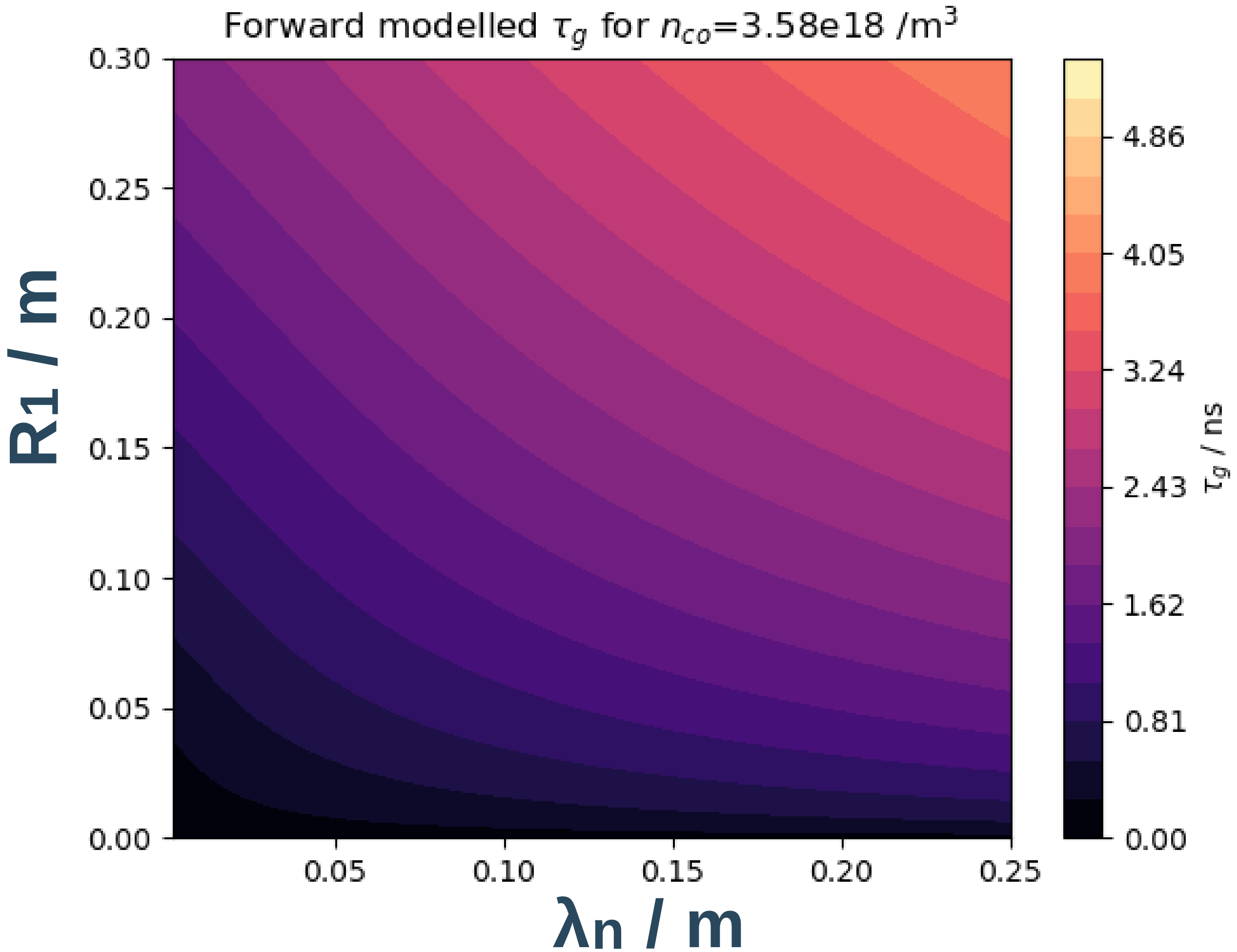


Initialisation: exponential density

Step 1: Define parameter space and compute expected data



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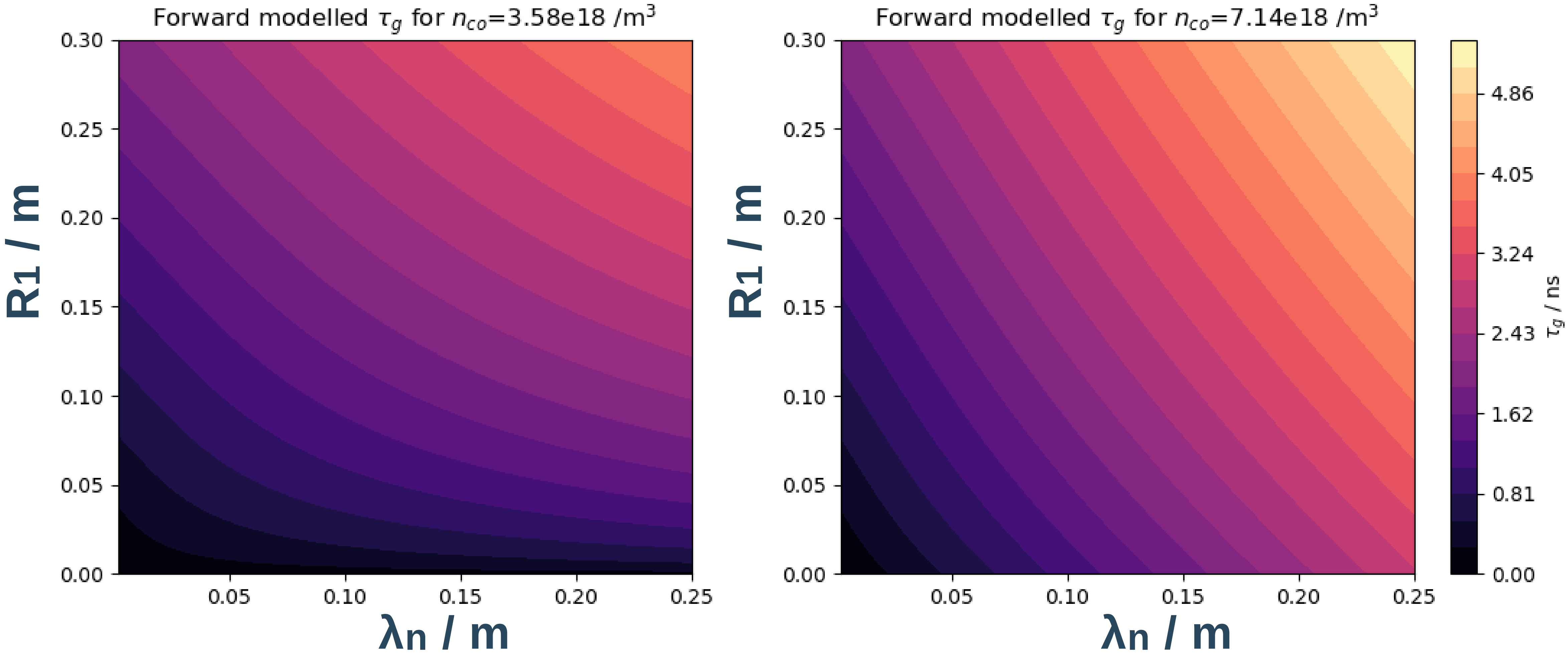
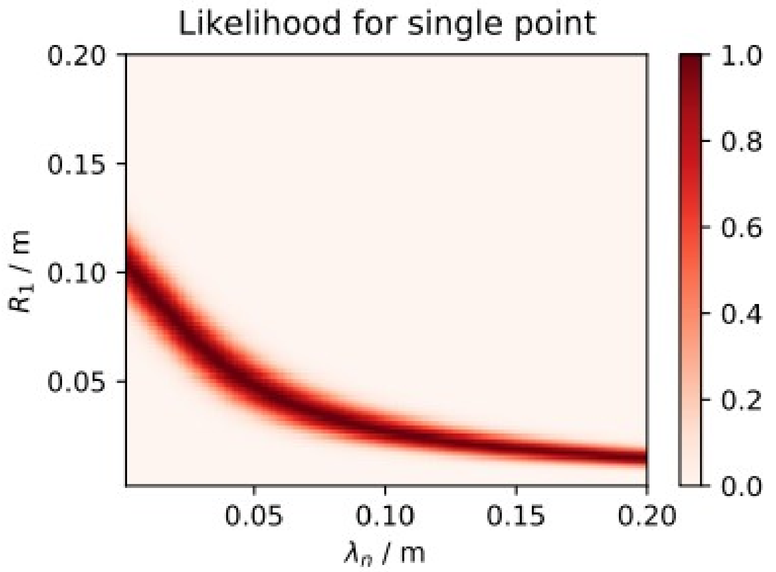


Initialisation: exponential density

1 point



Mathematically

Underdetermined

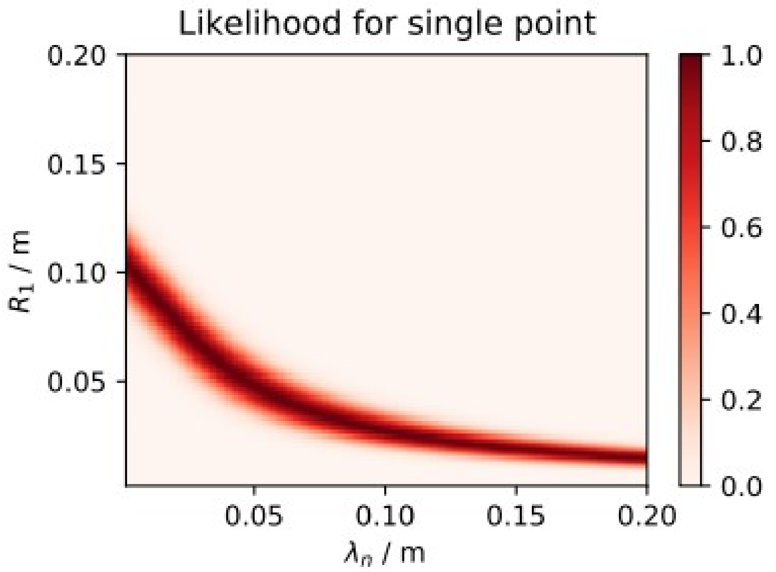
Any λ_n possible

Wide range of R_1

Initialisation: exponential density

$\Delta f = 0.5\text{GHz}$
$\Delta n = 2\text{e}17/\text{m}^3$

1 point



Mathematically
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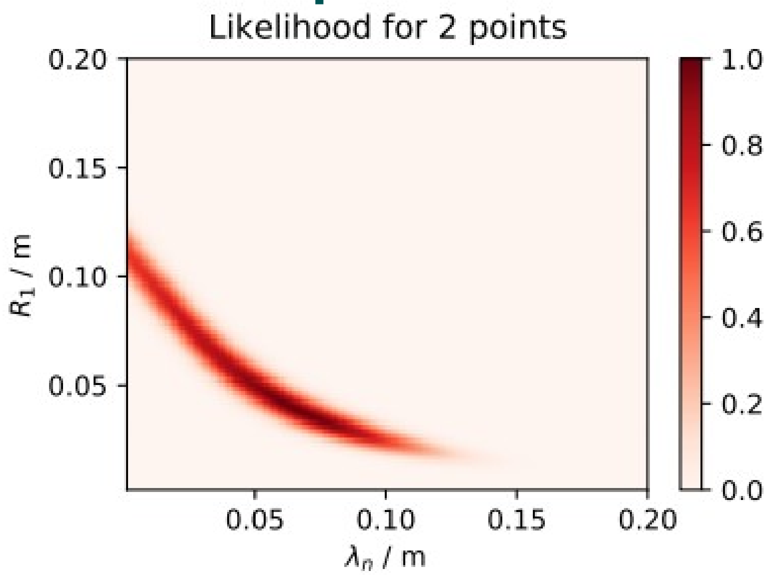
Any λ_n possible

Wide range of R_1

2 points



Very uncertain

Upper limit to λ_n

Lower limit to R_1

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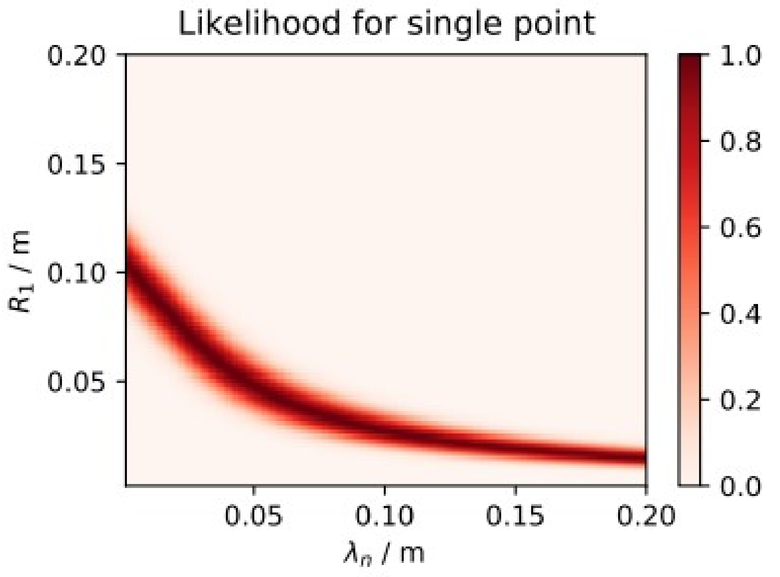

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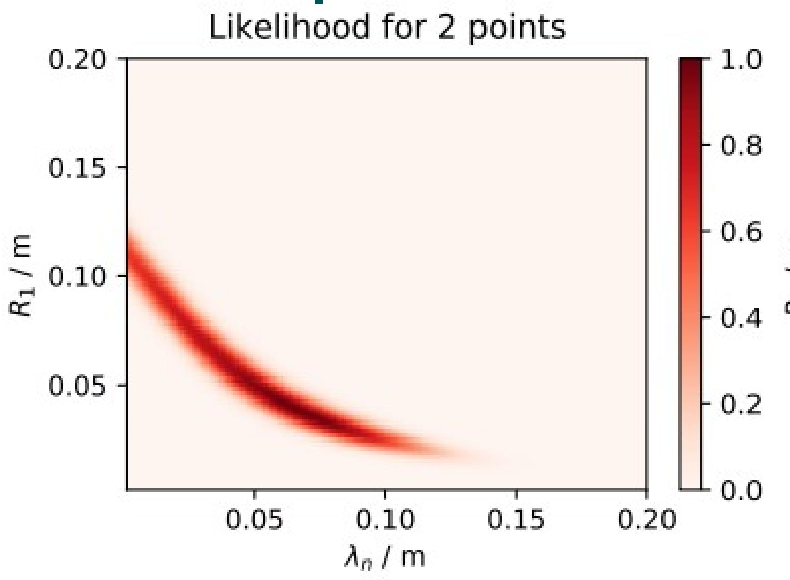
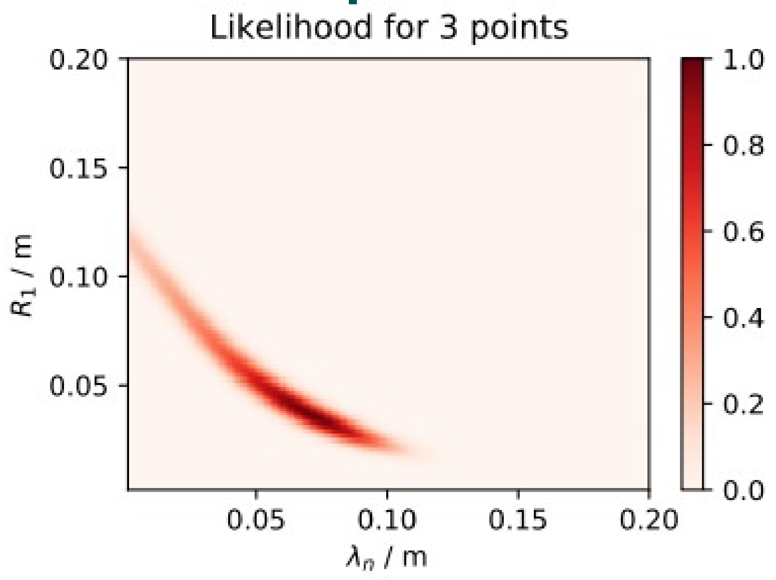


Very uncertain

Upper limit to λ_n

Lower limit to R_1

3 points



Less uncertain

Narrower range for λ_n

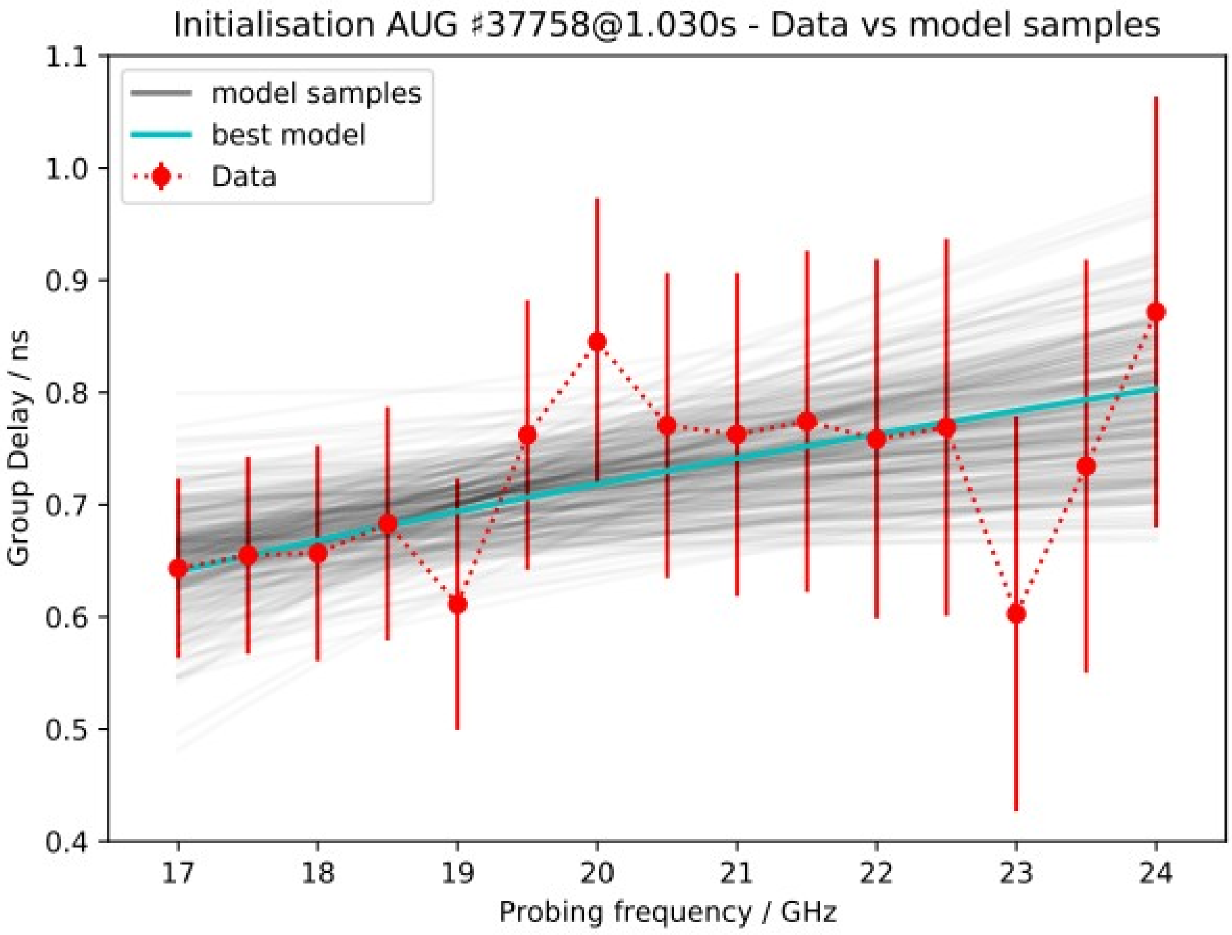
Narrower range of R_1

Initialisation in practice

Example from AUG

- 15 frequencies
- Cauchy likelihood (outlier resistant)
- Uncertainty increases with frequency

  → n ~ exp(x) is assumption

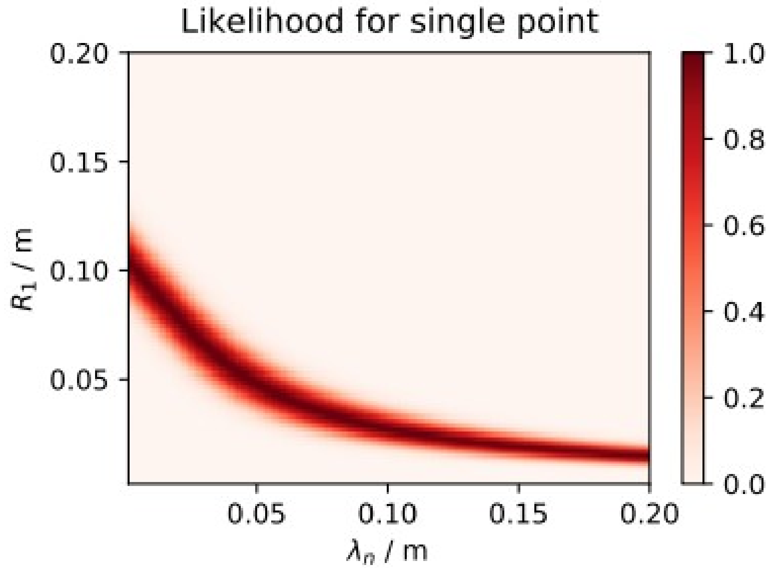


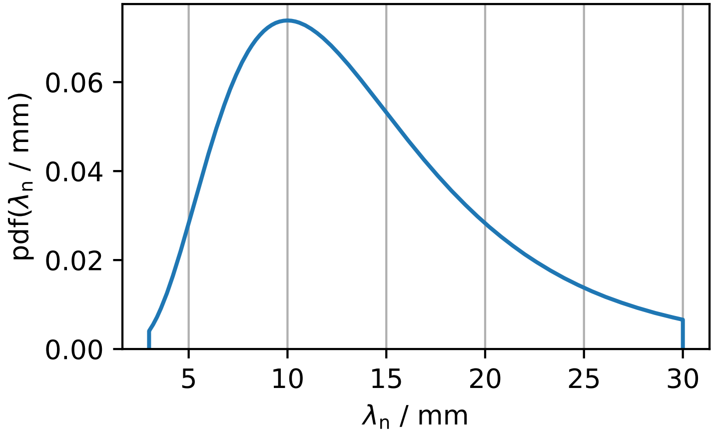
Comment on priors

- λ_n affects density strongly
- Log-normal for λ_n → log(λ_n) normally distributed
  e.g. peak at 10mm, same probability at 5 & 20mm
- For reliable estimates with little bias → strict limits
- Ideal source for prior → Actual data

At AUG λ_n ~ (5 to 20)mm
H-Mode ~ (5:10) mm
L-Mode ~ (10:20) mm

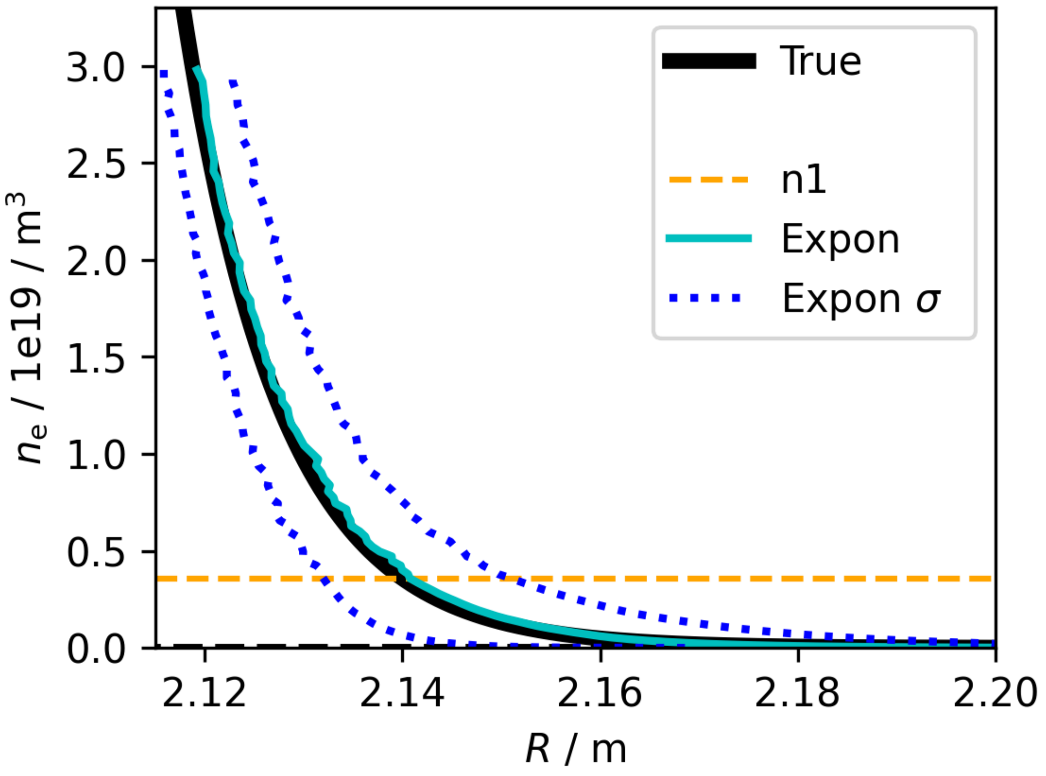
H J Sun PPCF 2020,
DOI 10.1088/1361-6587/ab5259





Synthetic examples: 1) exponential basis

Both exponential: trivial



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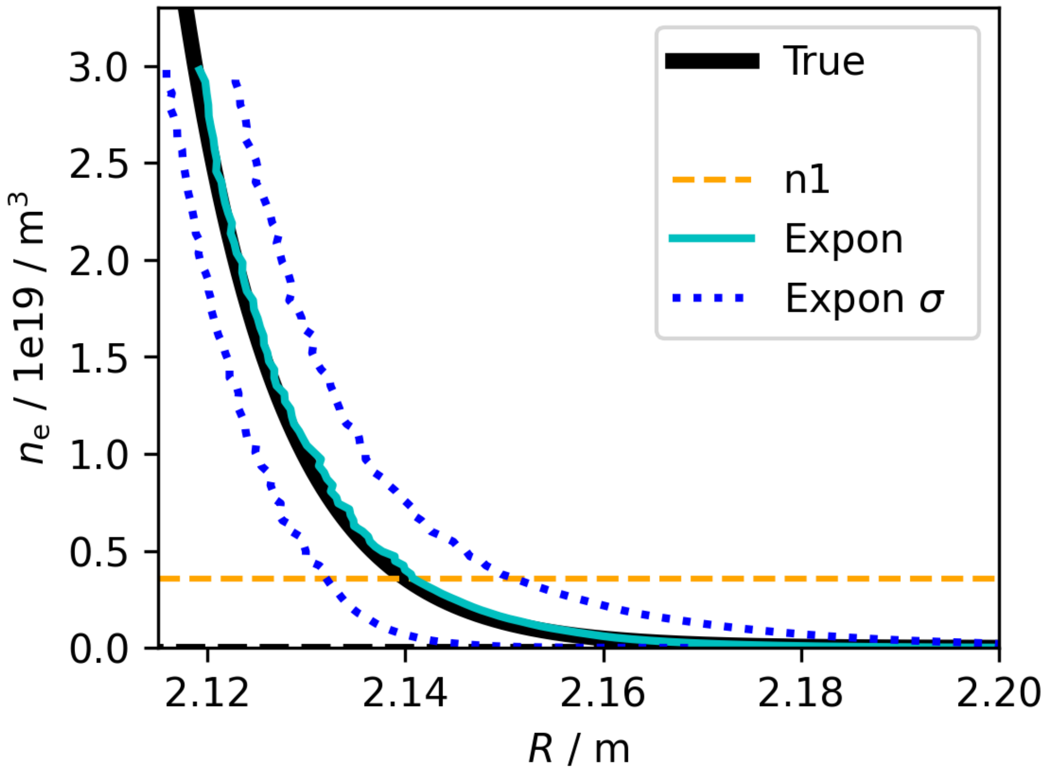
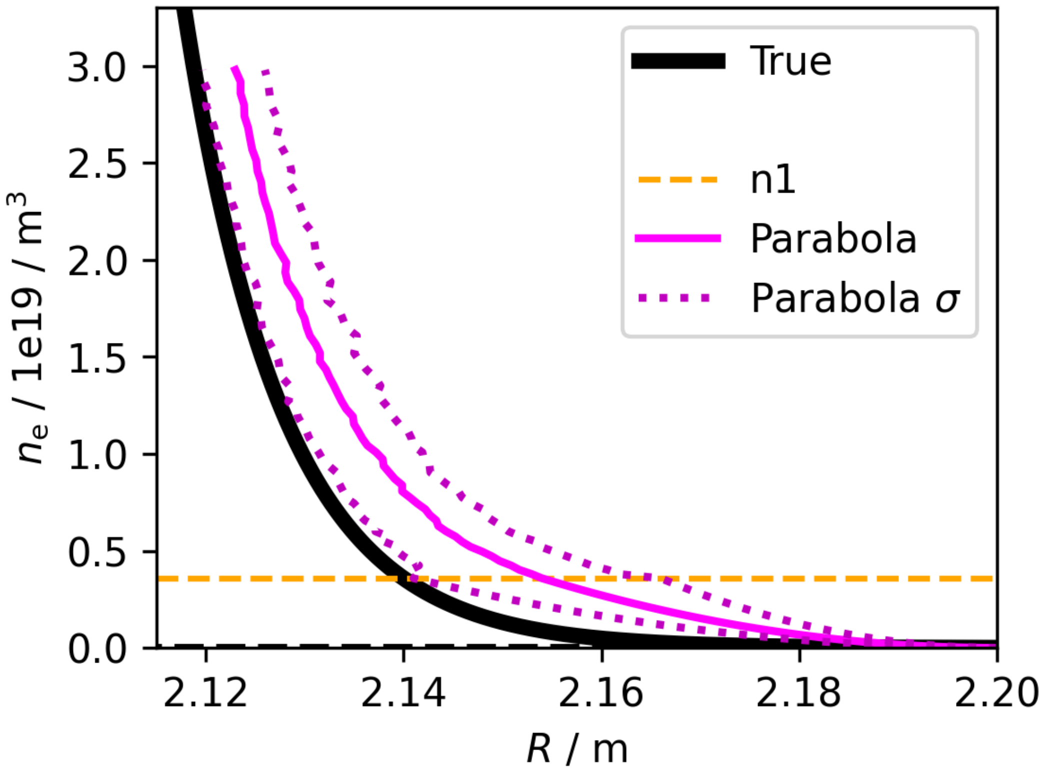


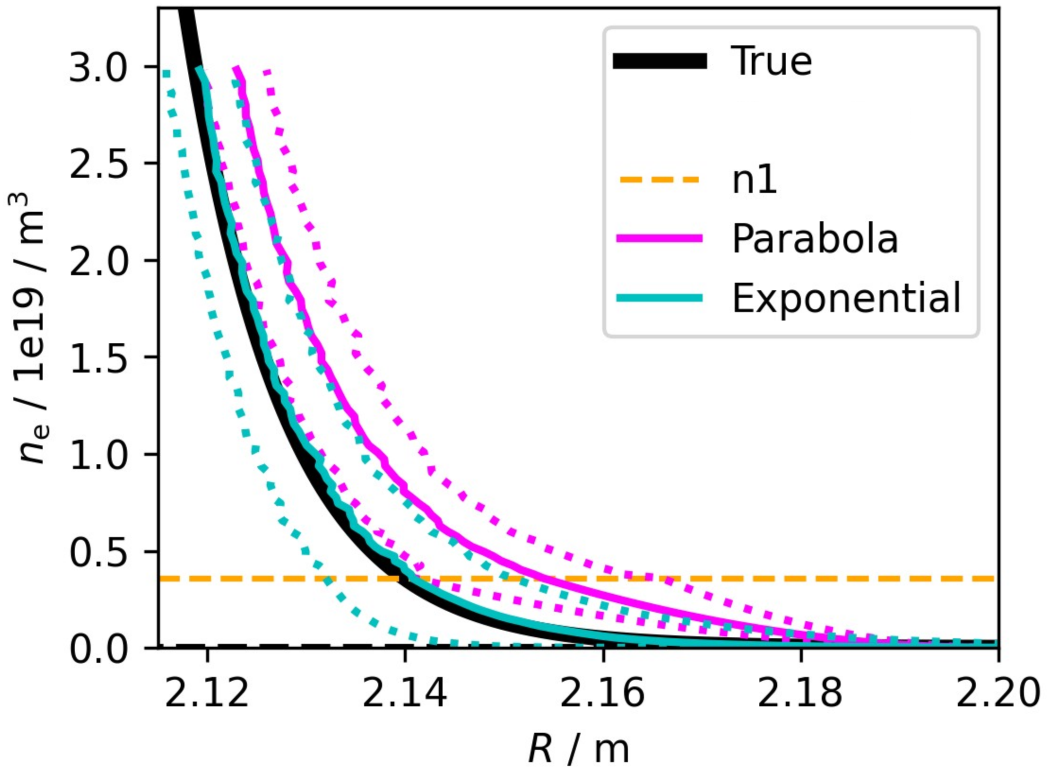
Parabolic model is biased



Synthetic examples: 2) exponential vs parabola

Exponential Basis

→ Parabola model biased



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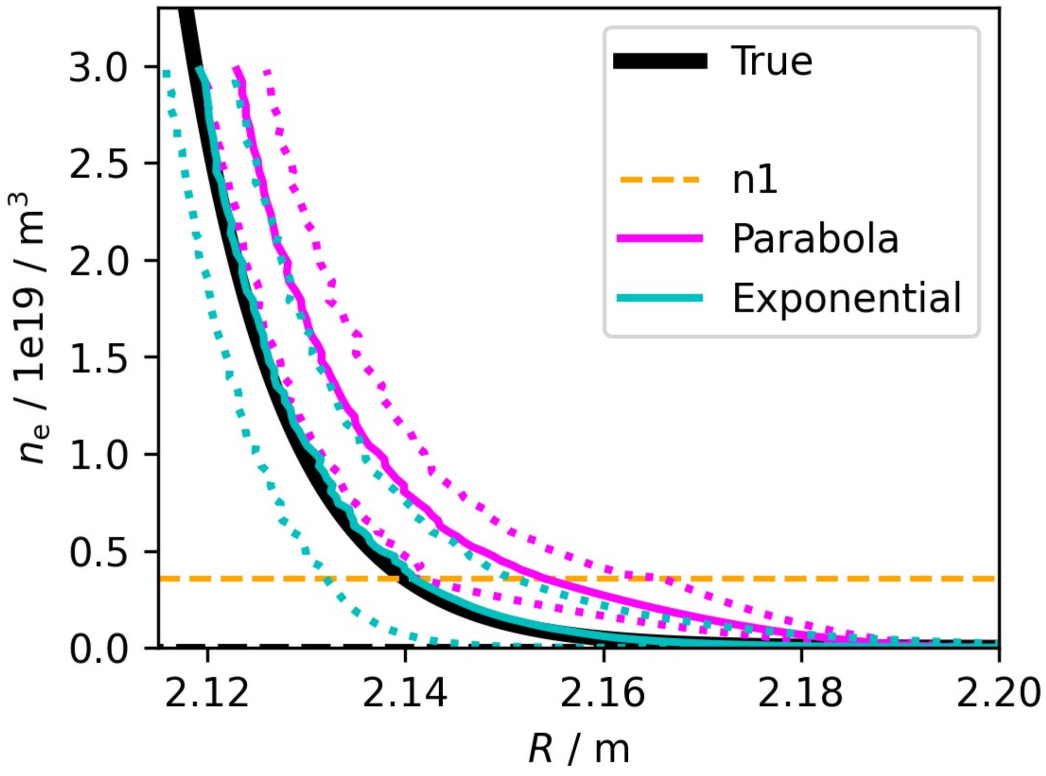

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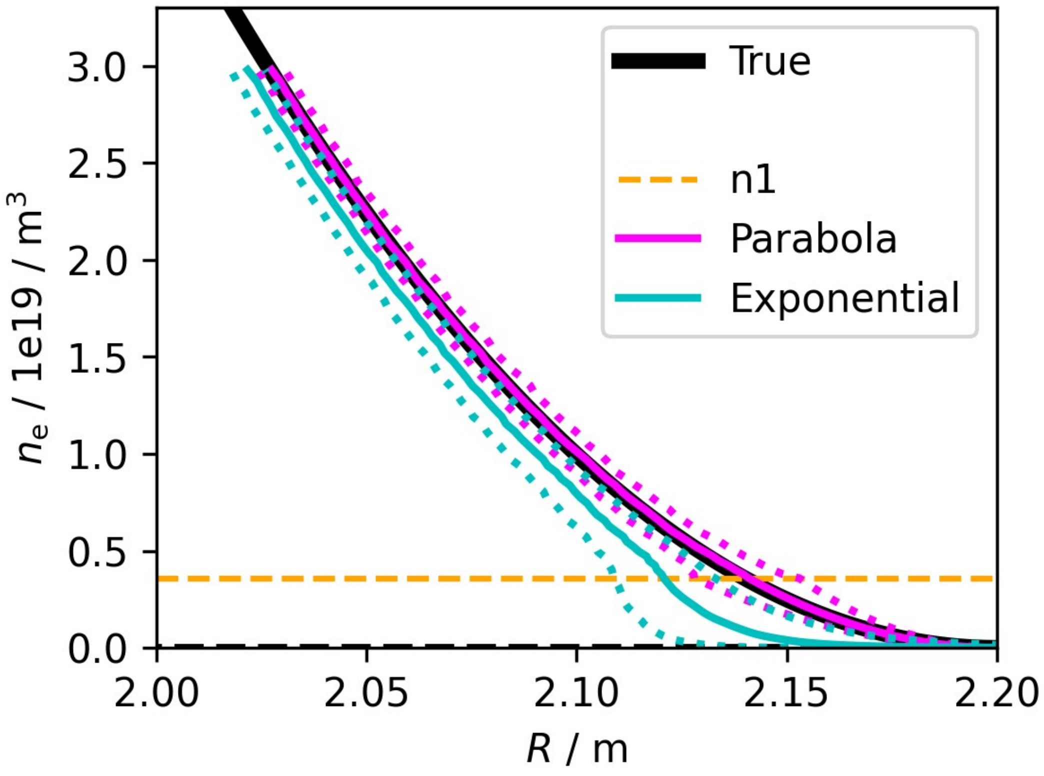
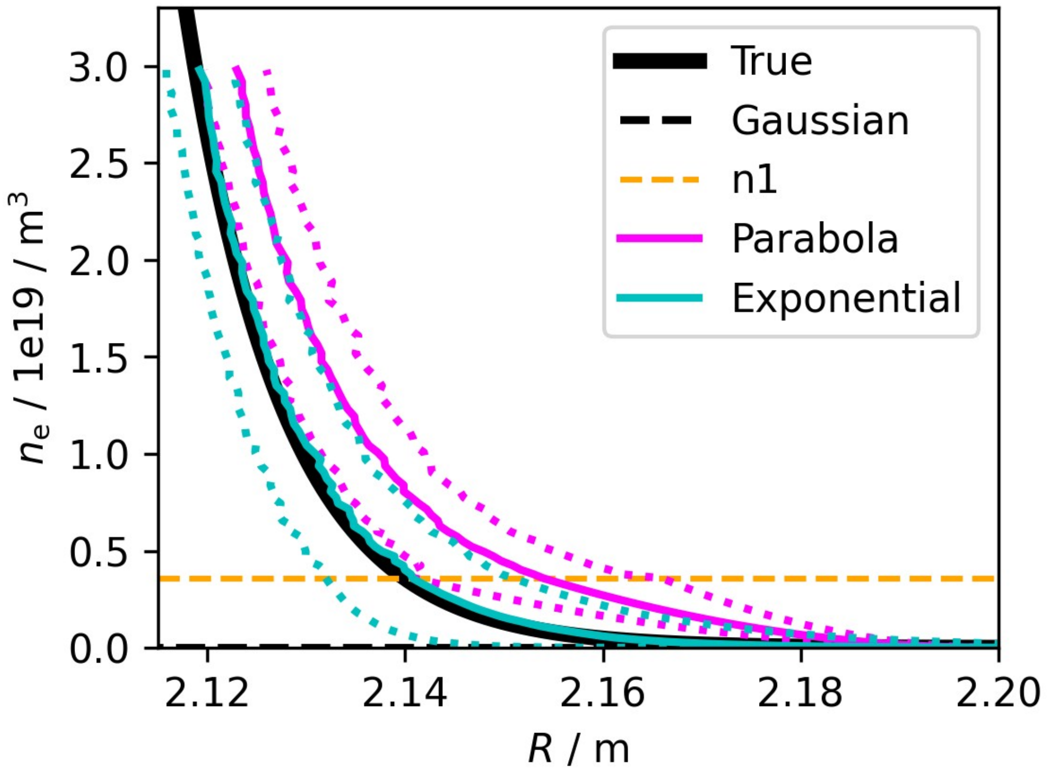
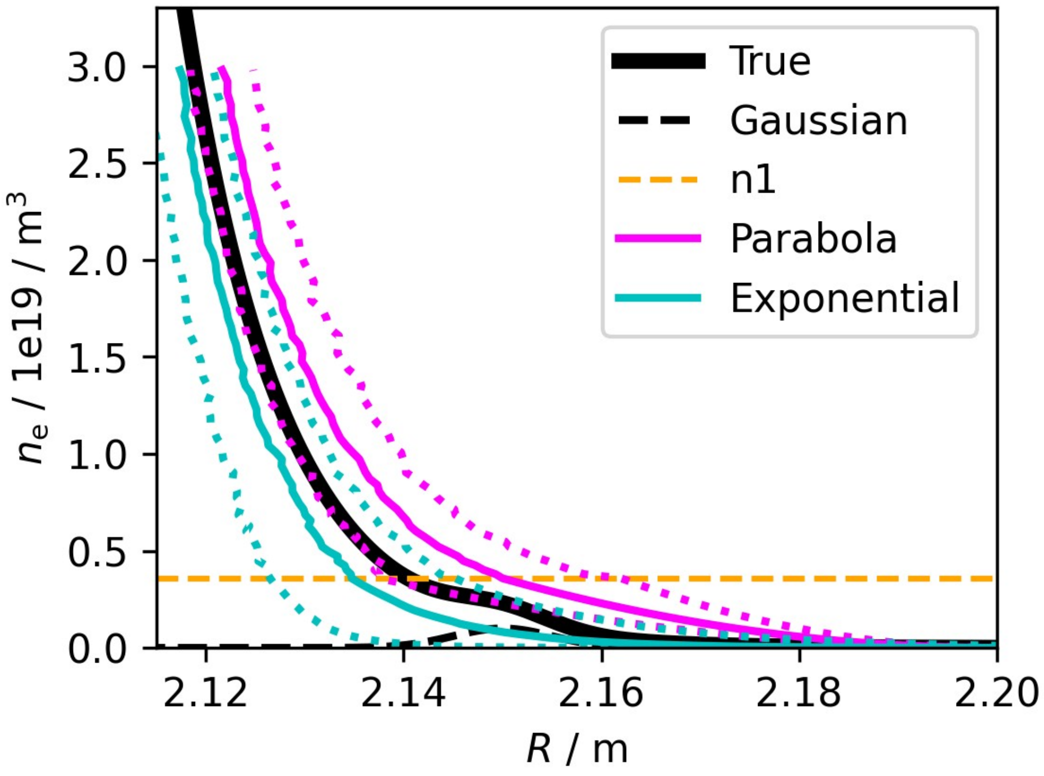


Synthetic examples: 2) exponential with shoulder

Exponential Basis ...



... with Gaussian „shoulder“



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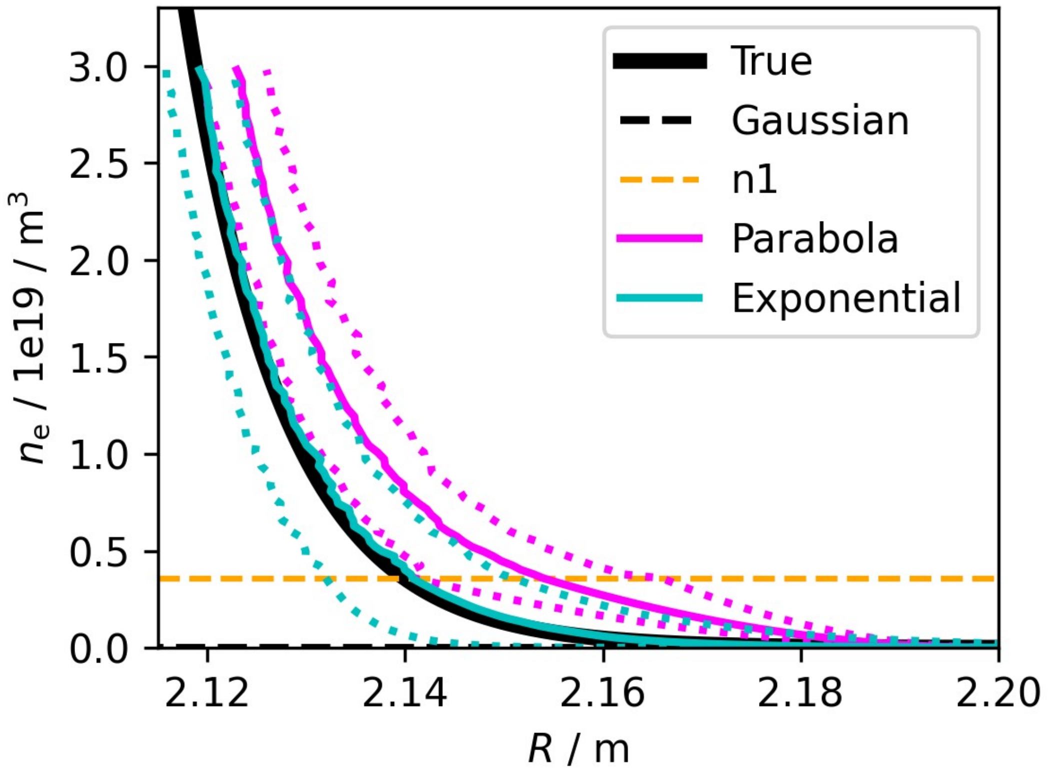

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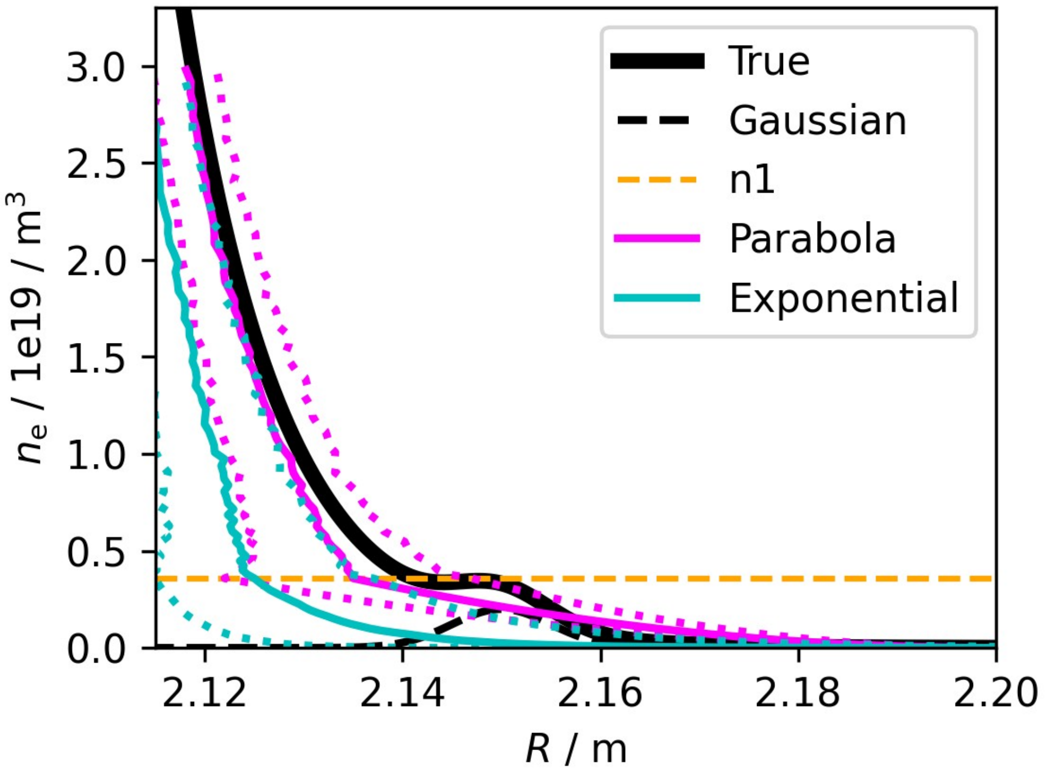


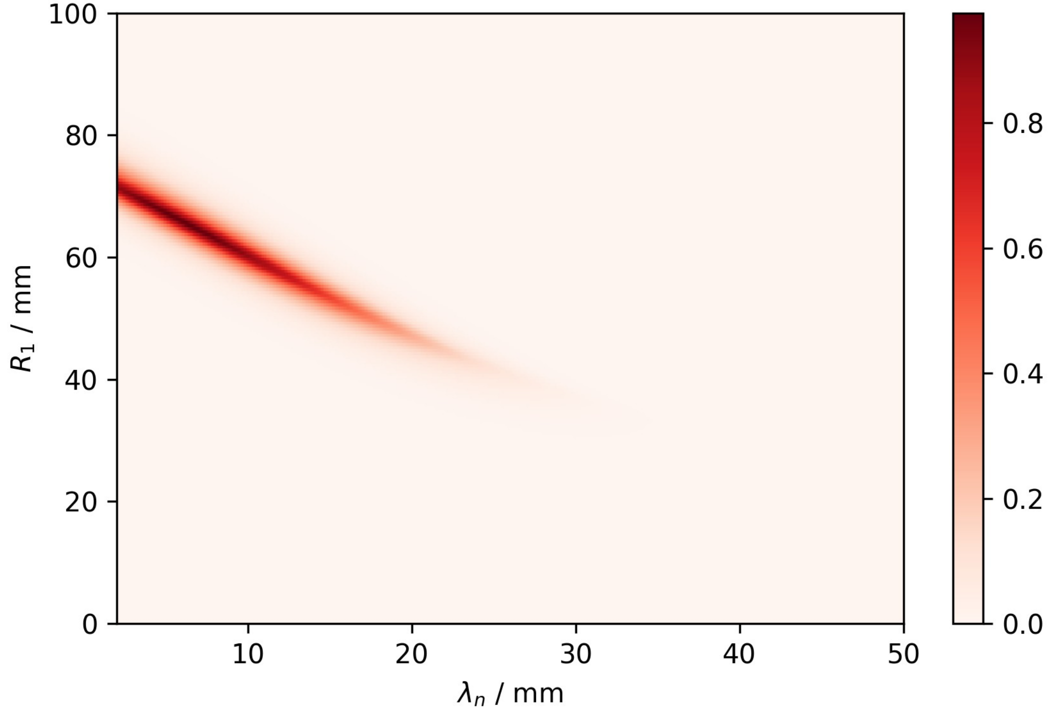
Wrong guess → wrong result

Synthetic examples: 3) Fit quality

Fit quality seen in discrepancy model vs data → likelihood

Alternative: calculate Evidence to compare models

Good fit



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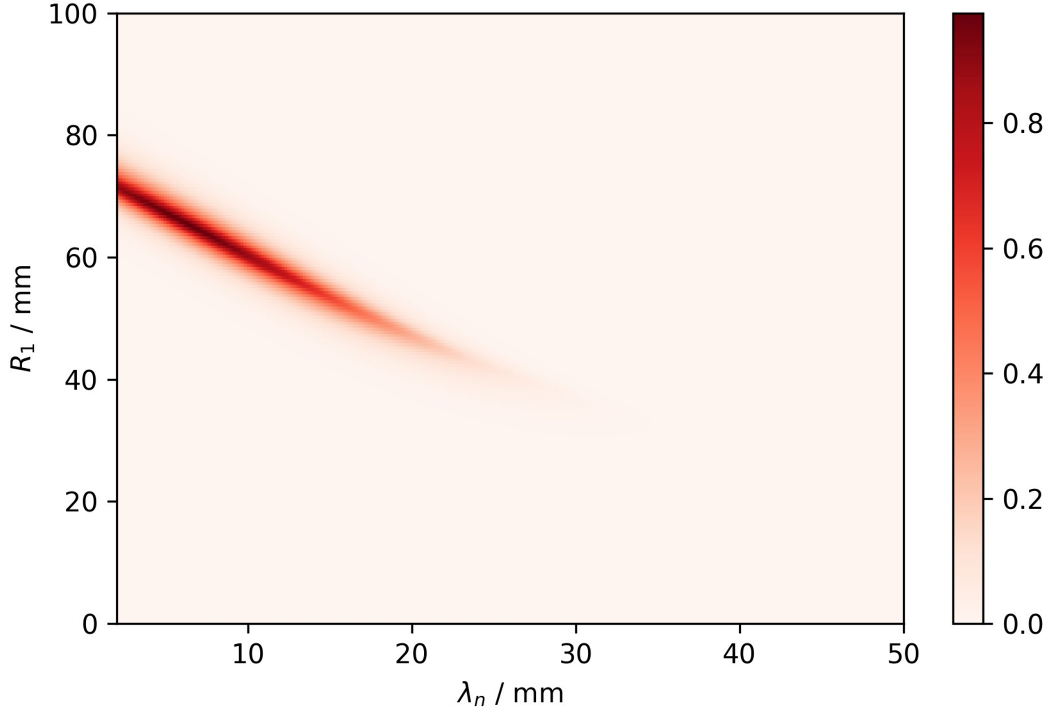

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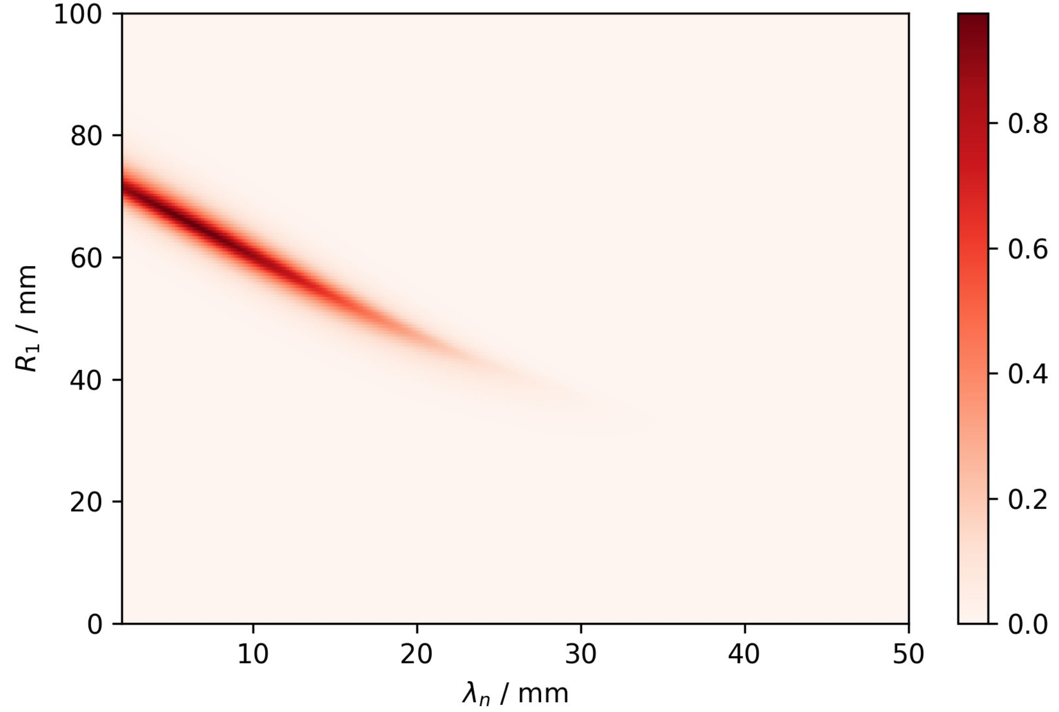
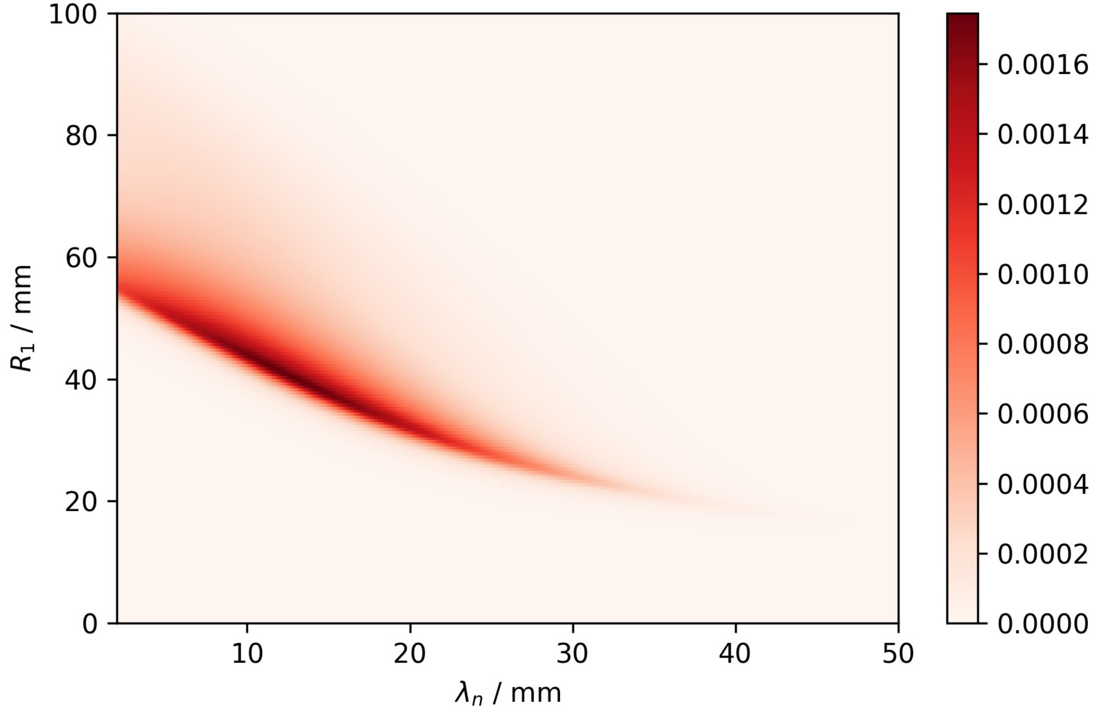


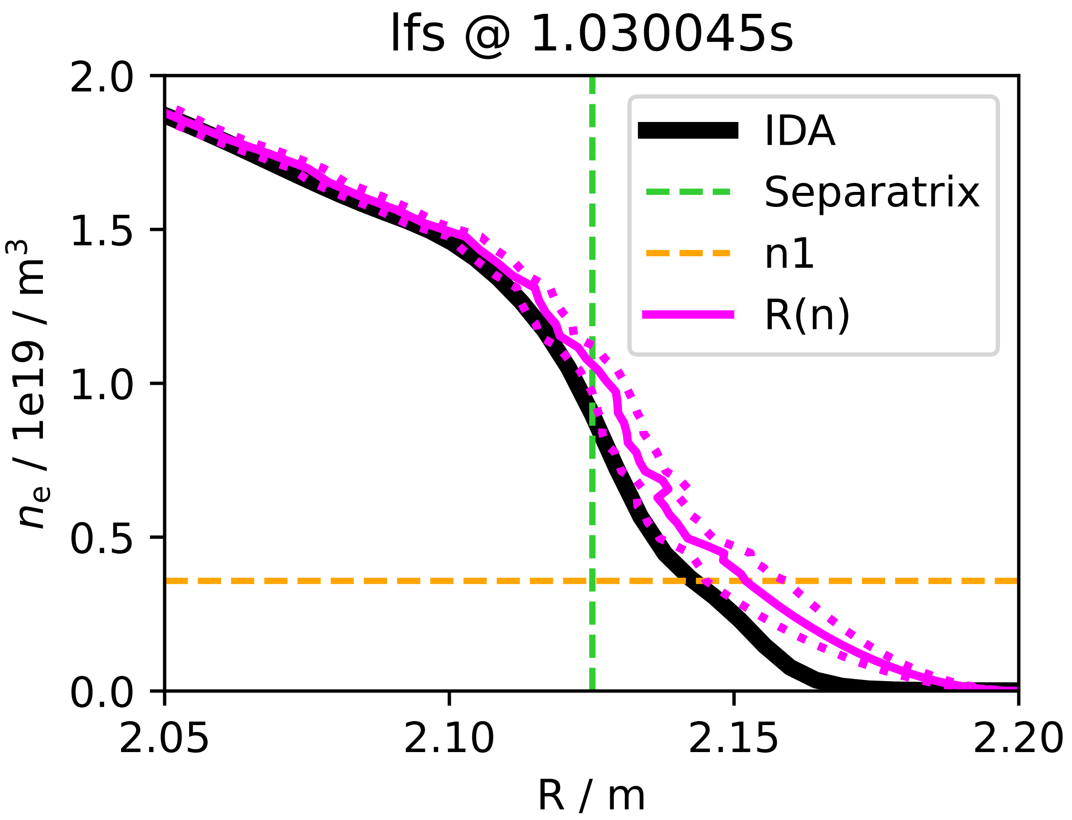
Bad Fit



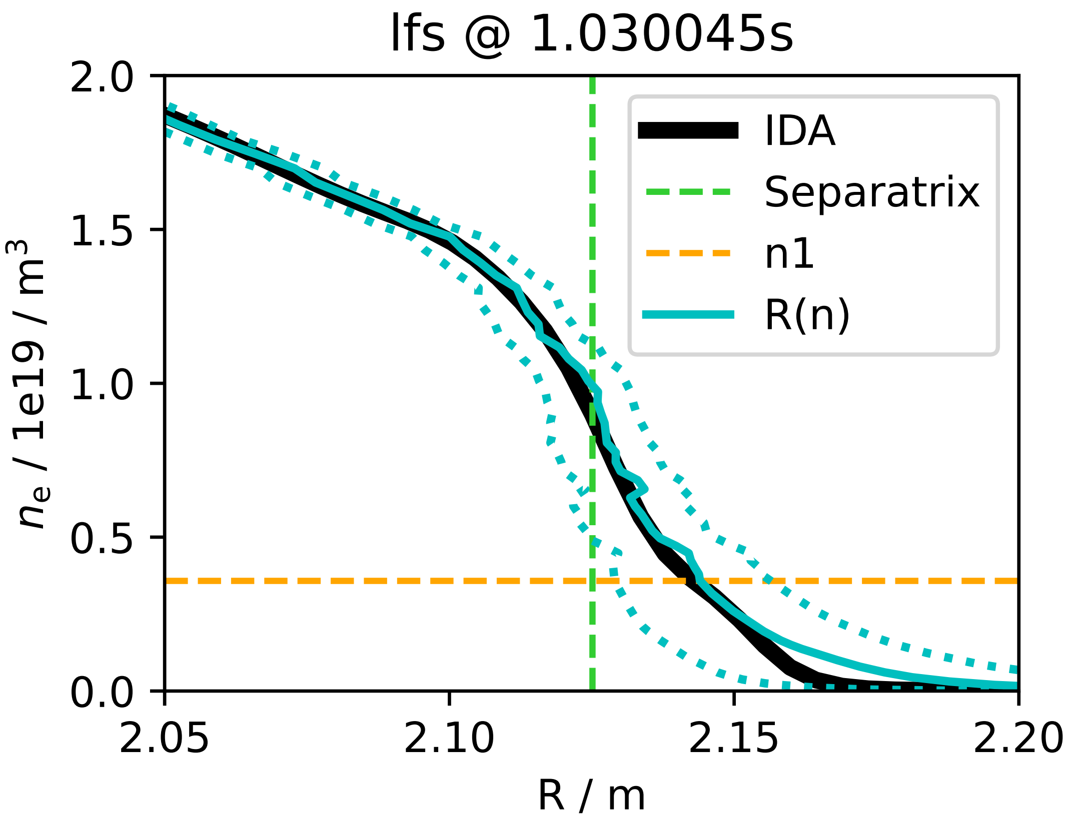
Application in Practice

Comparison to IDA with Lithium Beam & Reflectometry

Start just inside Limit: 2.20m



Prior λ_n = 10mm +- 1.0



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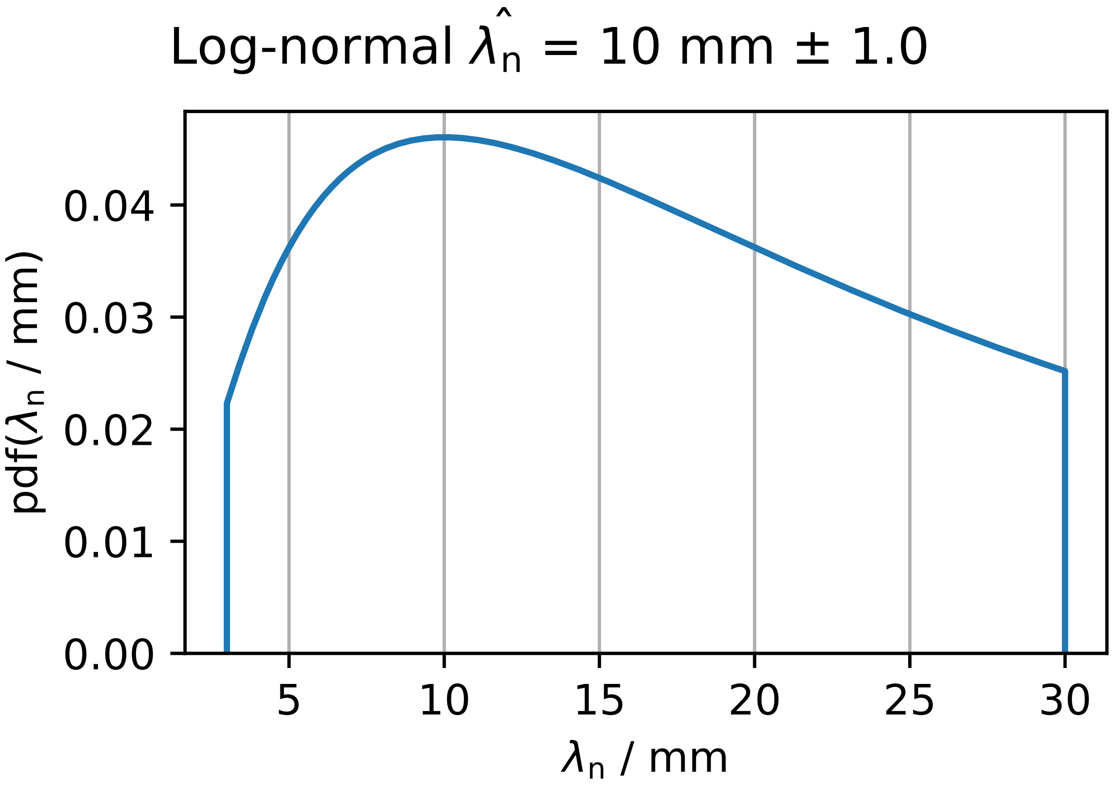
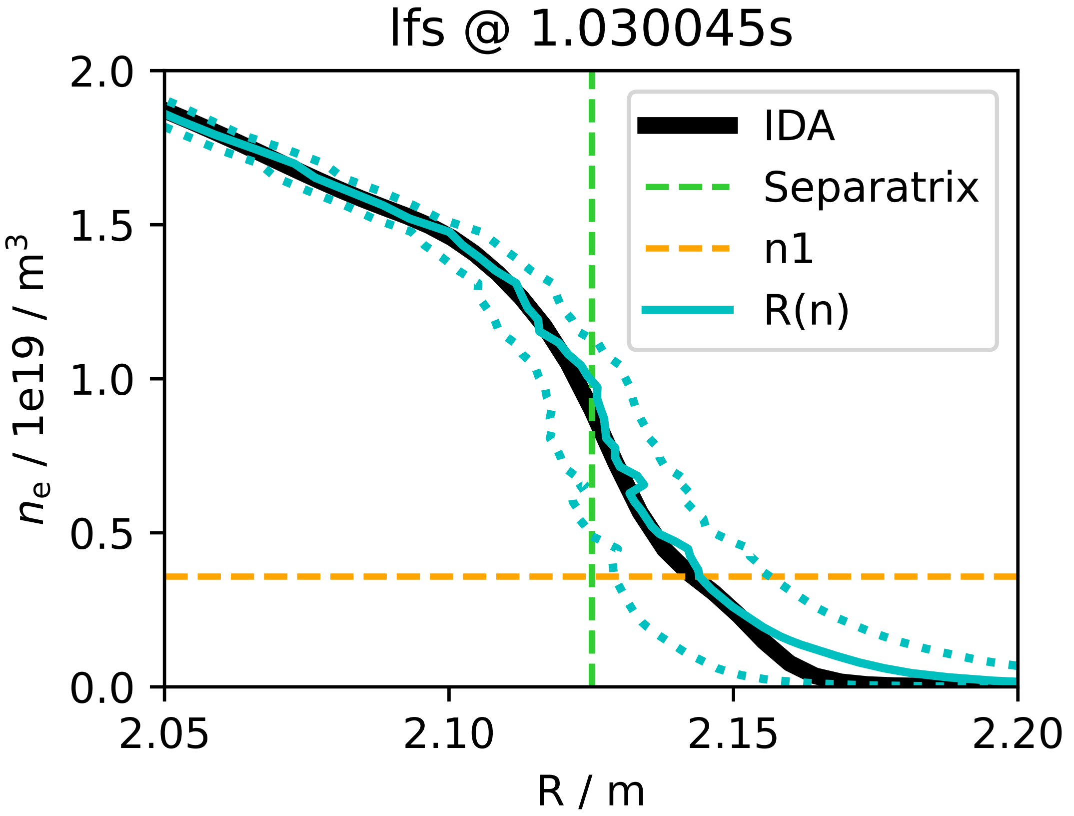
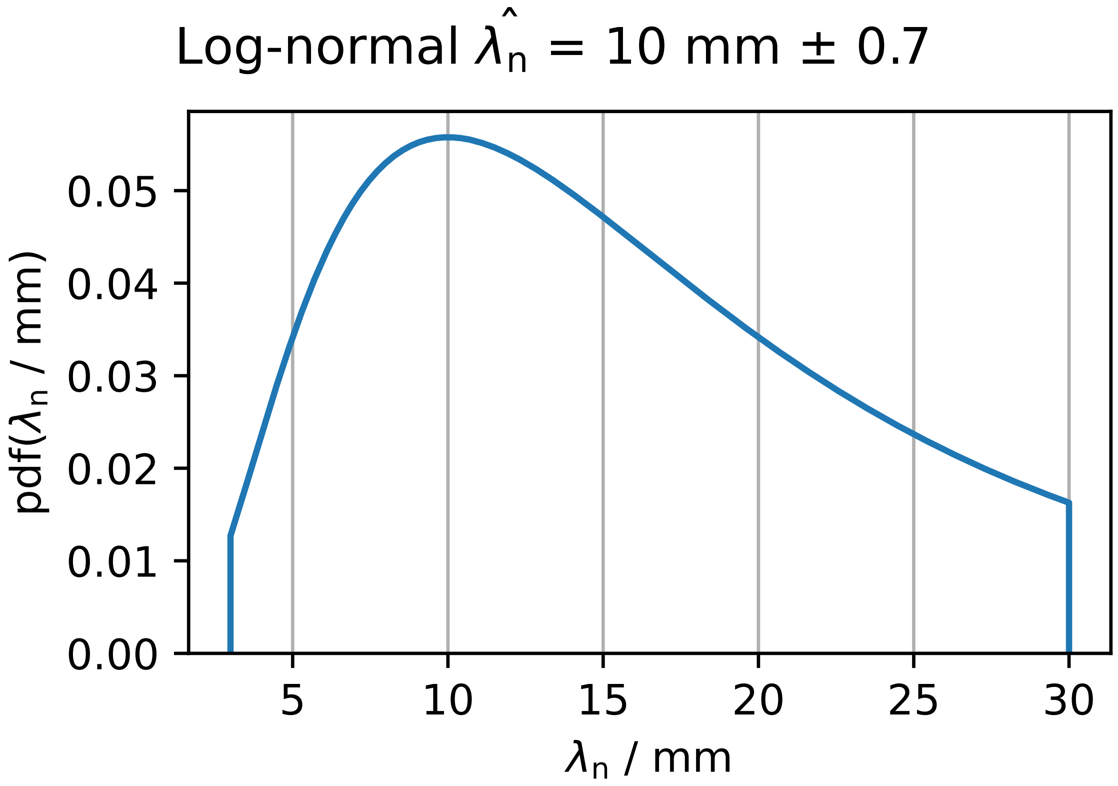


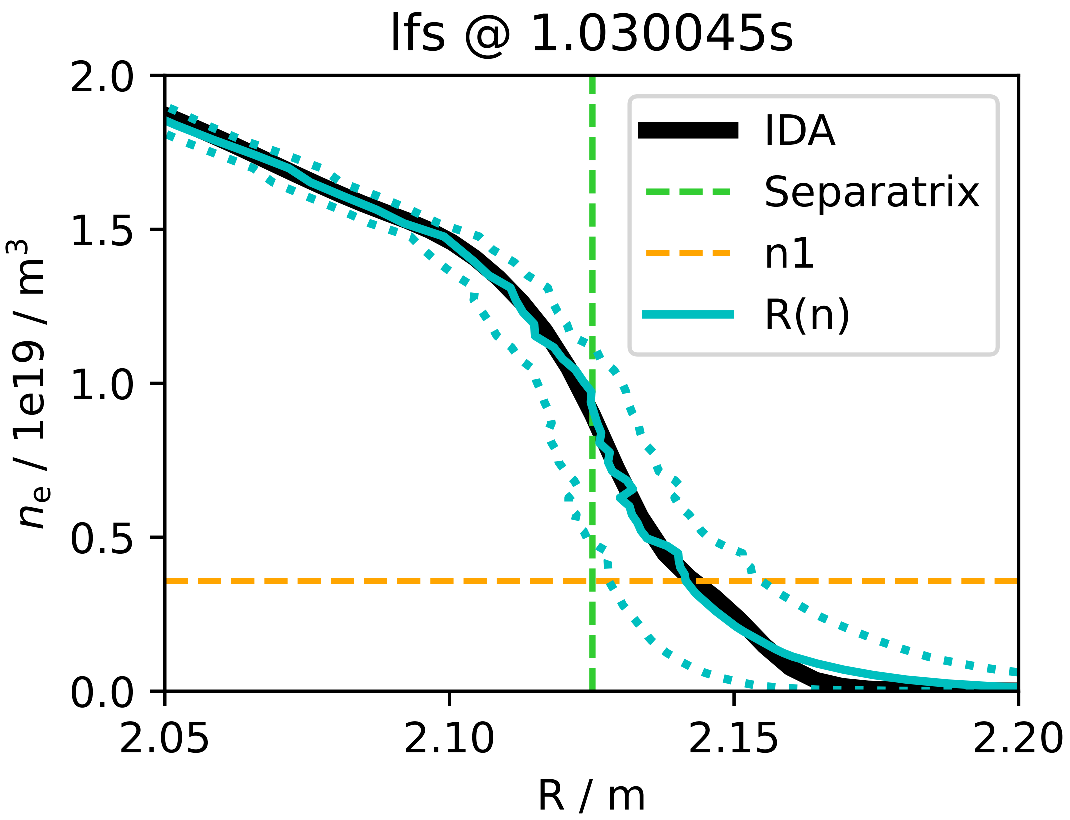


Application in Practice

Comparison to IDA with Lithium Beam & Reflectometry

Prior λ_n = 10mm +- 0.7

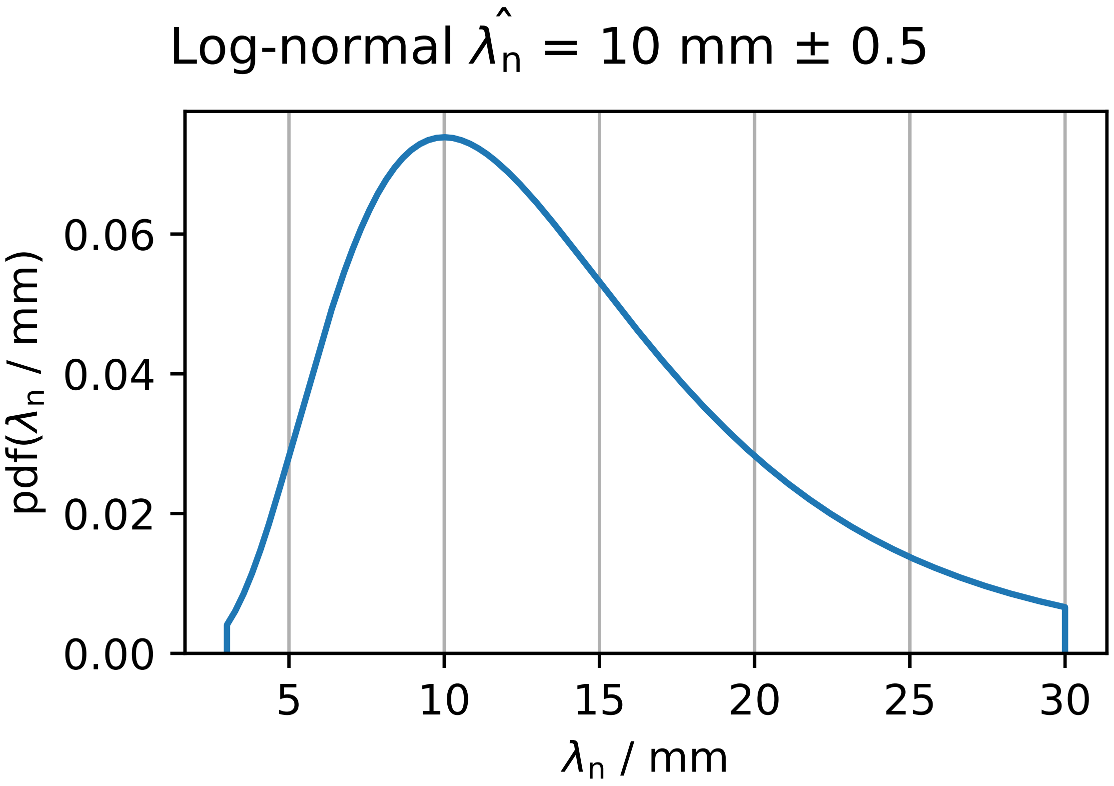


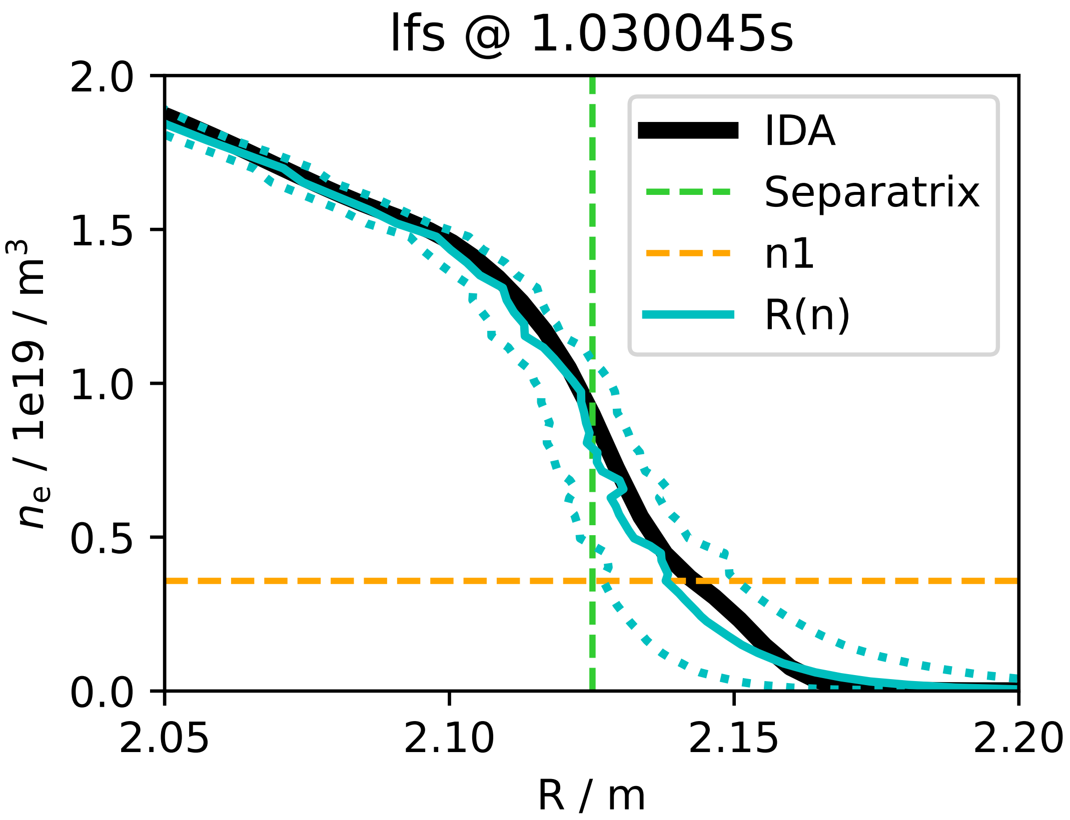


Application in Practice

Comparison to IDA with Lithium Beam & Reflectometry

Prior λ_n = 10mm +- 0.5

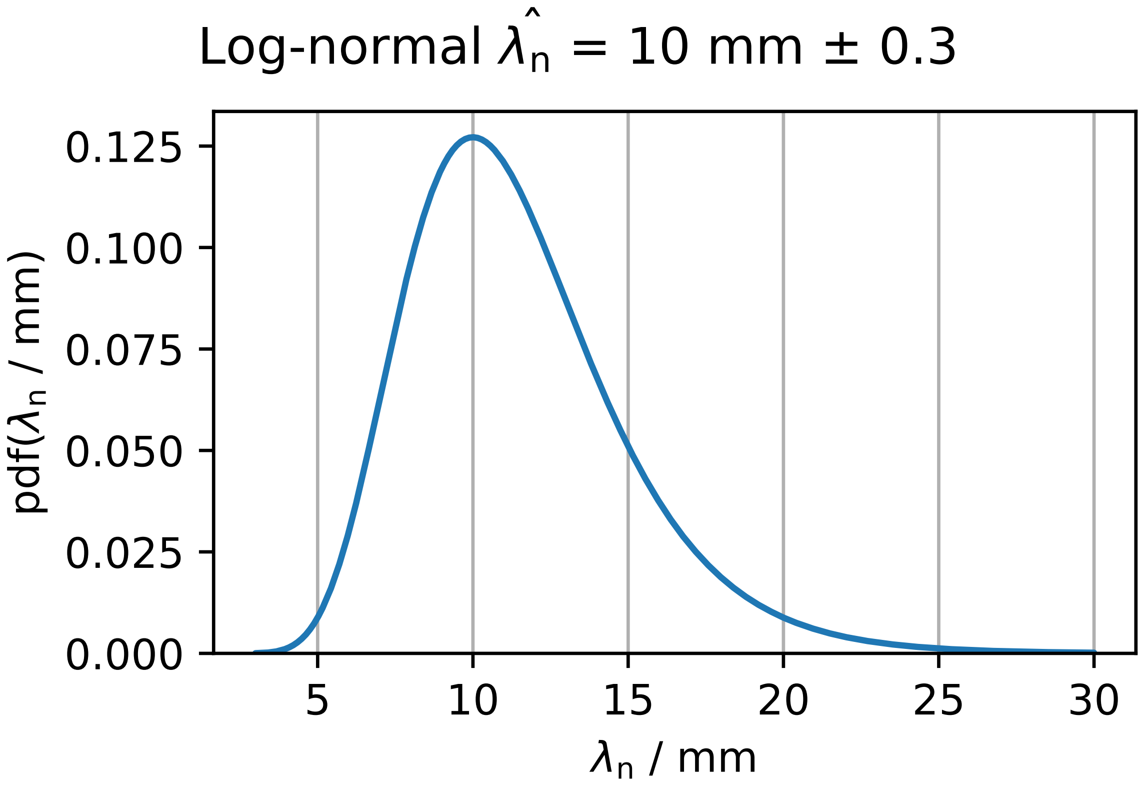


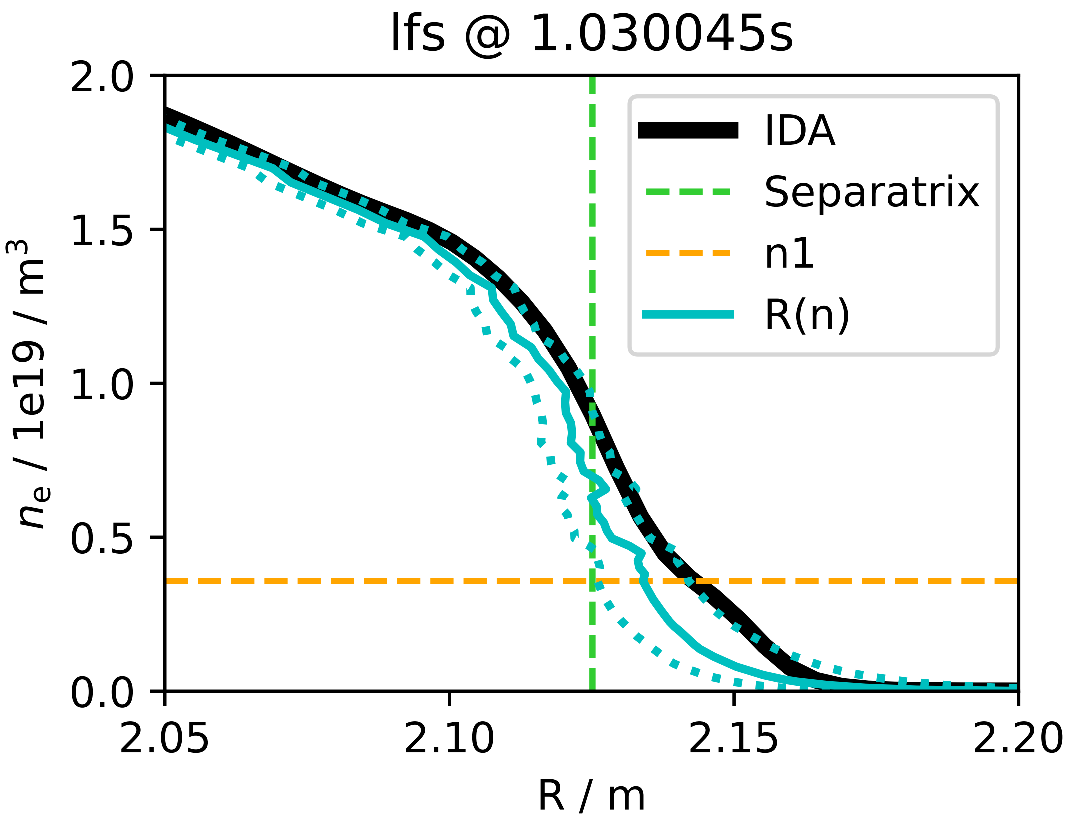


Application in Practice

Comparison to IDA with Lithium Beam & Reflectometry

Prior λ_n = 10mm +- 0.3

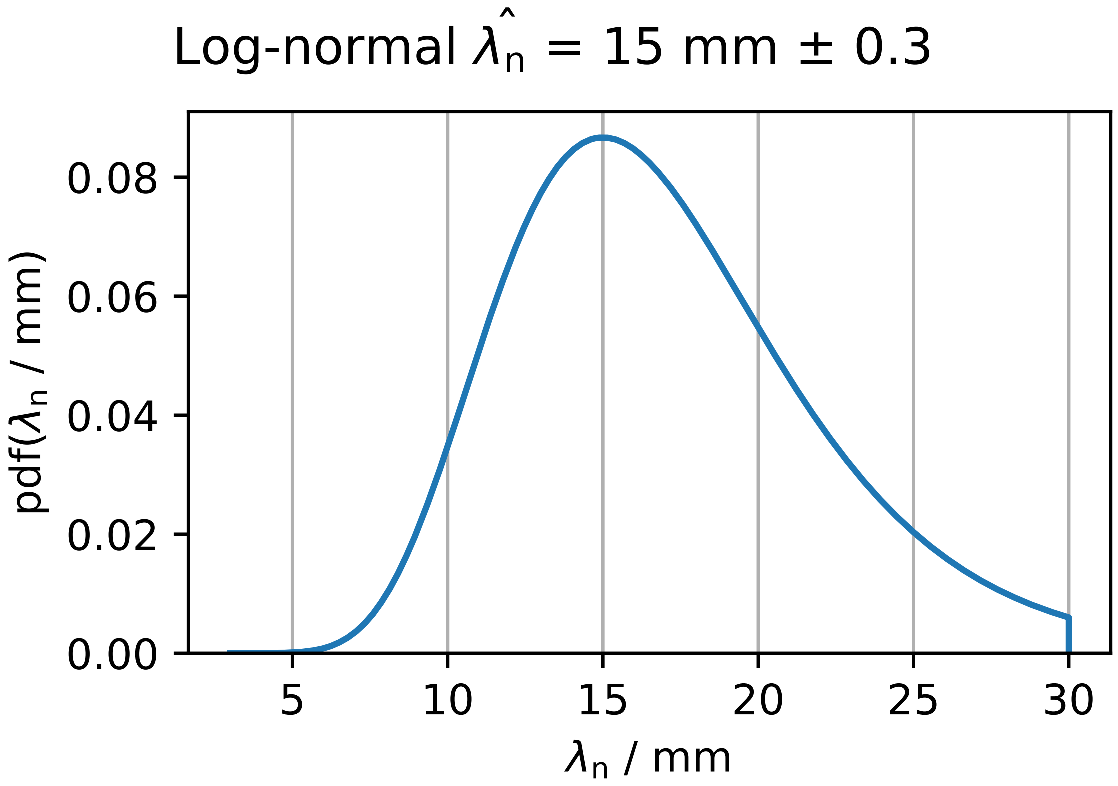


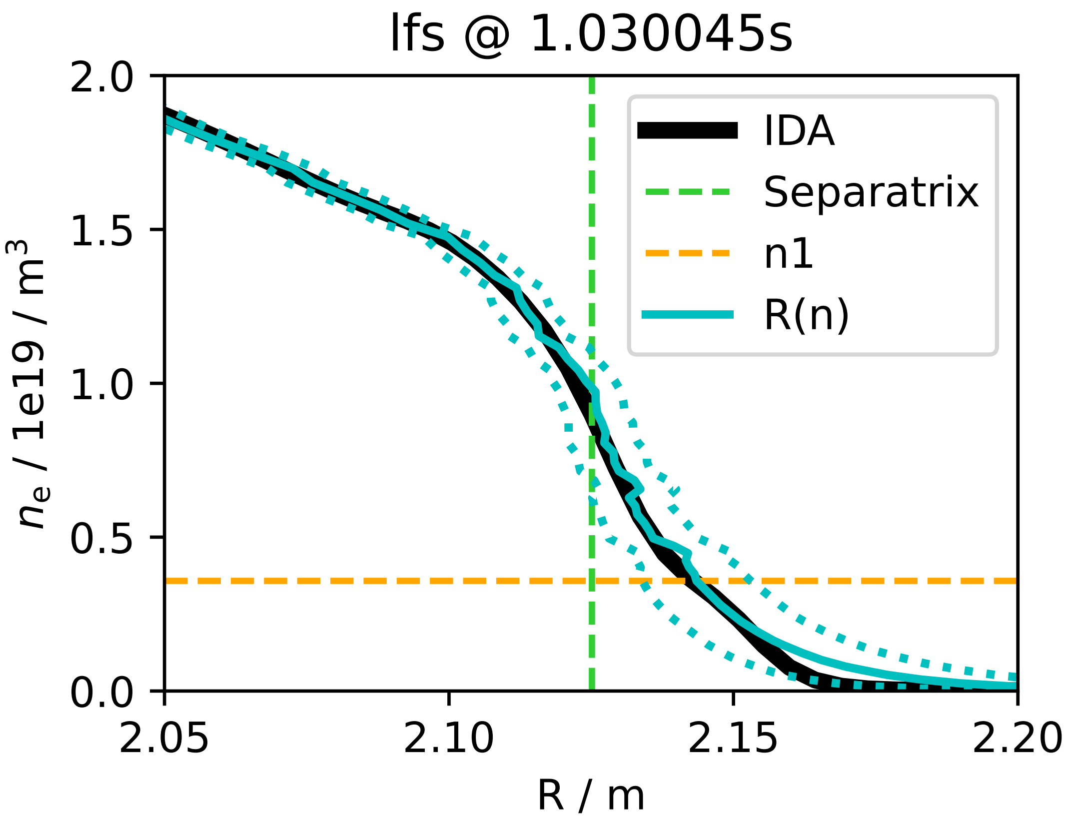


Application in Practice

Comparison to IDA with Lithium Beam & Reflectometry

Prior λn = 15mm +- 0.3

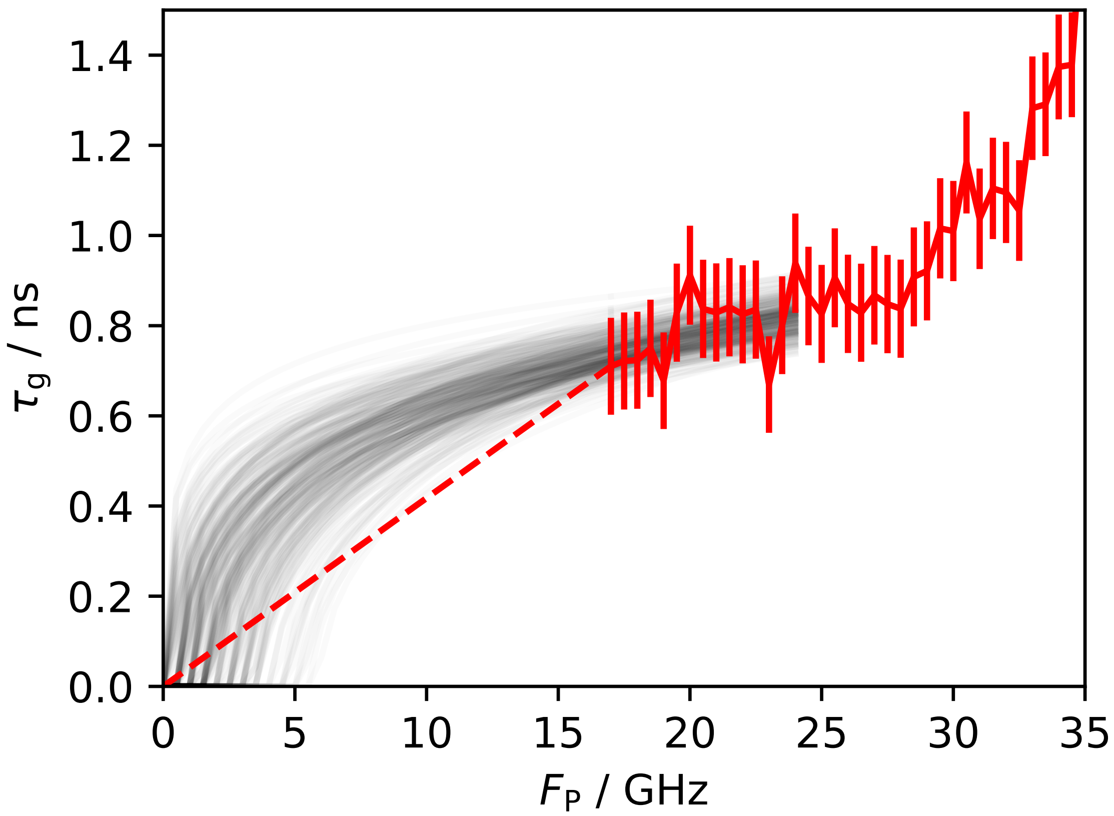




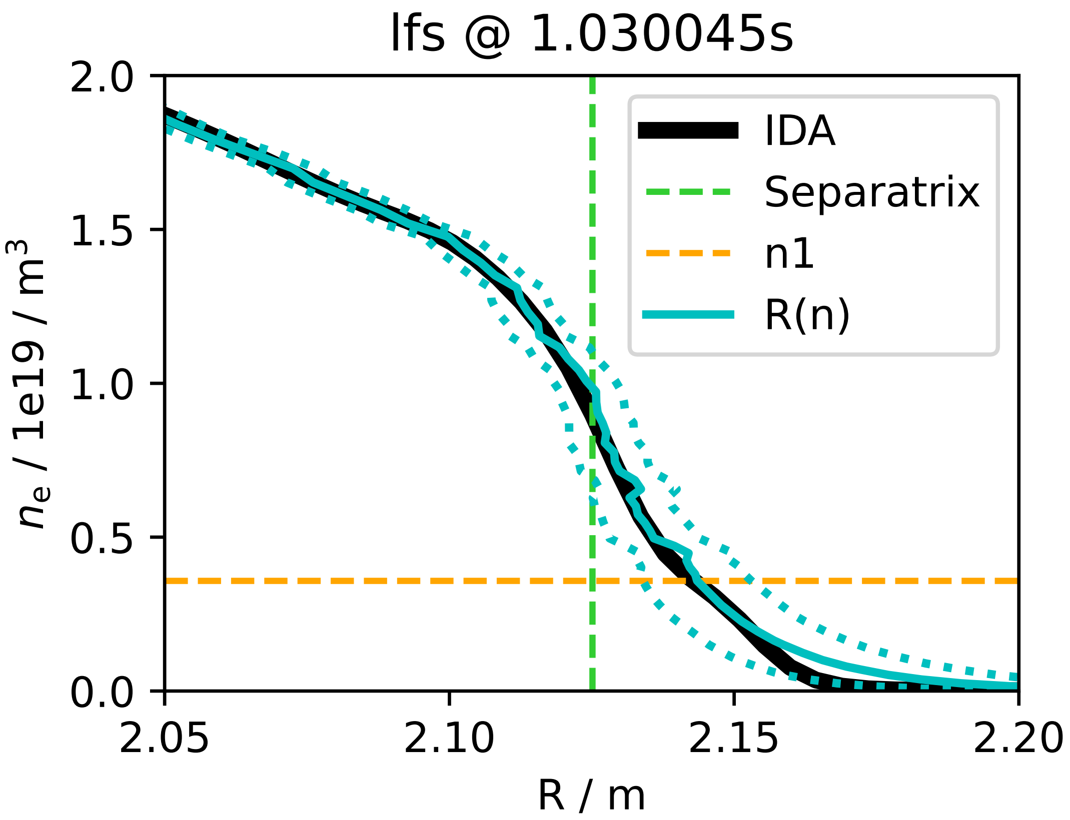
Application in Practice

In L-Mode 10-20mm → adjust prior to 15mm

15 frequencies for initialisation



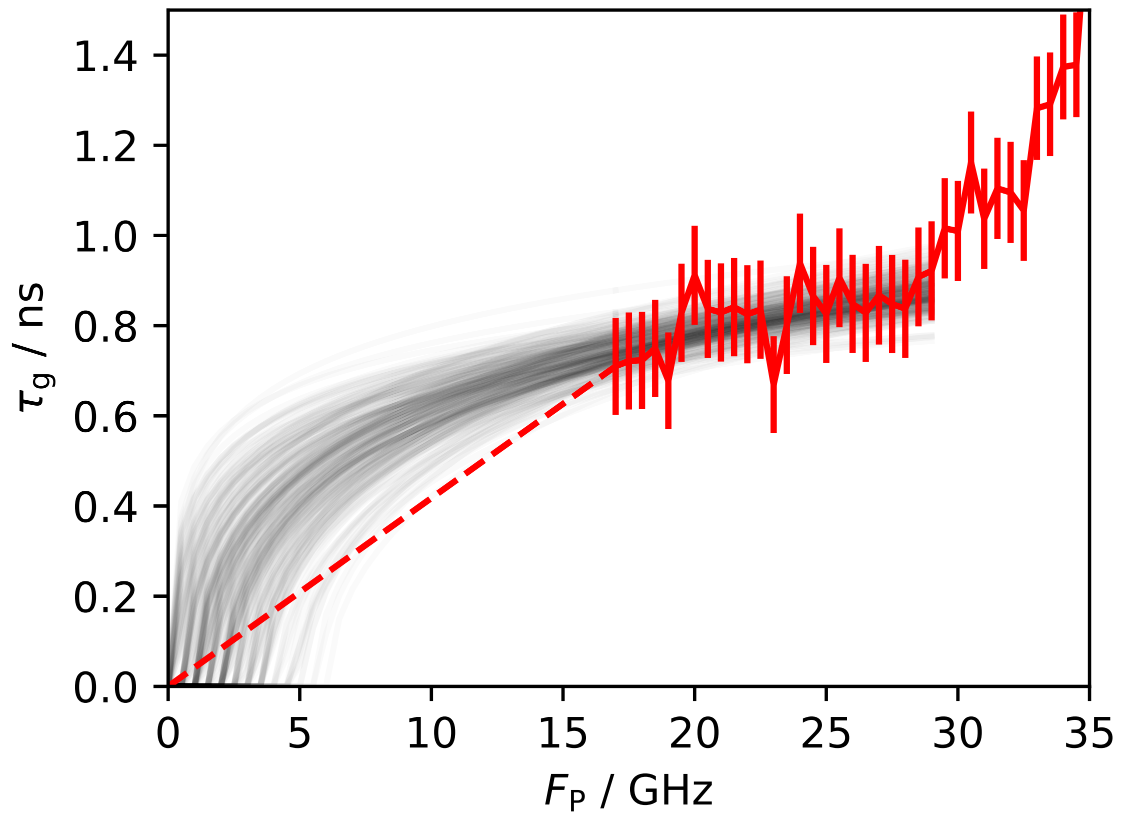
Prior λ_n = 15mm +- 0.3



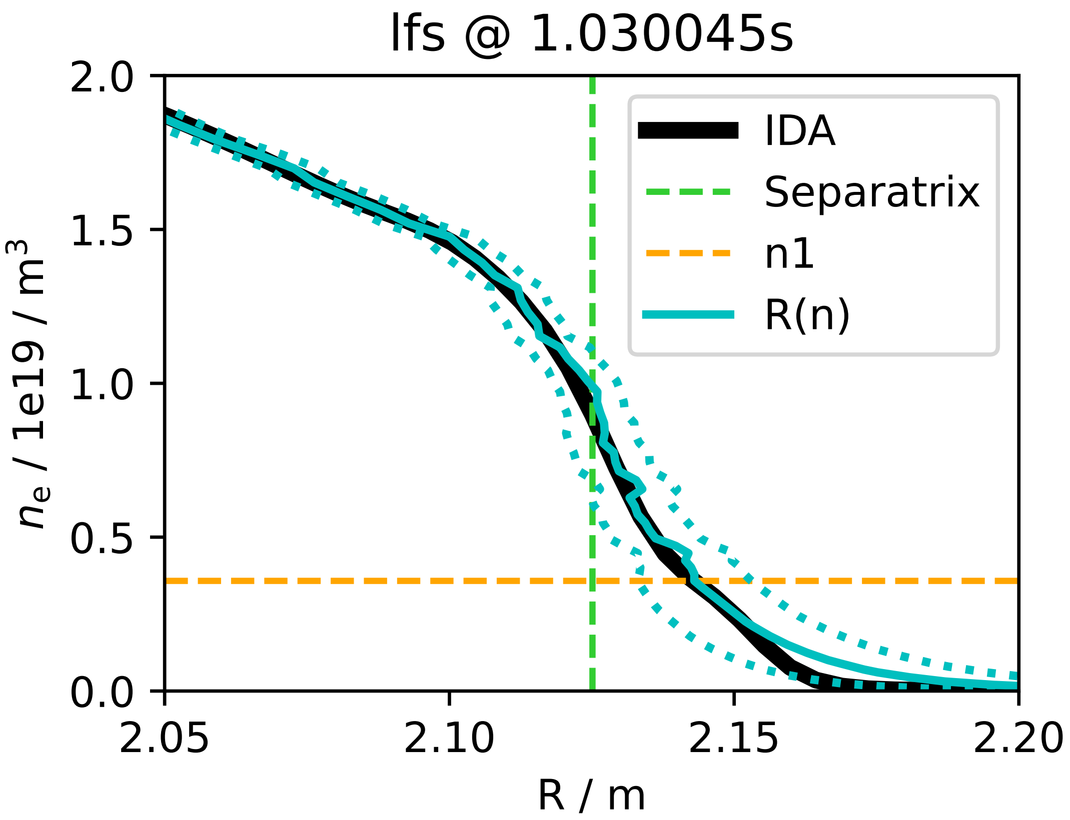
Application in Practice

In L-Mode 10-20mm → adjust prior to 15mm

25 frequencies for initialisation



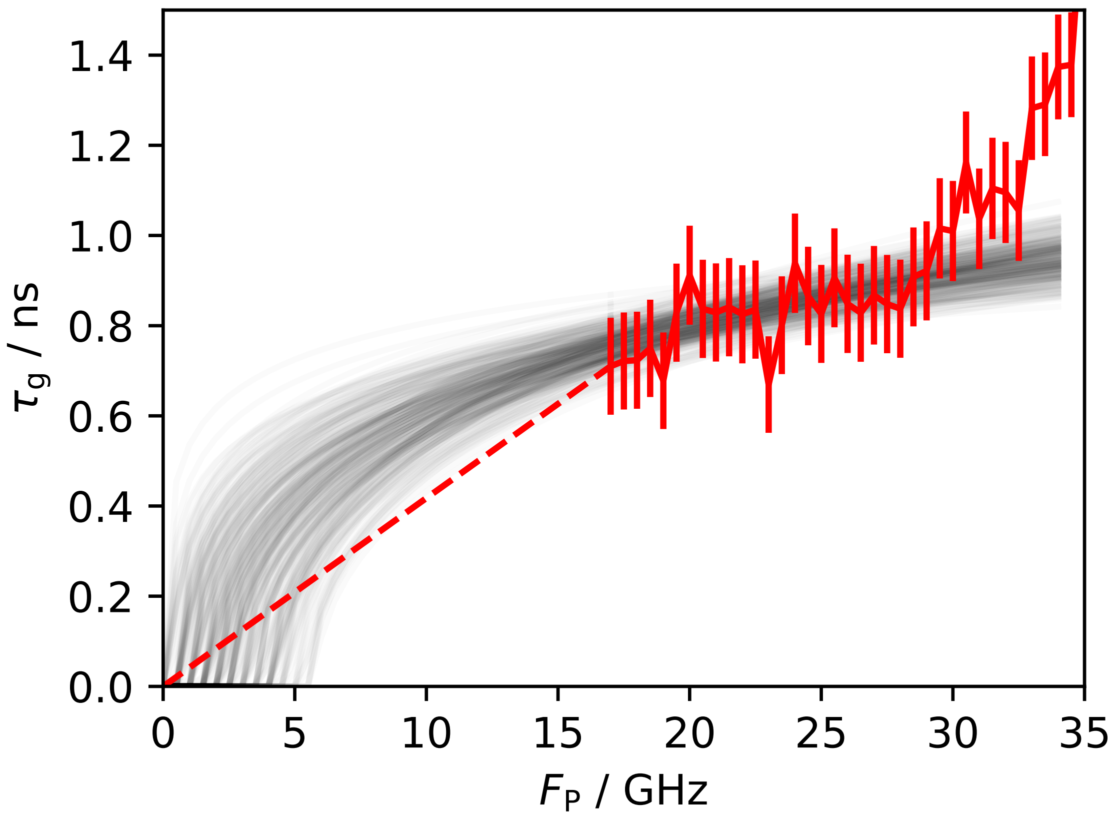
Prior λ_n = 15mm +- 0.3



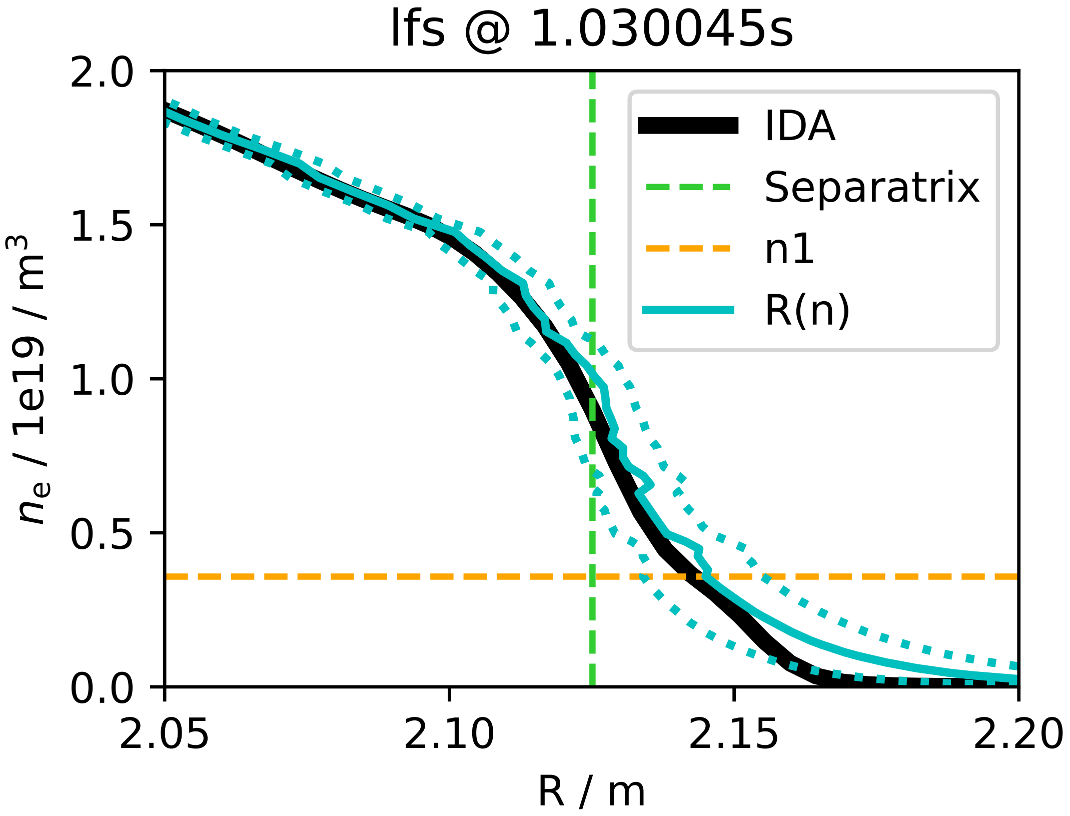
Application in Practice

In L-Mode 10-20mm → adjust prior to 15mm

35 frequencies for initialisation



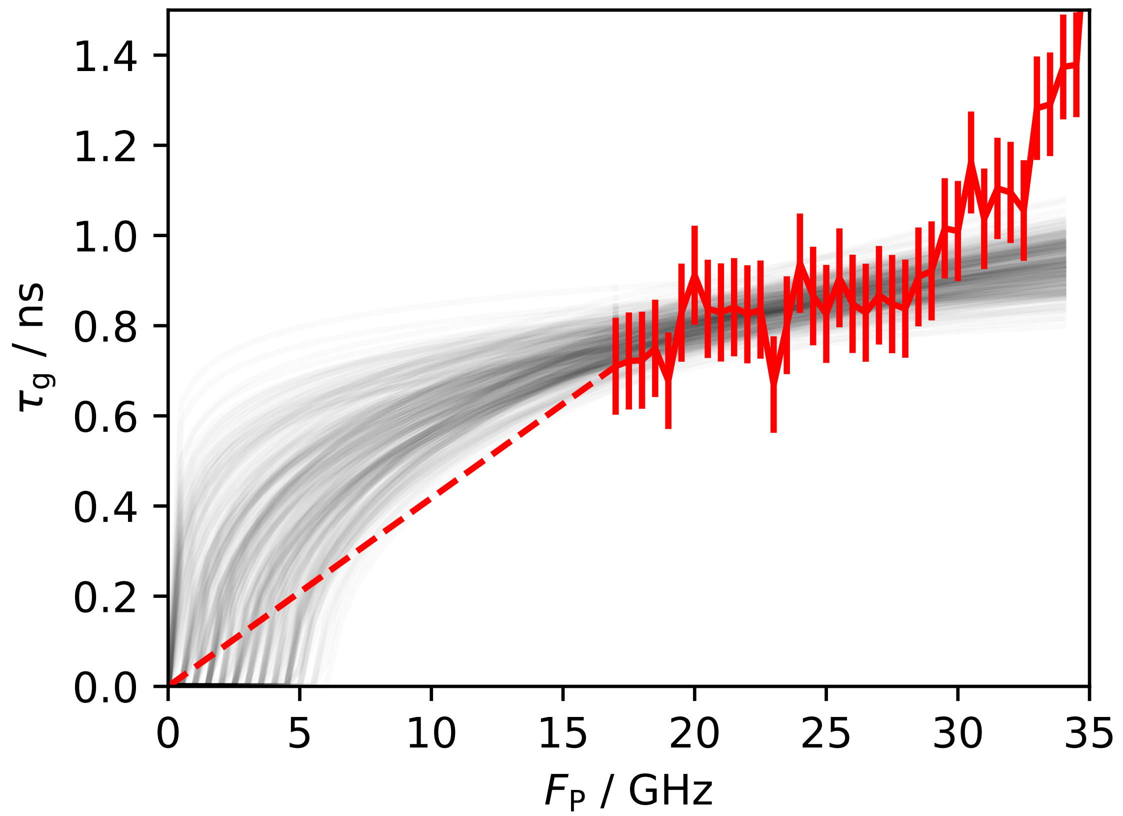
Prior λ_n = 15mm +- 0.3



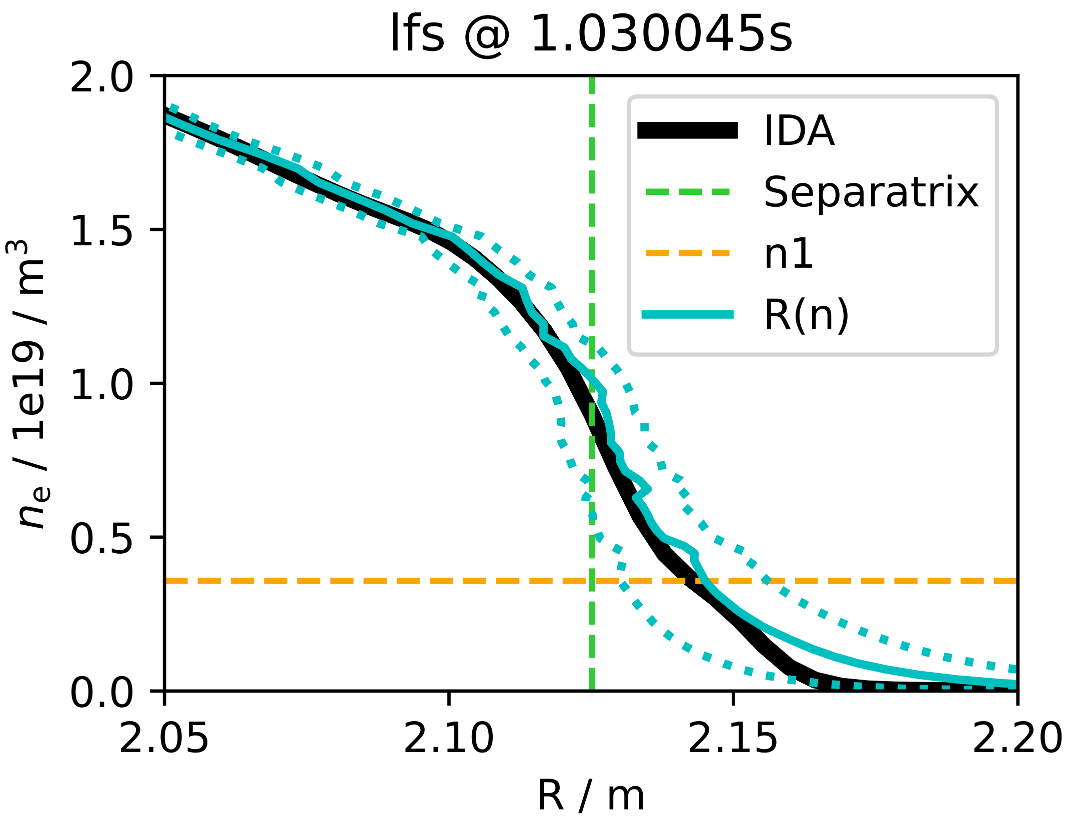
Application in Practice

More initialisation frequencies generally useful, prior can be relaxed

35 frequencies for initialisation

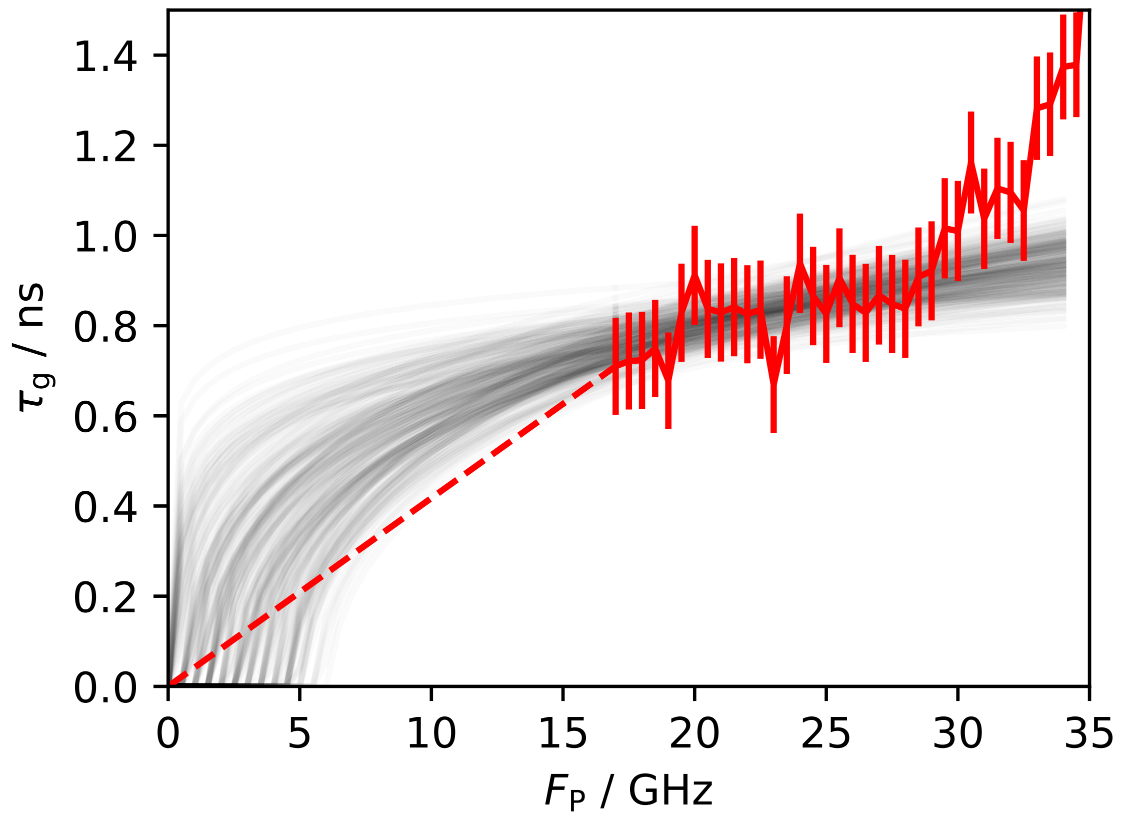


Prior λ_n = 10mm +- 0.5



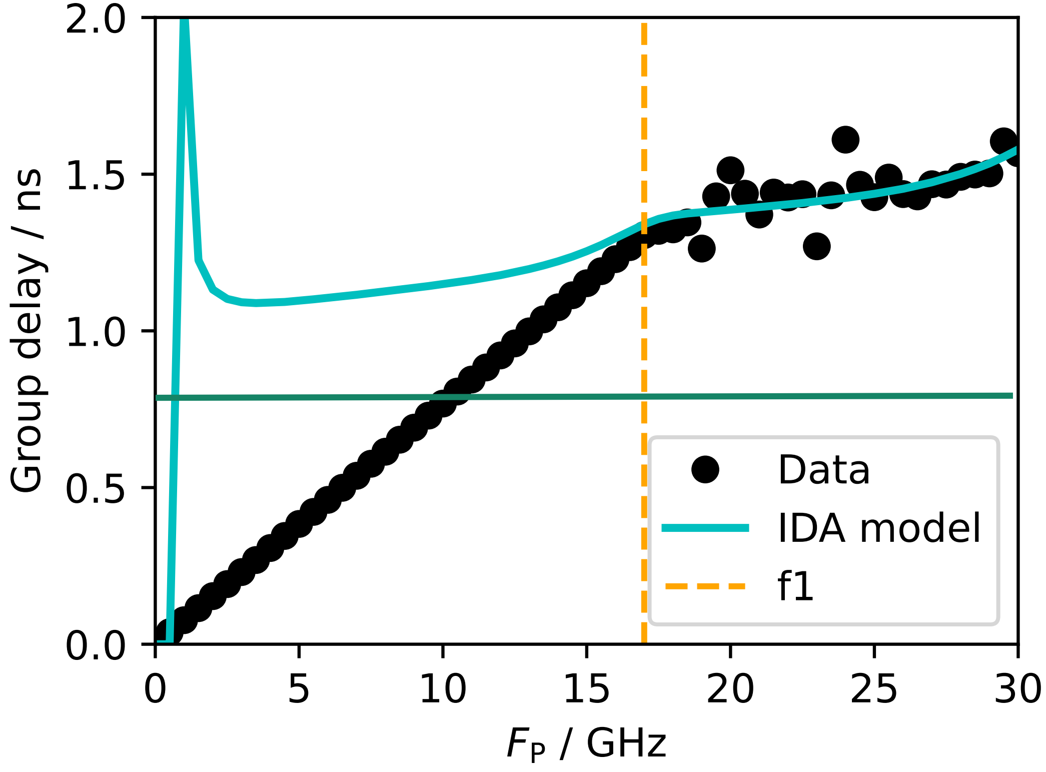
Application in Practice

35 frequencies for initialisation



IDA prediction with LIB

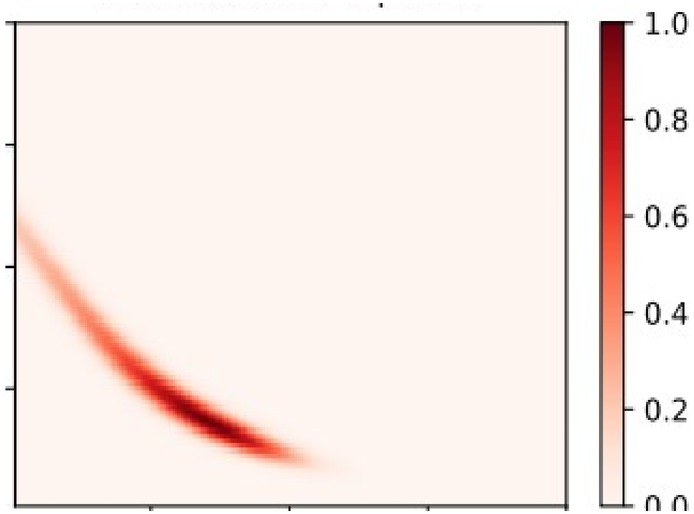
other R0 → +0.8ns

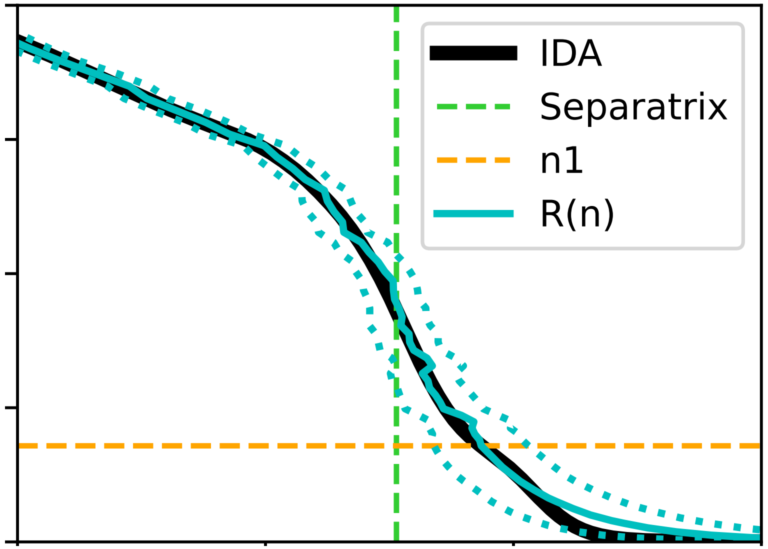


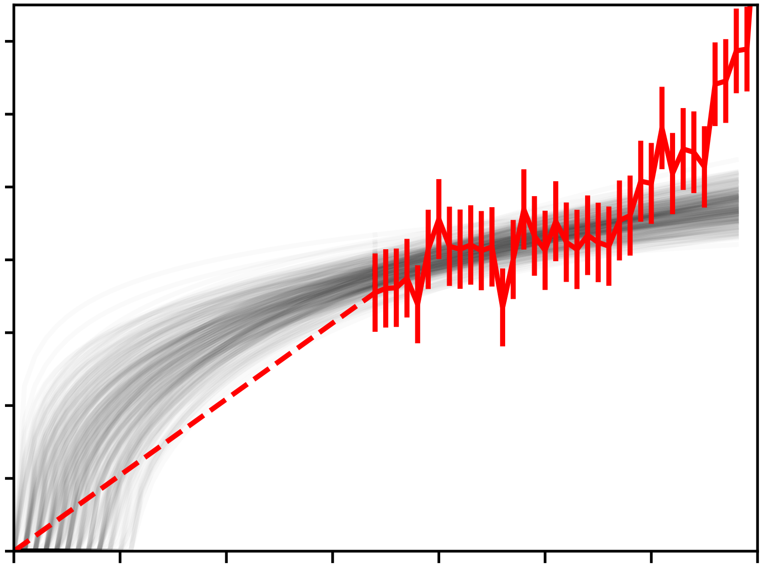
Summary

Initialisation

- One Parameter insufficient
- More data can be used







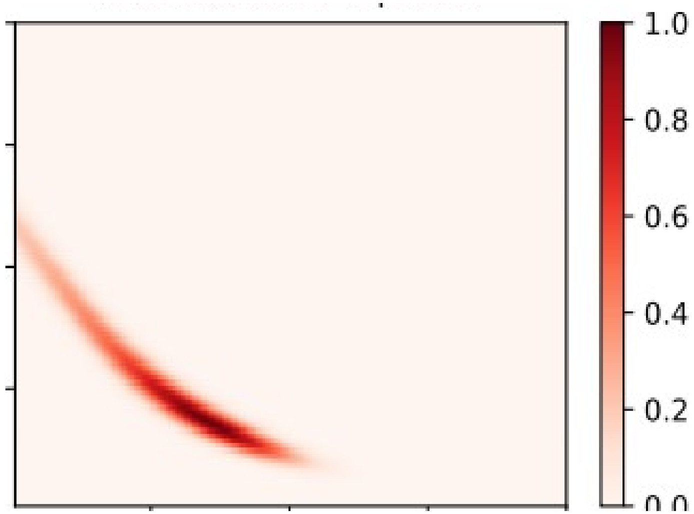
Summary

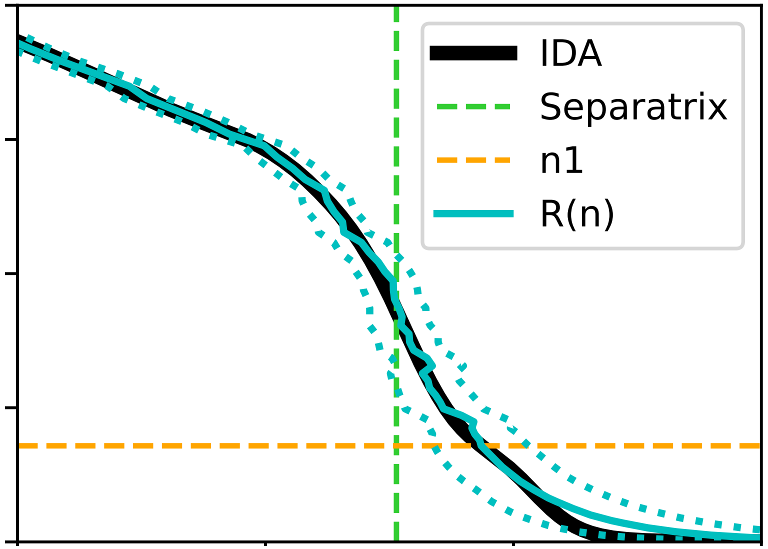
Initialisation

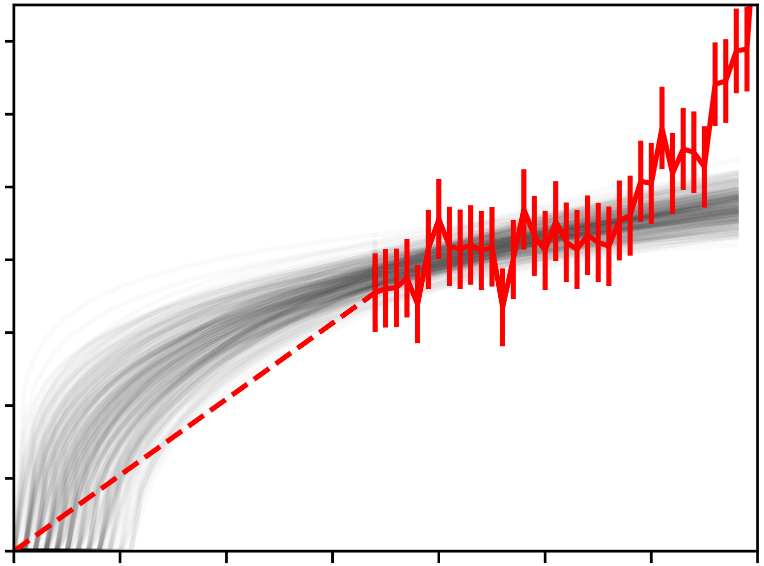
- One Parameter insufficient
- More data can be used

Exponential Model

- More flexible
- Closer to SOL plasma
- Prior very important







Summary

Initialisation

- One Parameter insufficient
- More data can be used

Exponential Model

- More flexible
- Closer to SOL plasma
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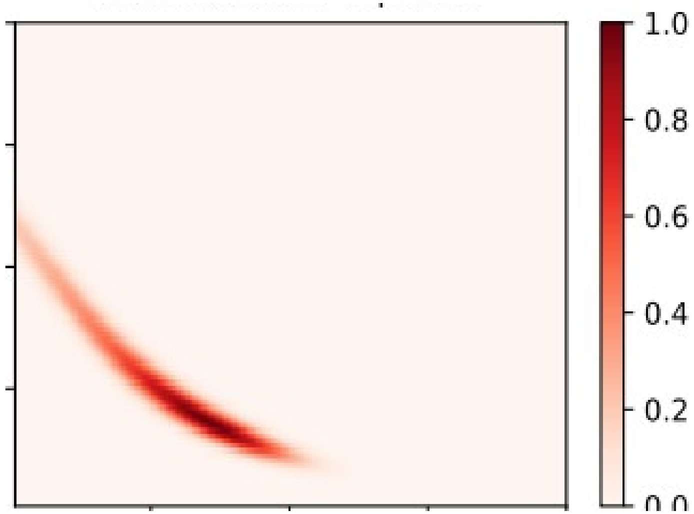
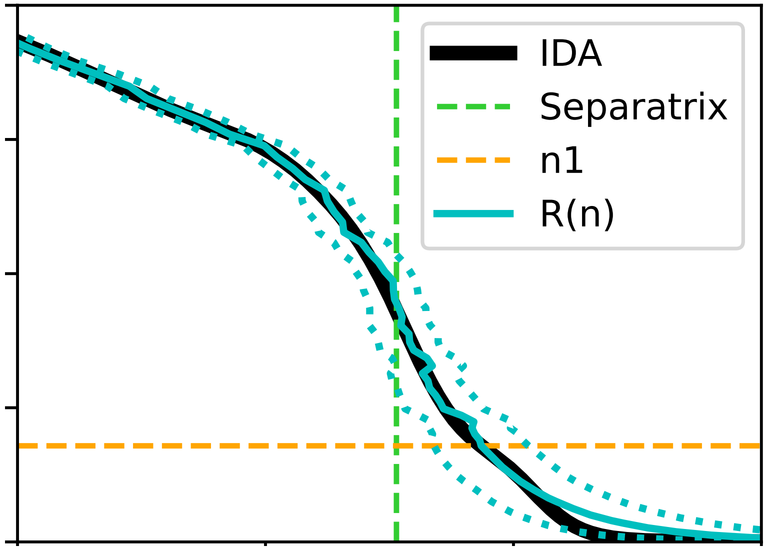
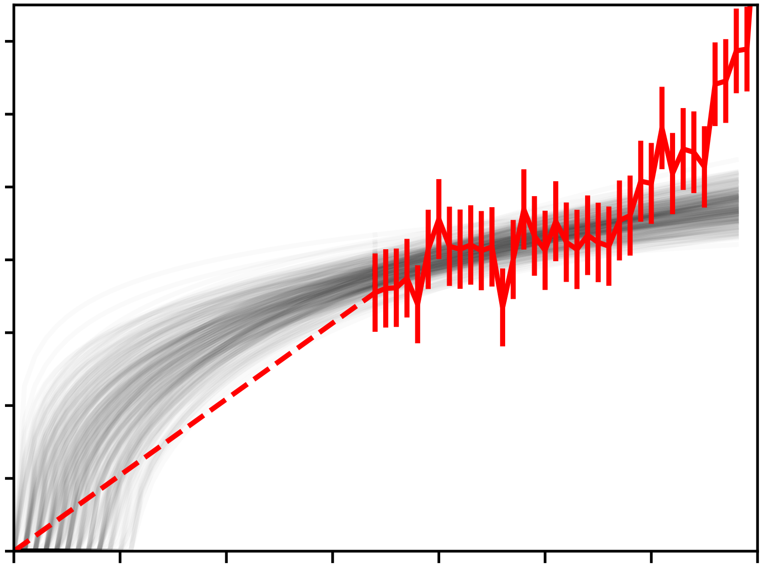
Fit approach tunable

- Number of data
- Outlier resistant likelihood
- Physics exploitation / Control







Backup Slides

Initialisation: Variability implications

Key-points:

- “Reasonable” profiles with flexibility needs 2 or more parameters
- Problem underdetermined with one data point
- Trend in group delays representative of local density gradient
- Initialisation choice affects density gradients at transition (smoothness desirable) and shifts the remaining profile (dominates uncertainty)

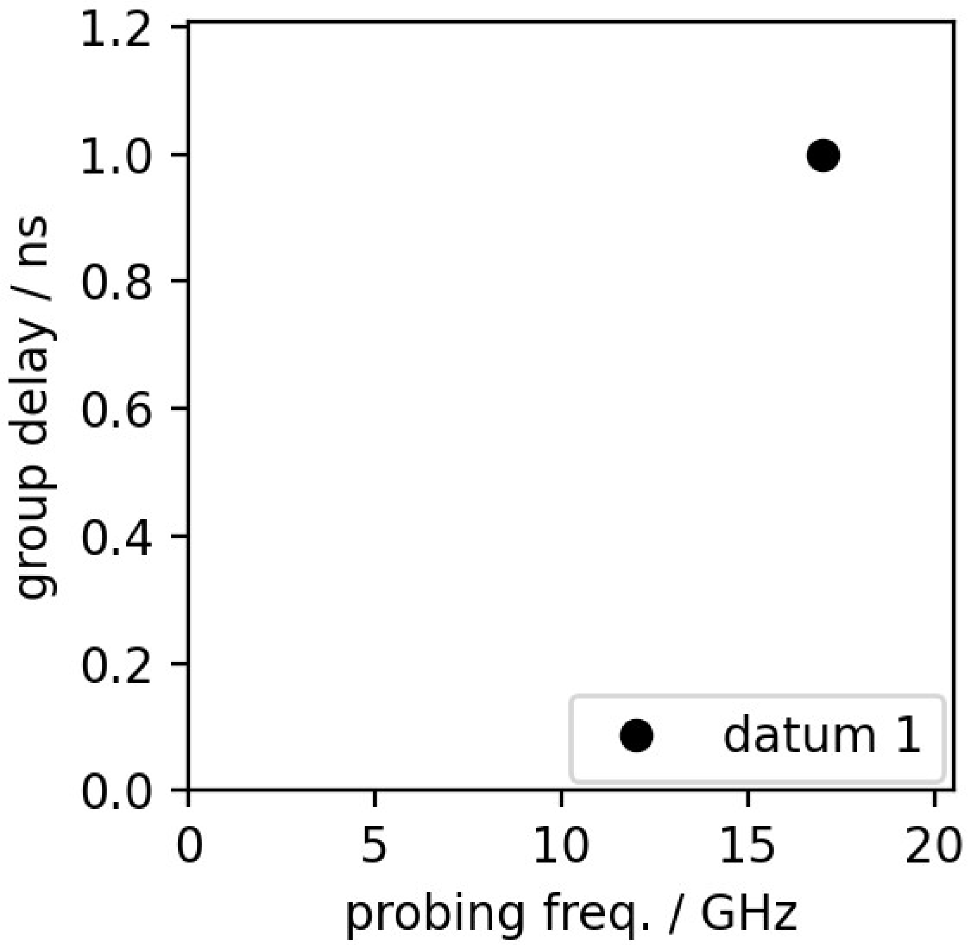
Initialisation: Variability implications

Key-points:

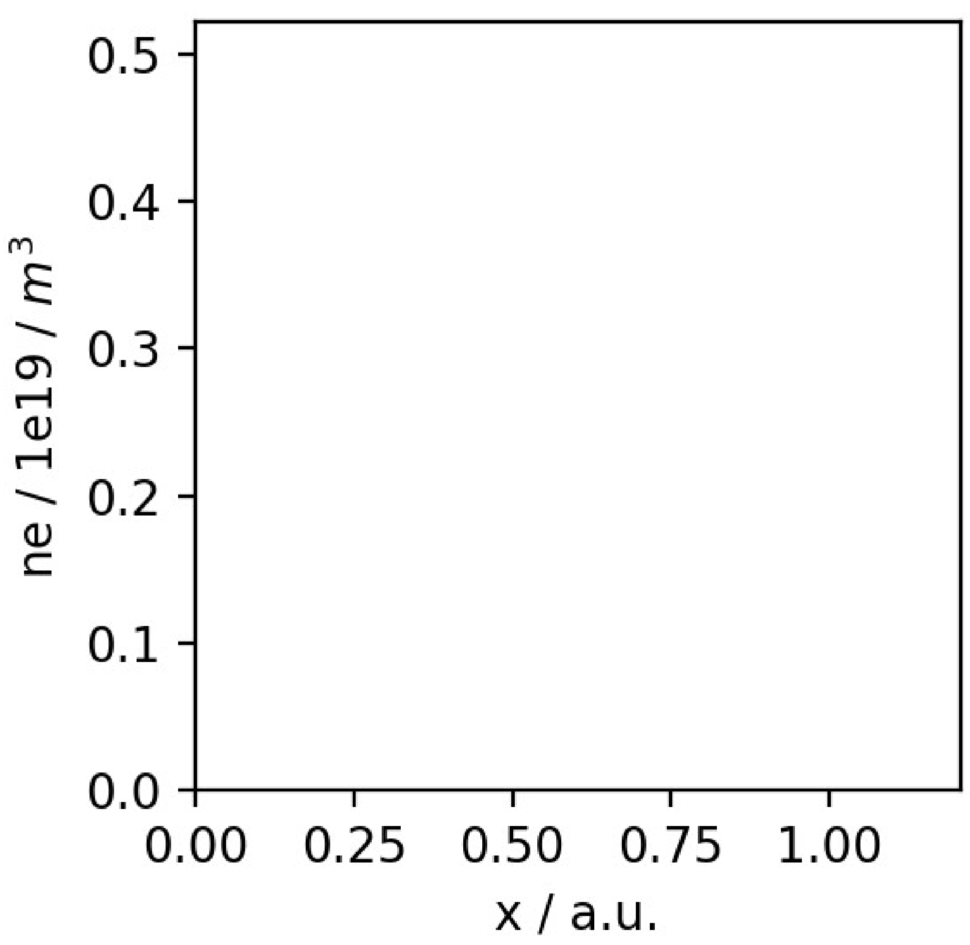
- “Reasonable” profiles with flexibility needs 2 or more parameters
- Problem underdetermined with one data point
- Trend in group delays representative of local density gradient
- Initialisation choice affects density gradients at transition (smoothness desirable) and shifts the remaining profile (dominates uncertainty)
- Effect on profile not linear → sampling for uncertainty quantification

Initialisation with linear group delay

Group Delay

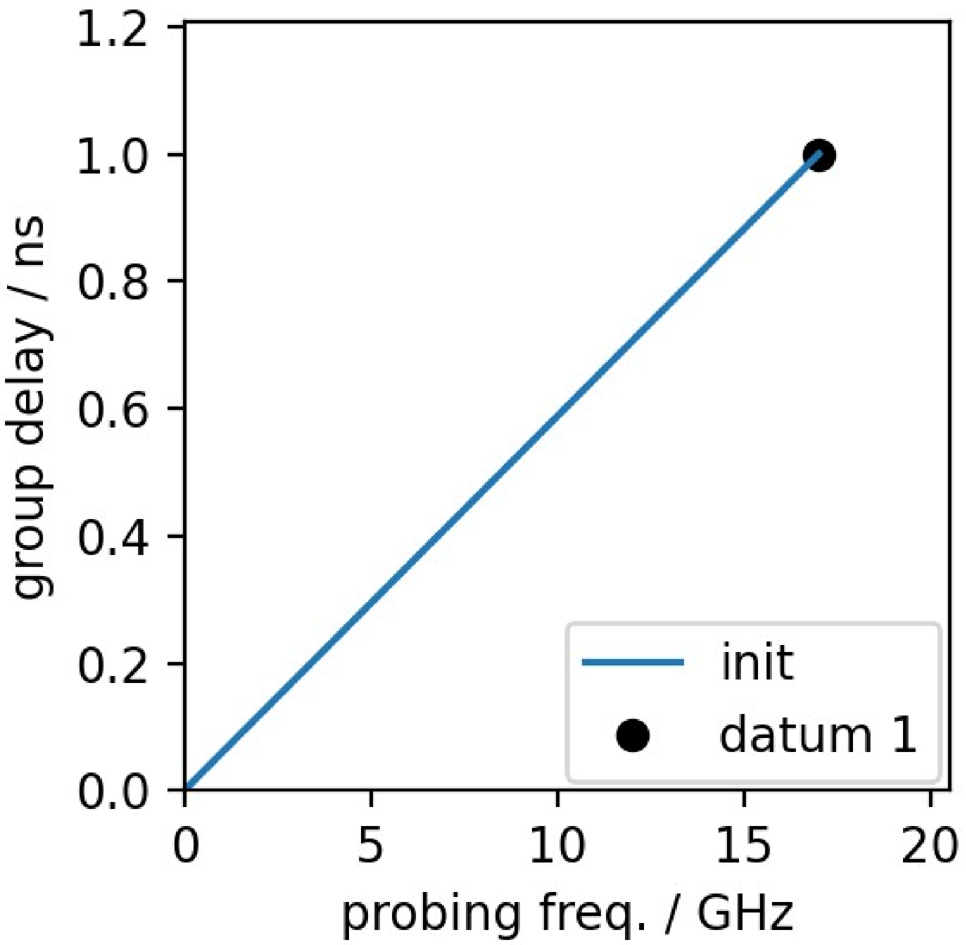


Density Profile

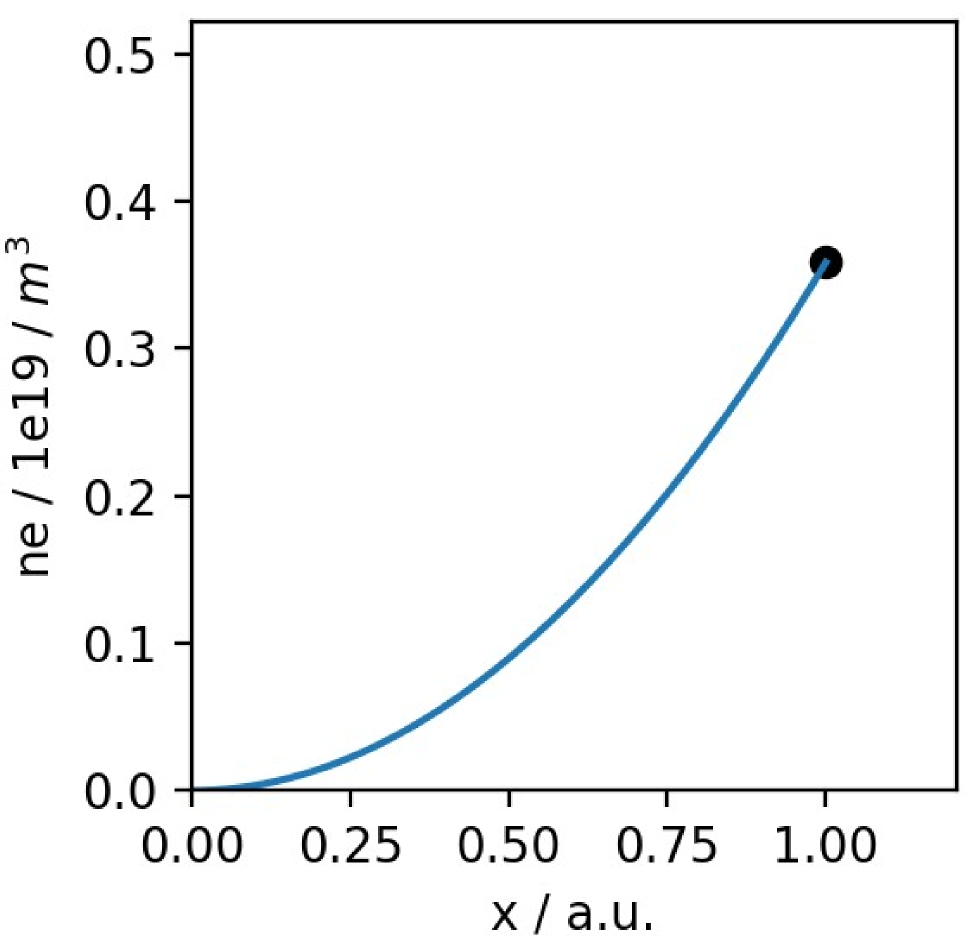


Initialisation with linear group delay

Group Delay

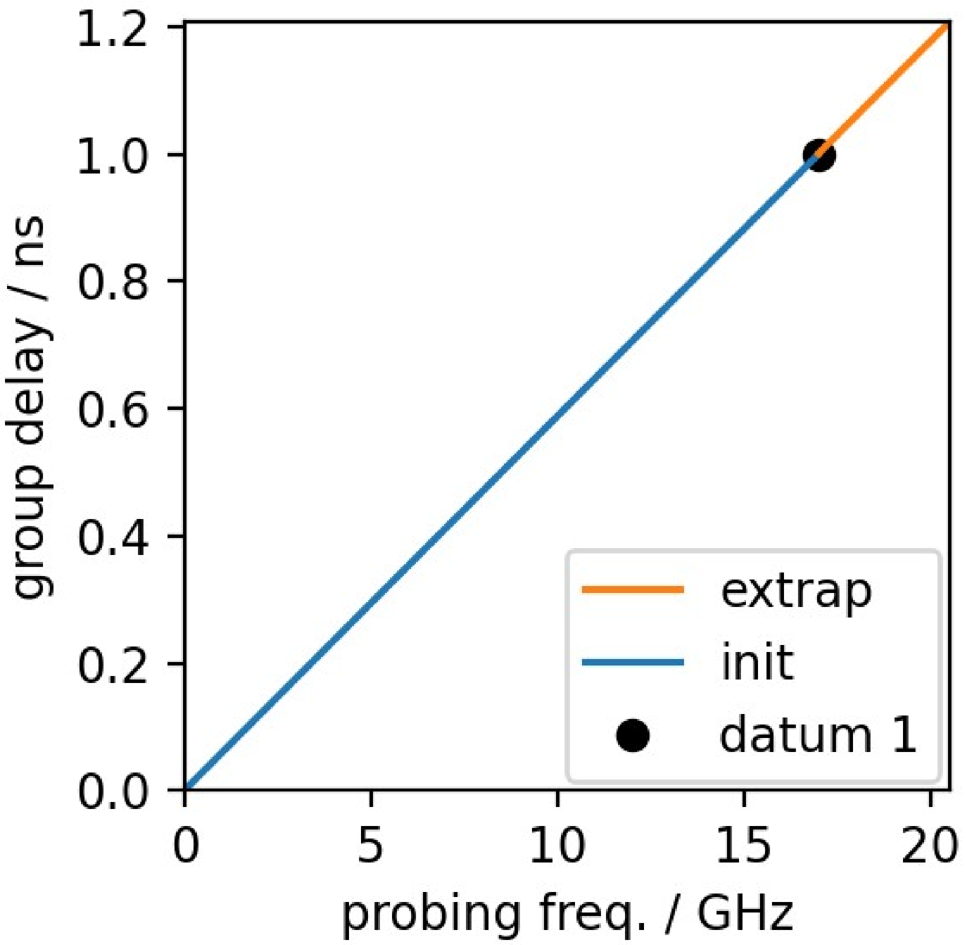


Density Profile

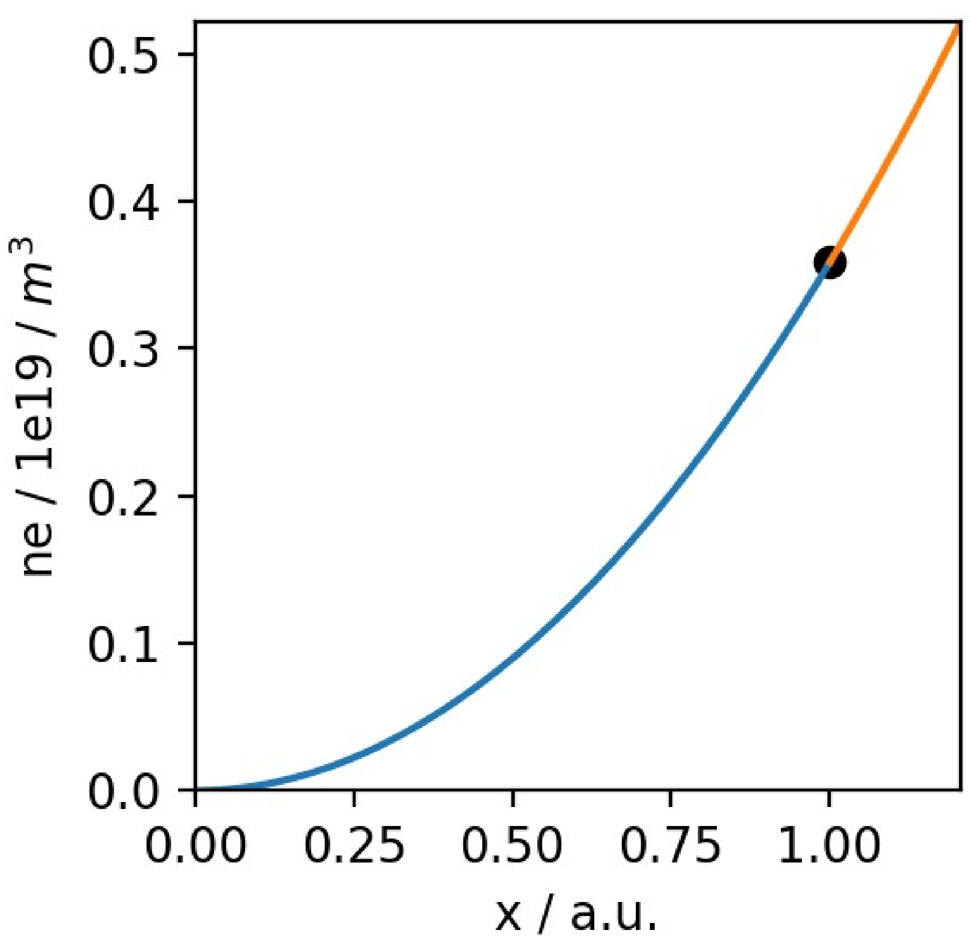


Initialisation with linear group delay

Group Delay

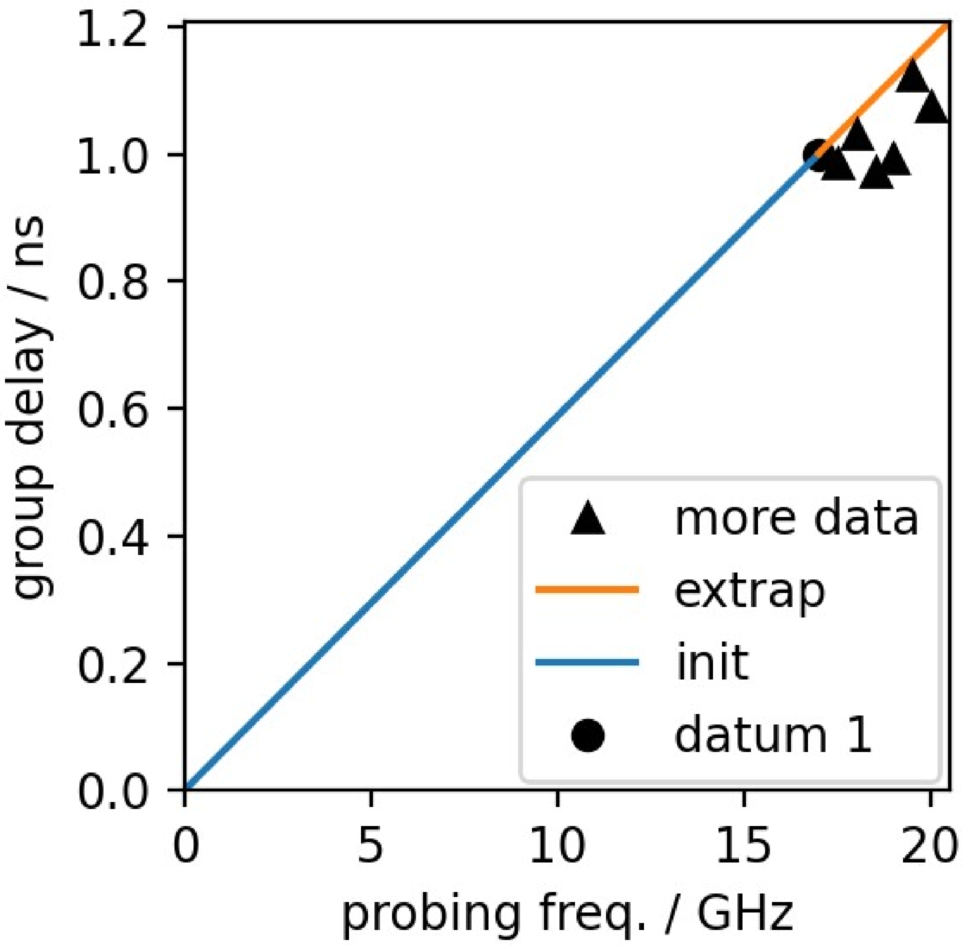


Density Profile

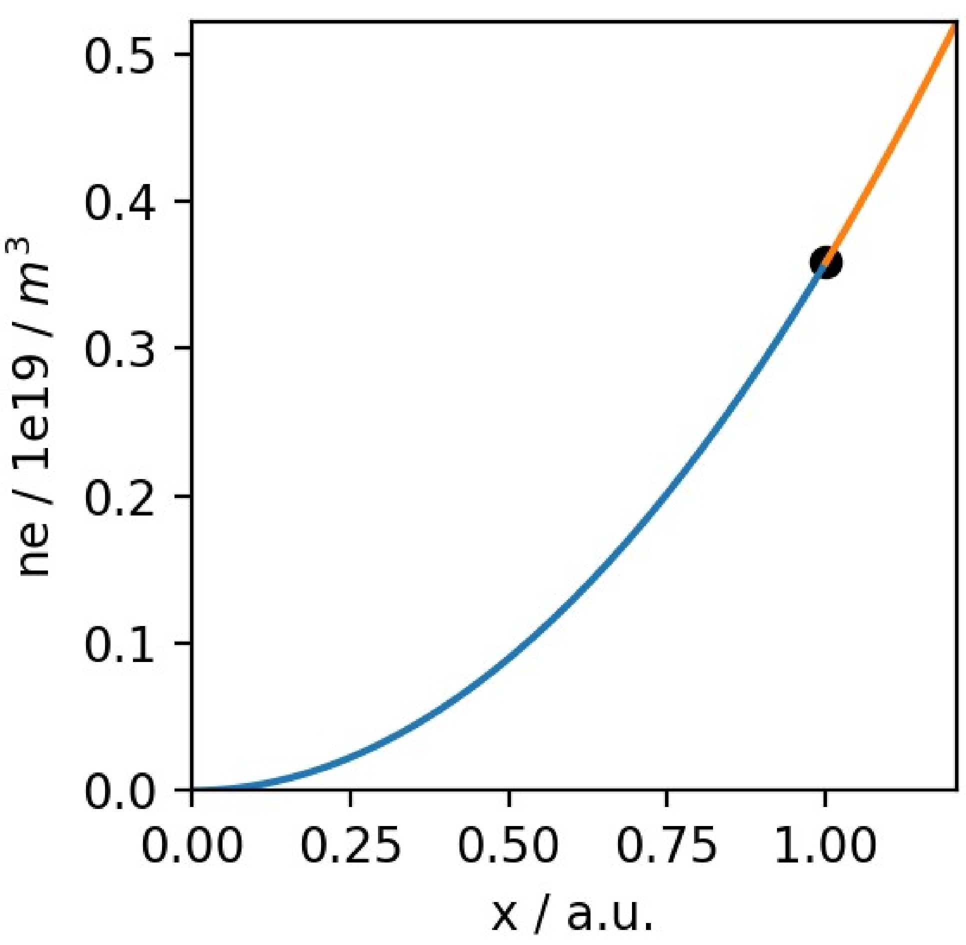


Initialisation with linear group delay

Group Delay

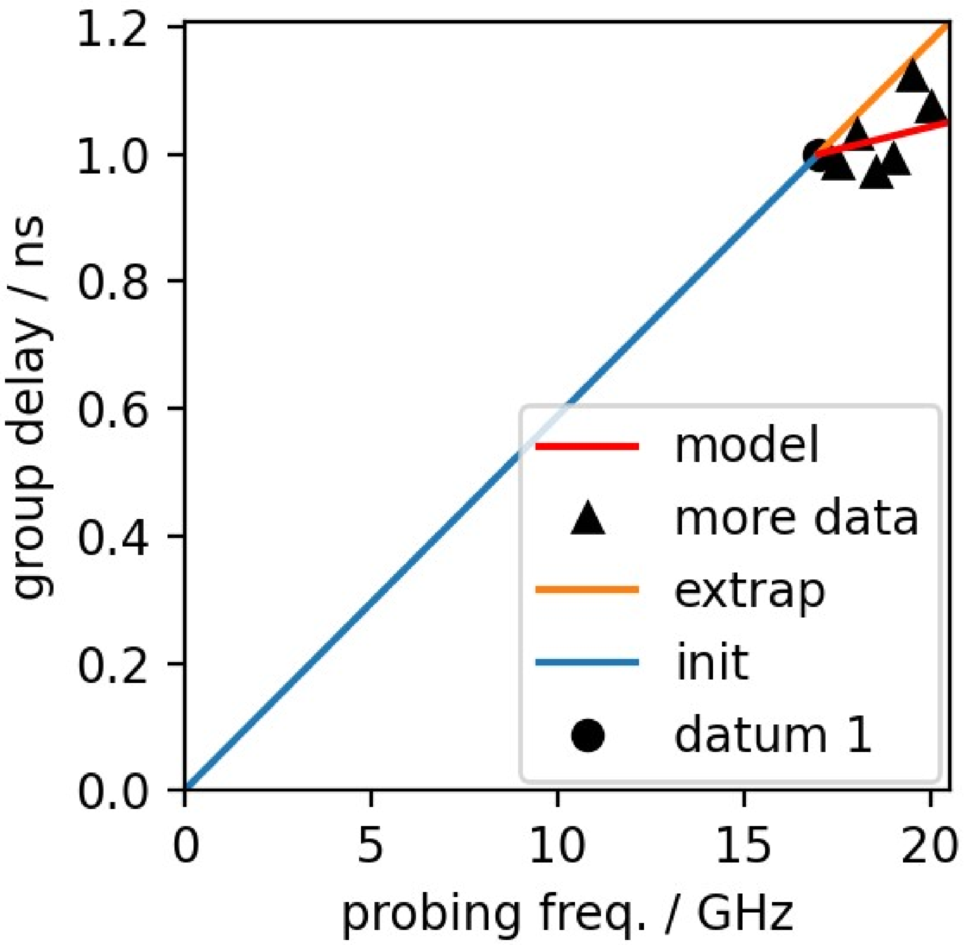


Density Profile

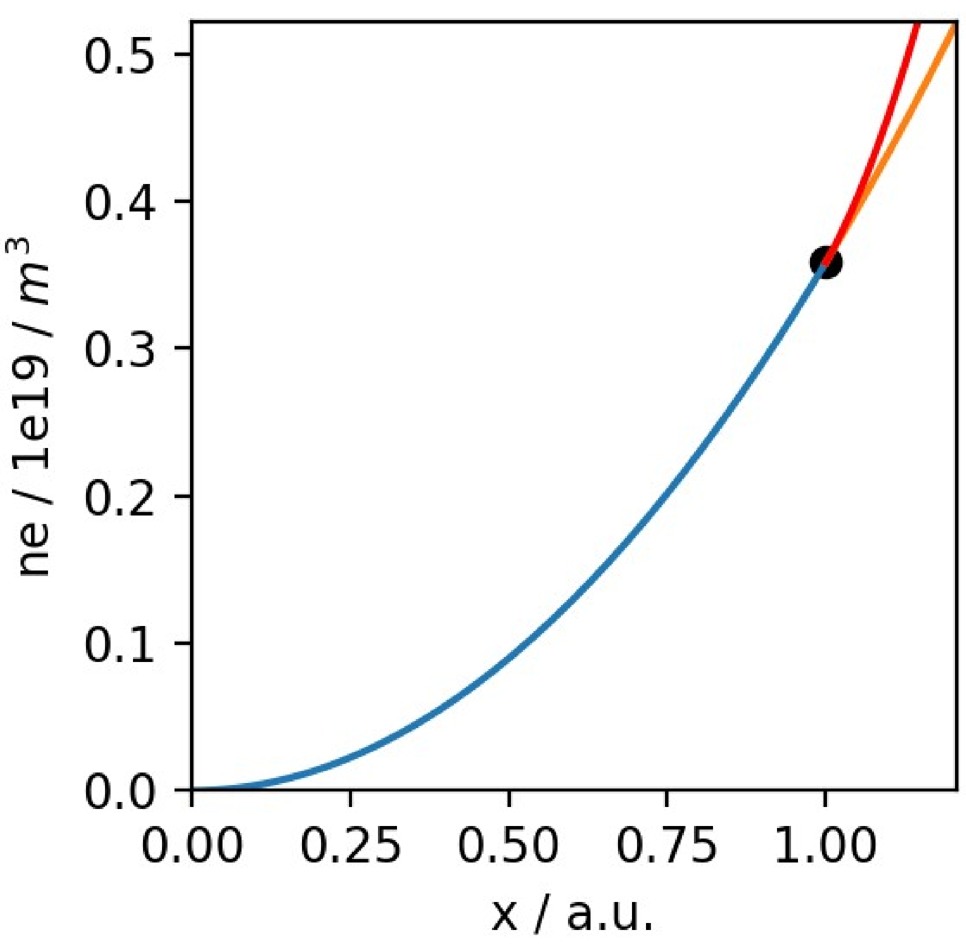


Initialisation with linear group delay

Group Delay

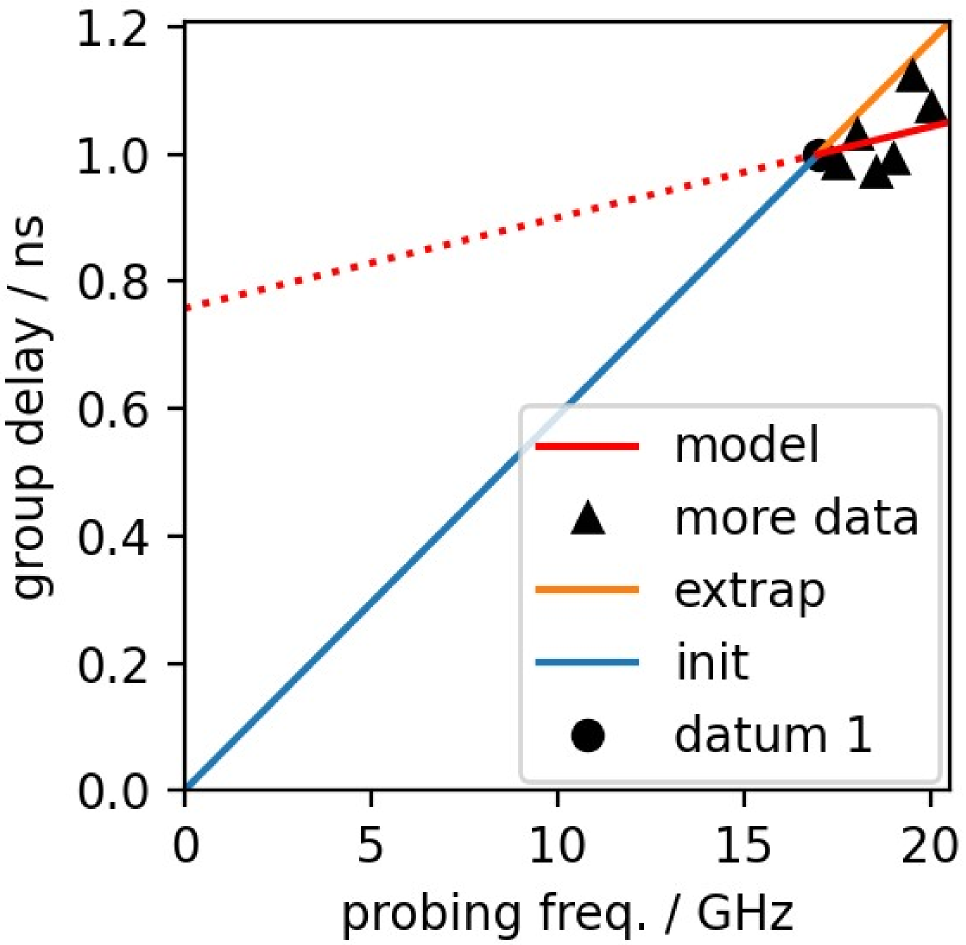


Density Profile

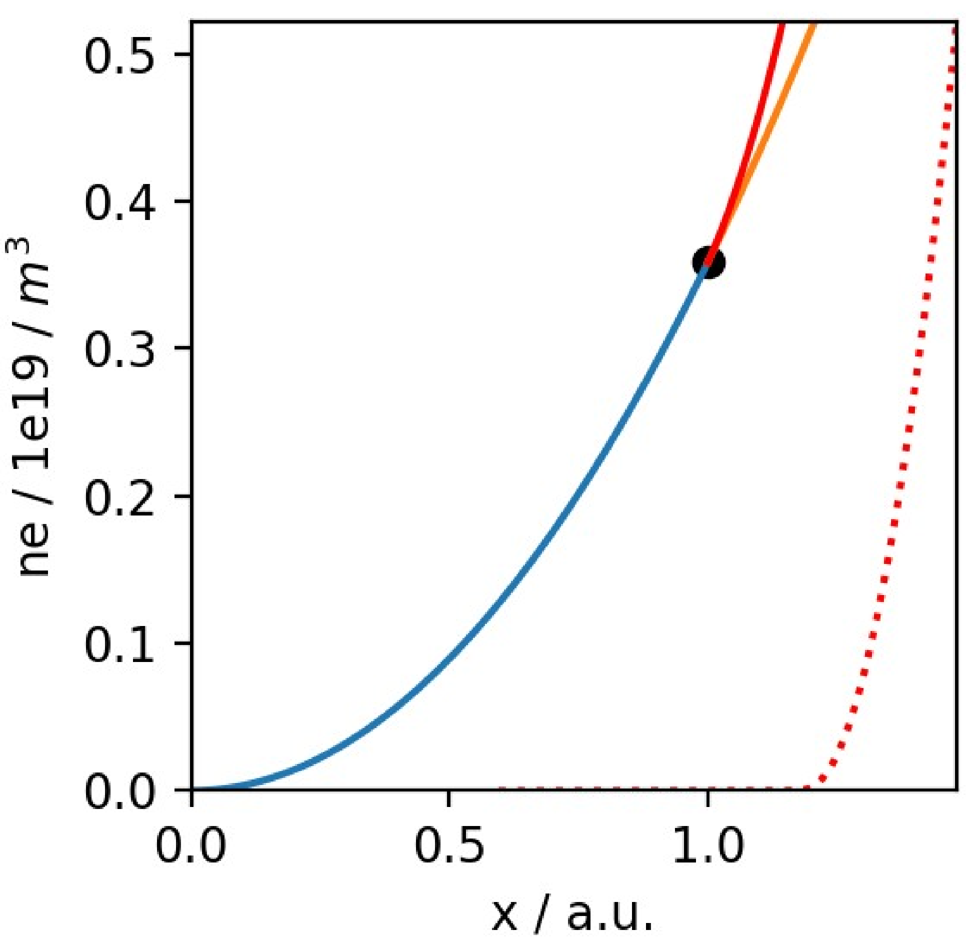


Initialisation with linear group delay

Group Delay



Density Profile

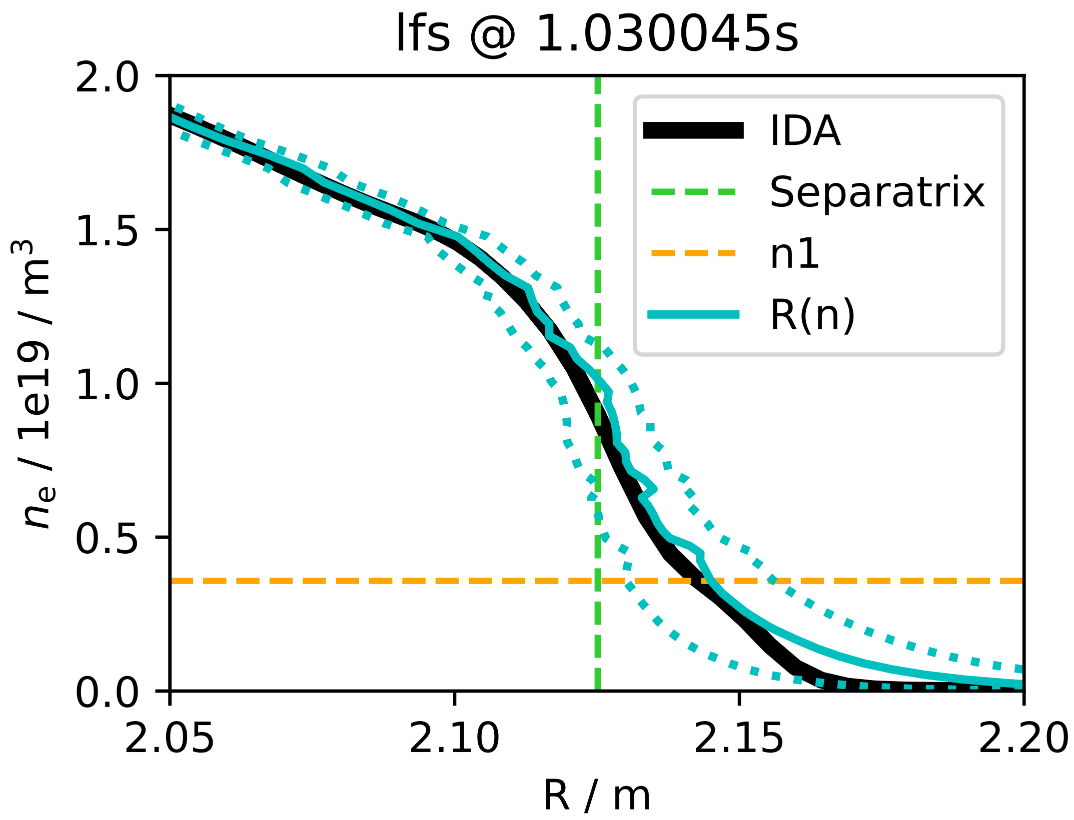


Application in Practice

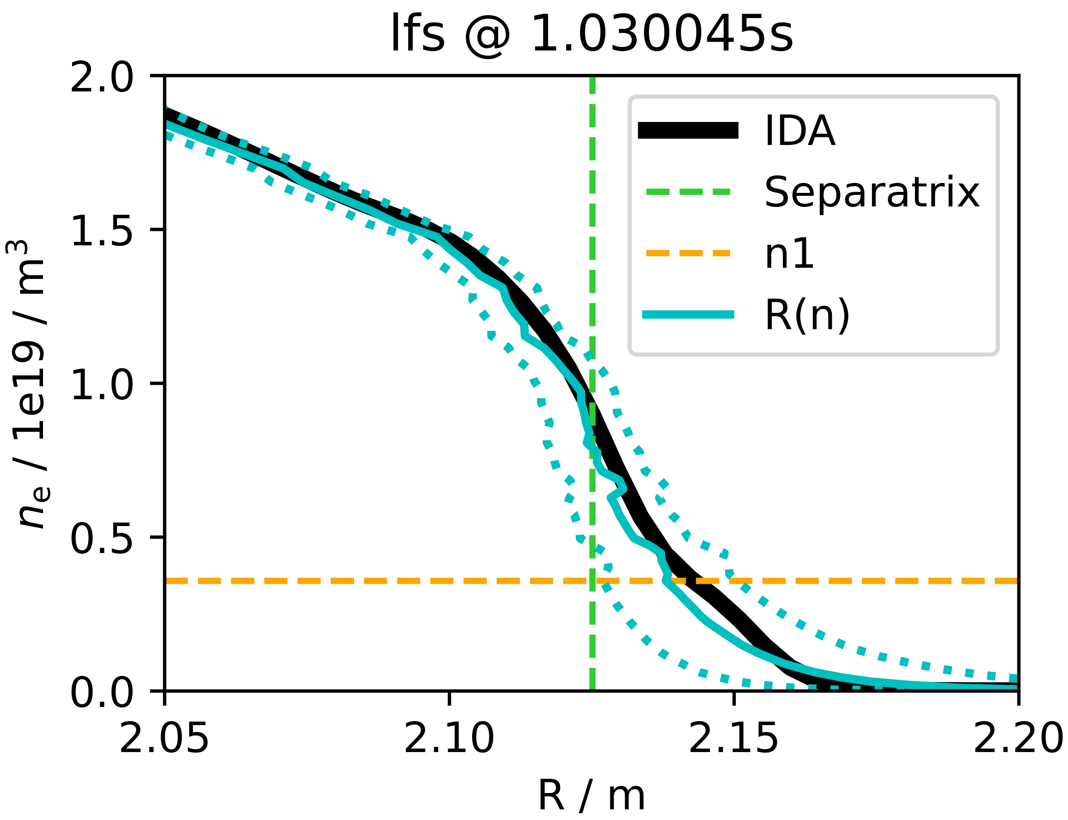
Comparison to IDA with Lithium Beam & Reflectometry

Prior λ_n = 10mm +- 0.5

35 initialisation frequencies

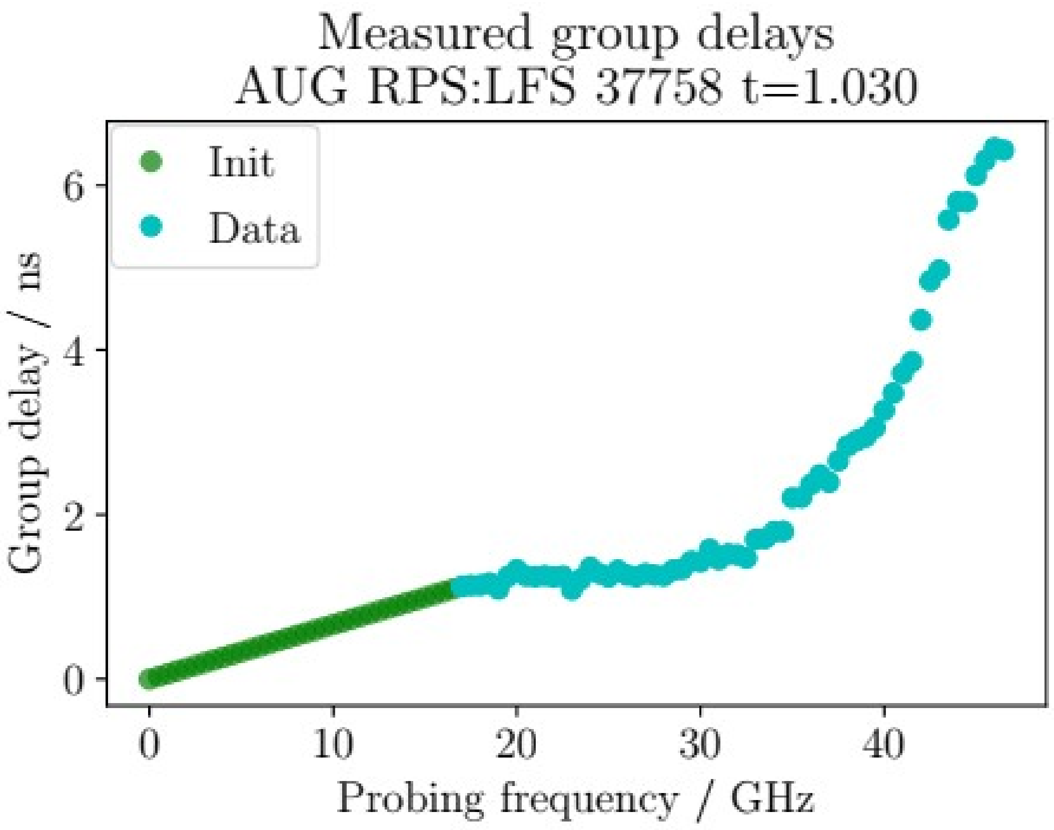


15 initialisation frequencies



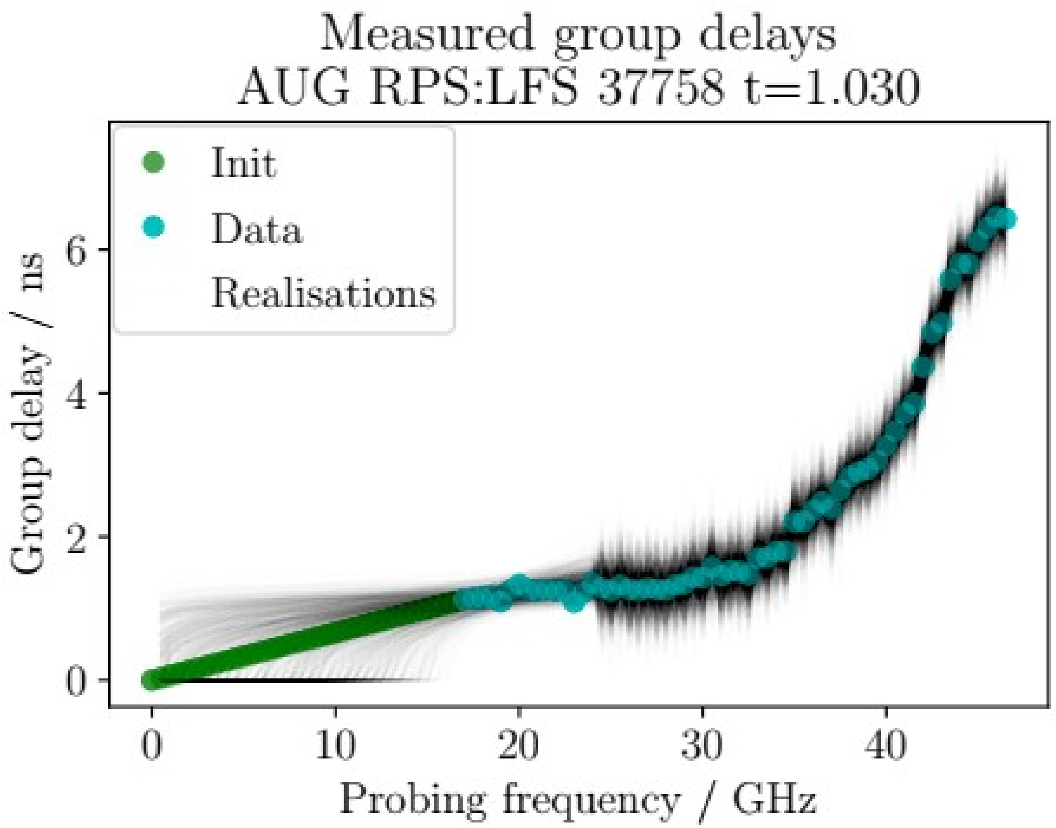
UQ for direct inversion: get n(R)

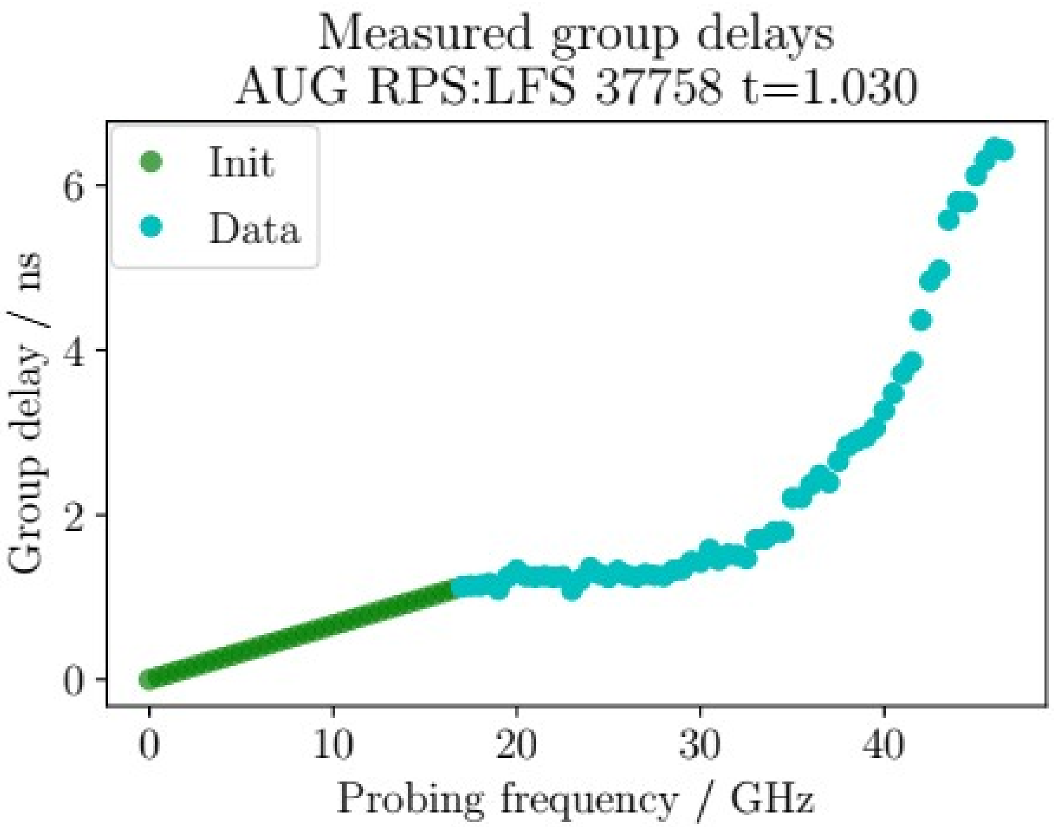
- What we would like to know:
  - R(n) with uncertainty
  - n(R) with uncertainty
  - dn(R)/dR (with uncertainty)



Inversion and Uncertainty Quantification

Get initialisation pdf and create *samples* for initialisation and data

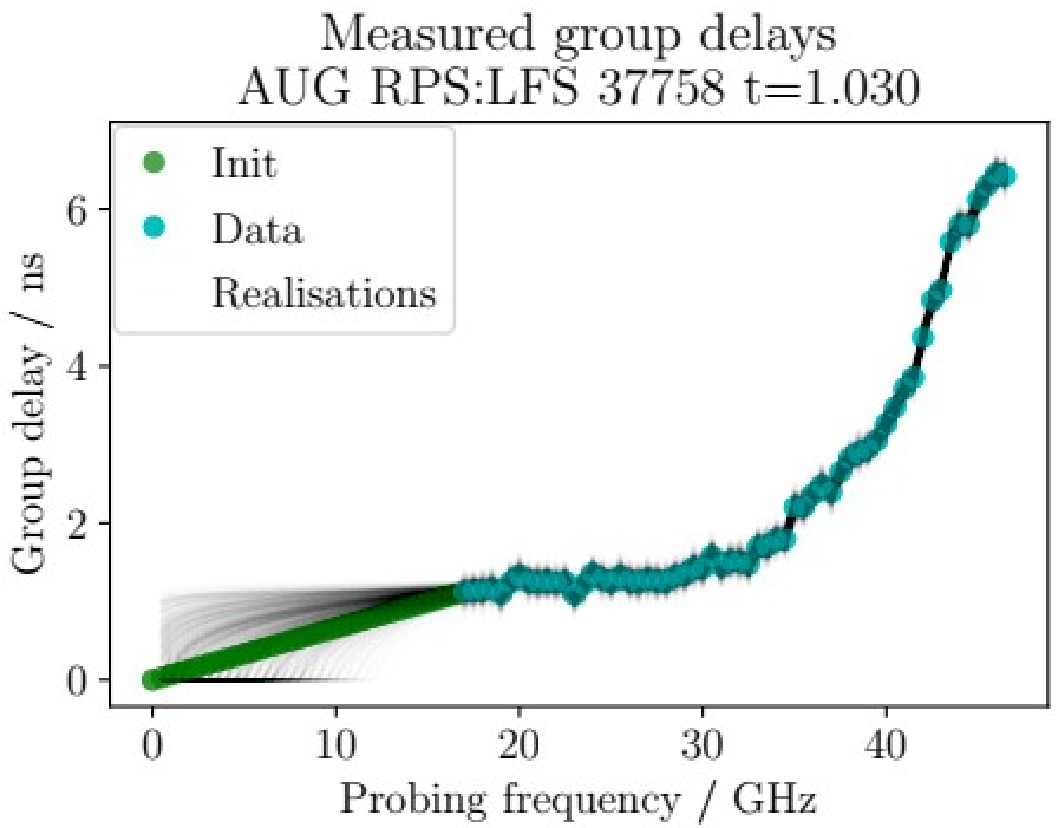


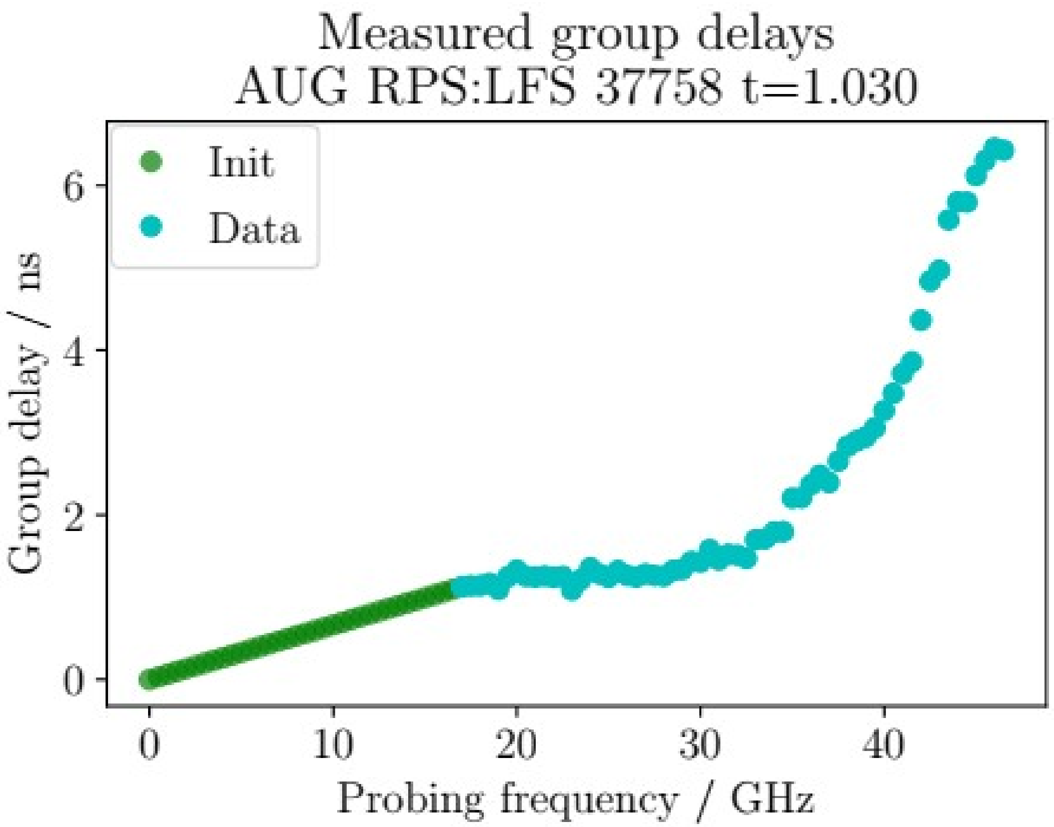


σ=0.3 ns

Inversion and Uncertainty Quantification

Get initialisation pdf and create *samples* for initialisation and data

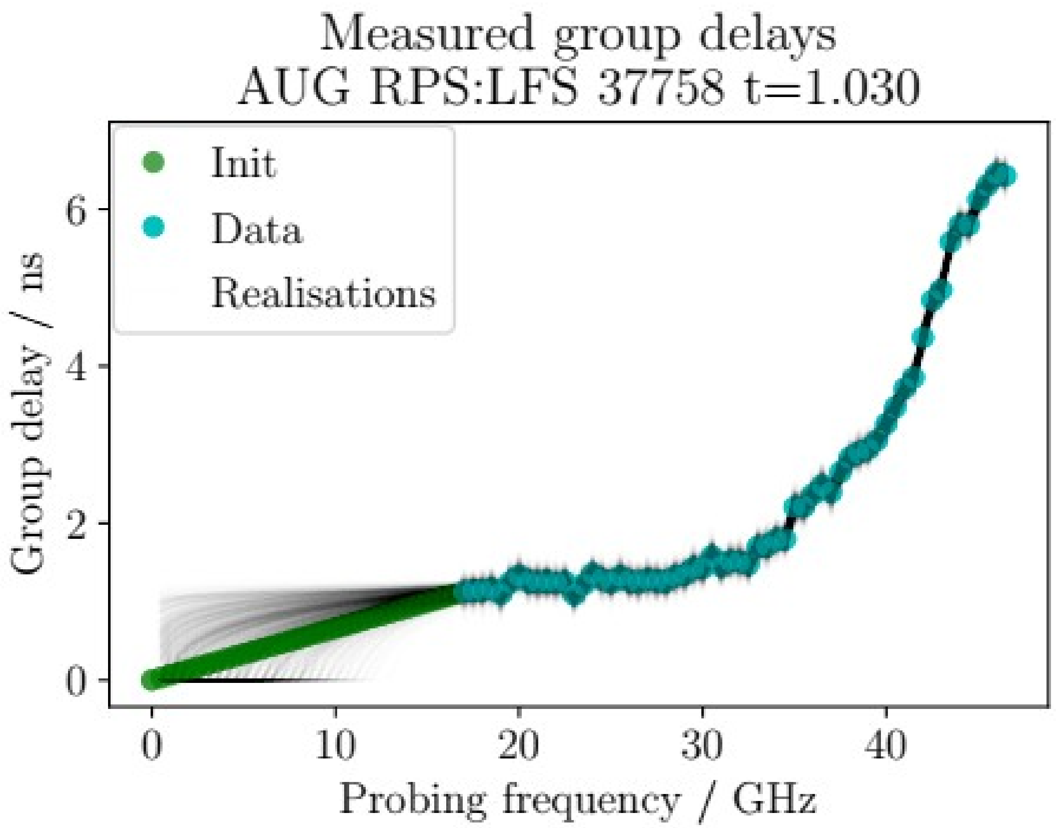


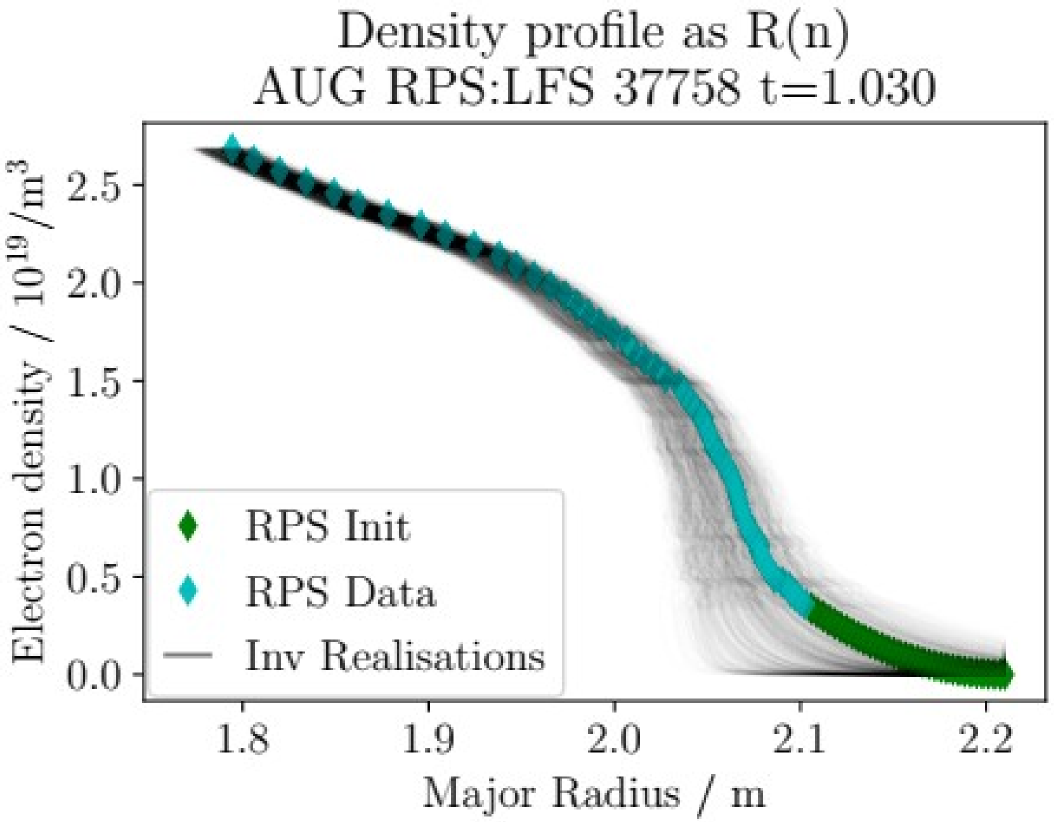


σ=0.1 ns

Inversion and Uncertainty Quantification

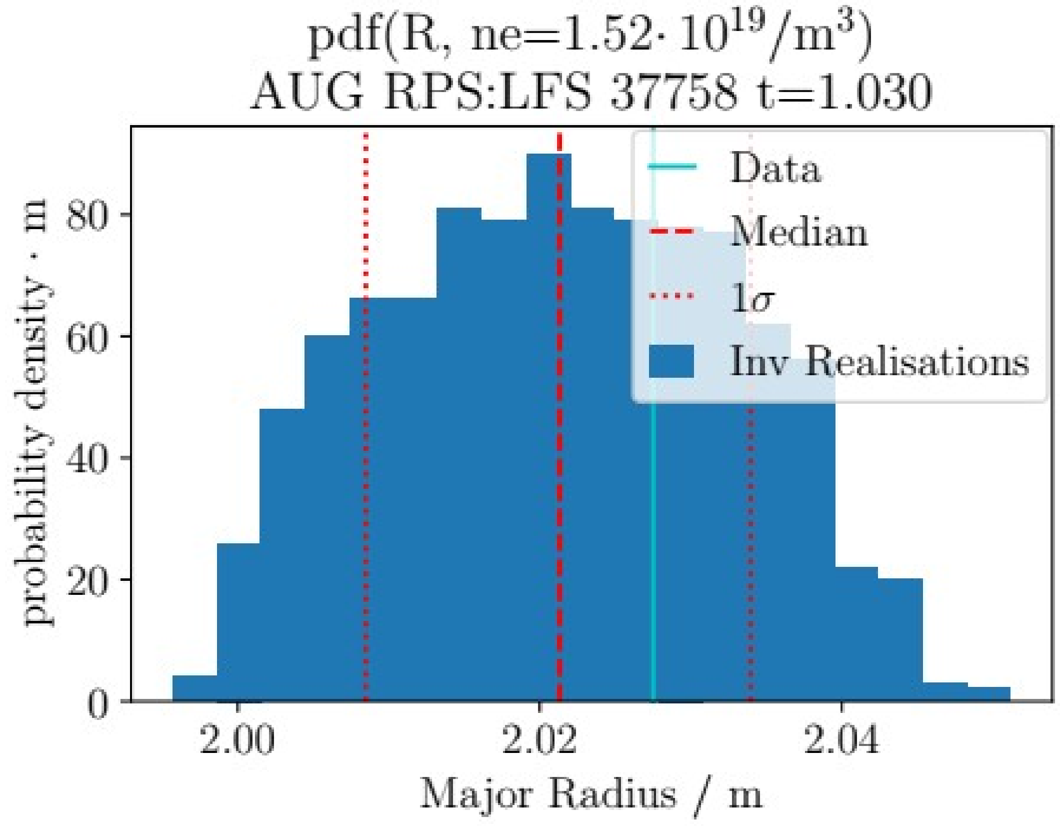
Apply (PWLD) inversion to realisations → distribution of R(n)

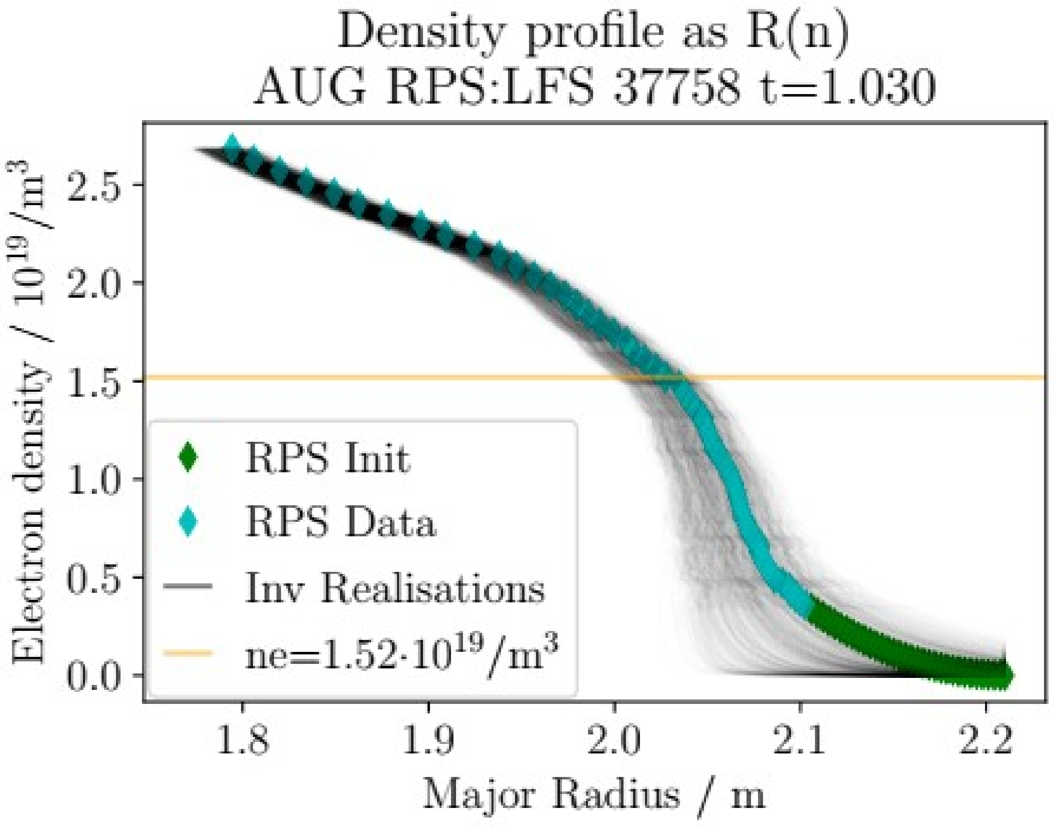




Inversion and Uncertainty Quantification

For an individual density, this means a distribution of locations → pdf with mean, median, std, etc

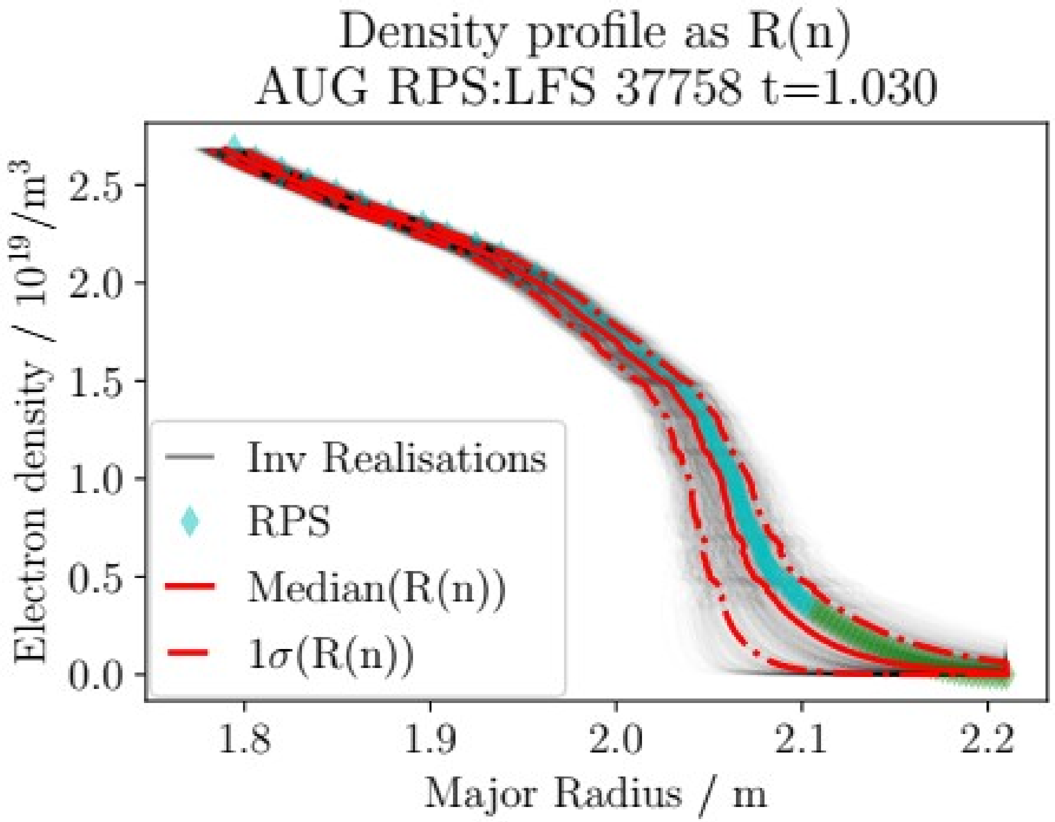


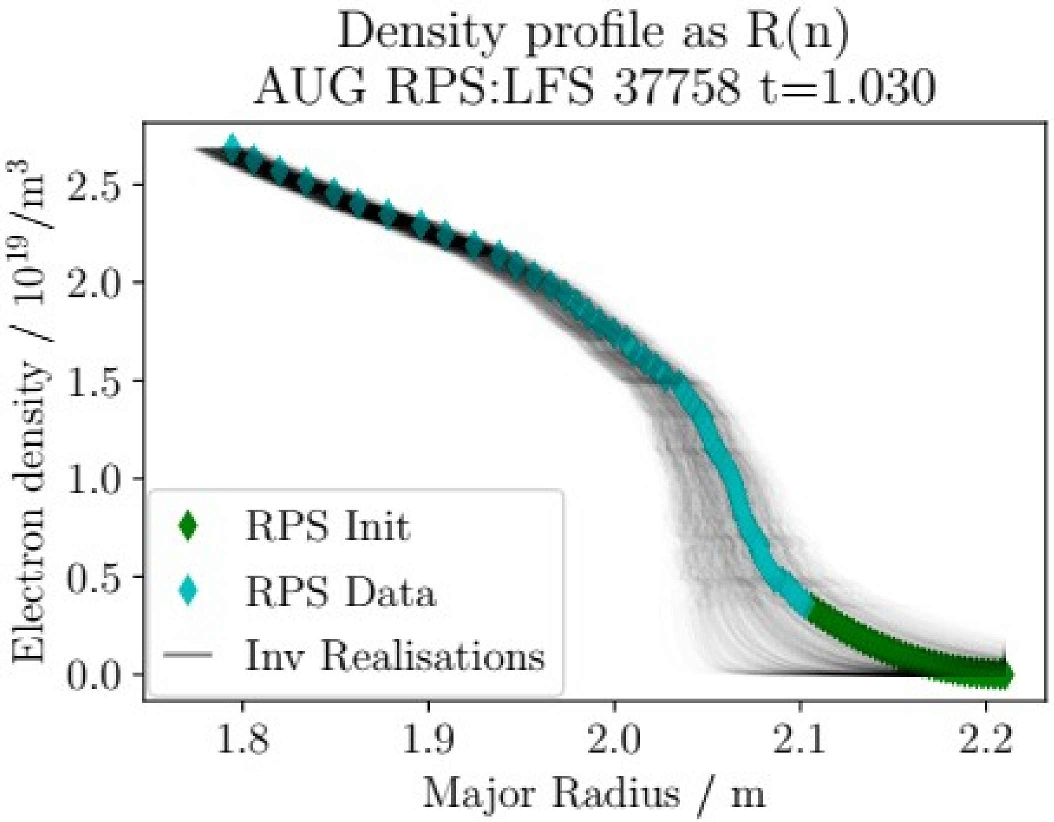


±1 σ = (50±34) percentile

Inversion and Uncertainty Quantification

If used for all densities → UQ for R(n)

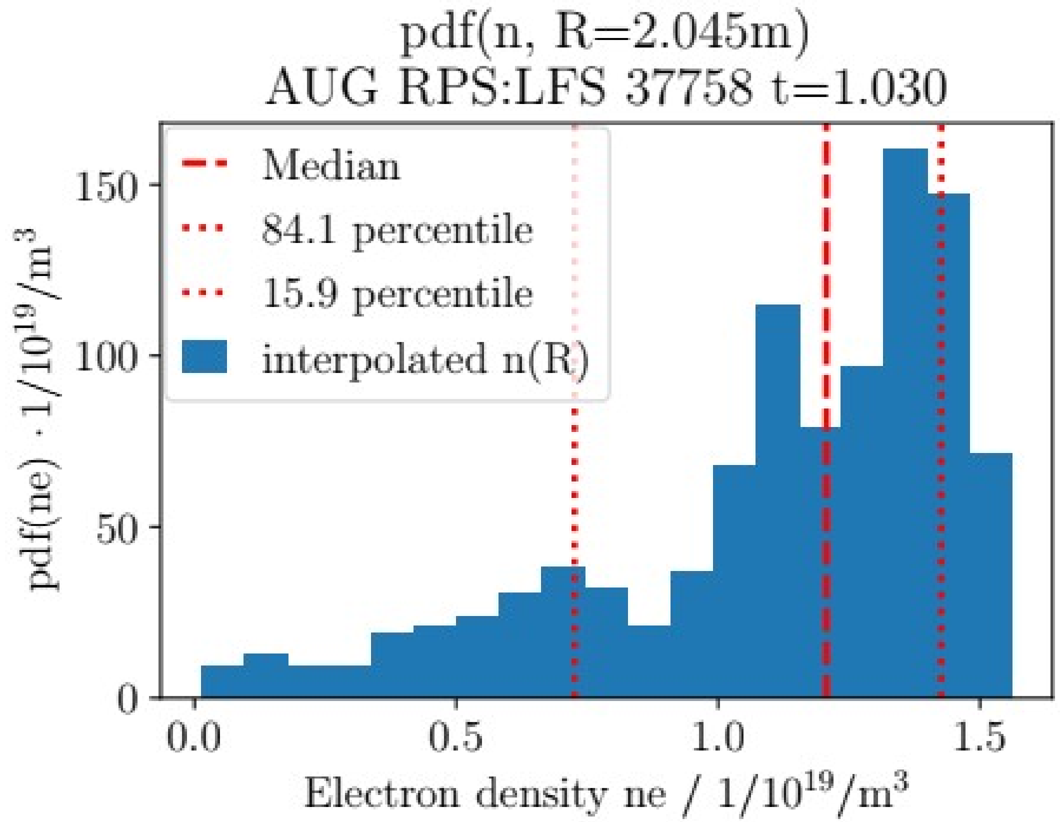


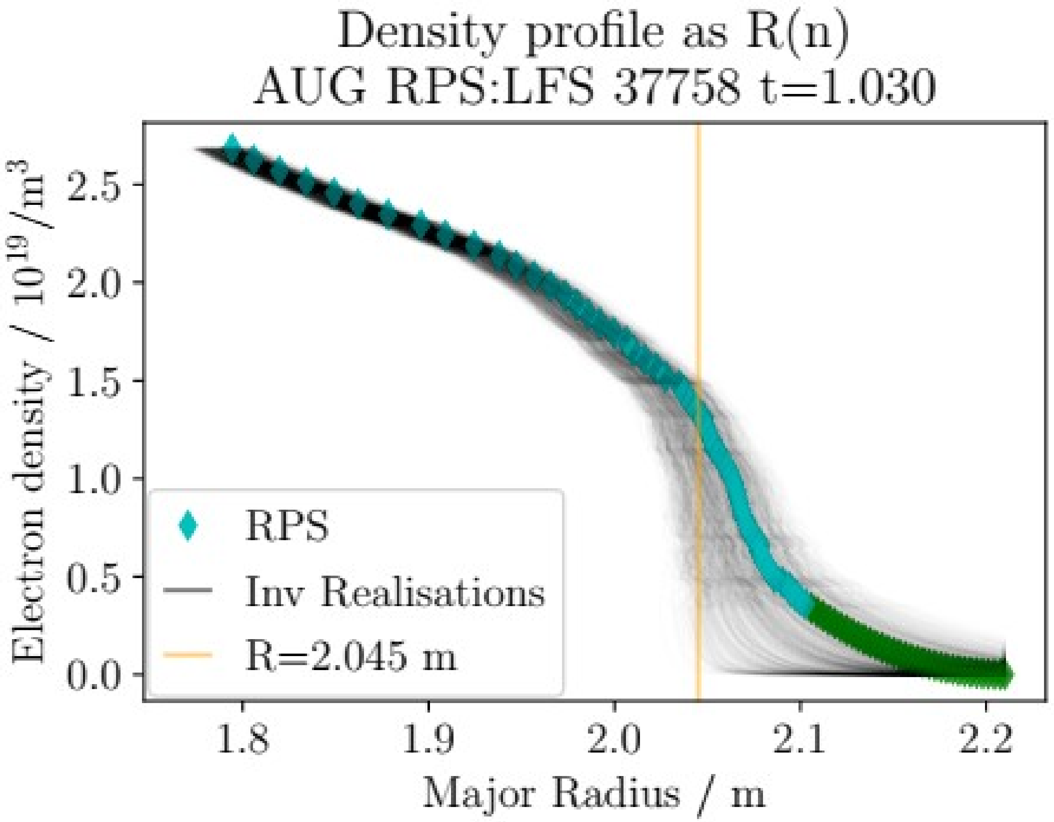


±1 σ = (50±34) percentile

Inversion and Uncertainty Quantification

Actually samples are 2D distribution in location and density → look at one location for pdf n(R)

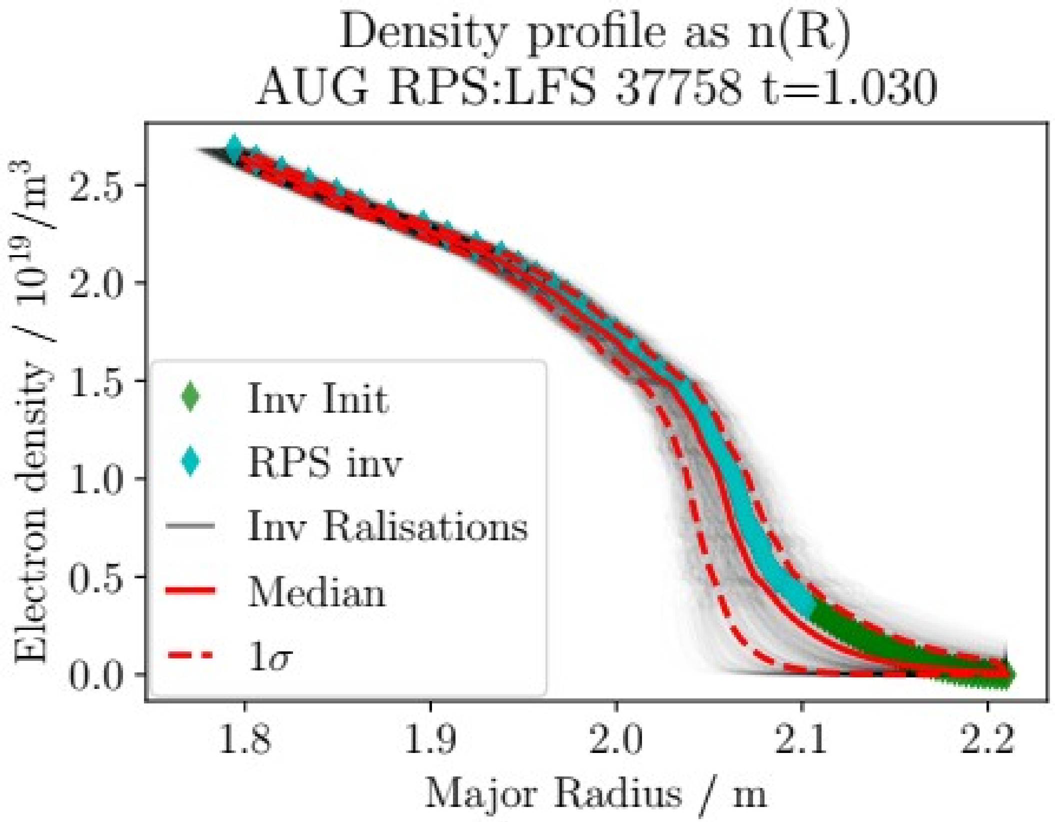


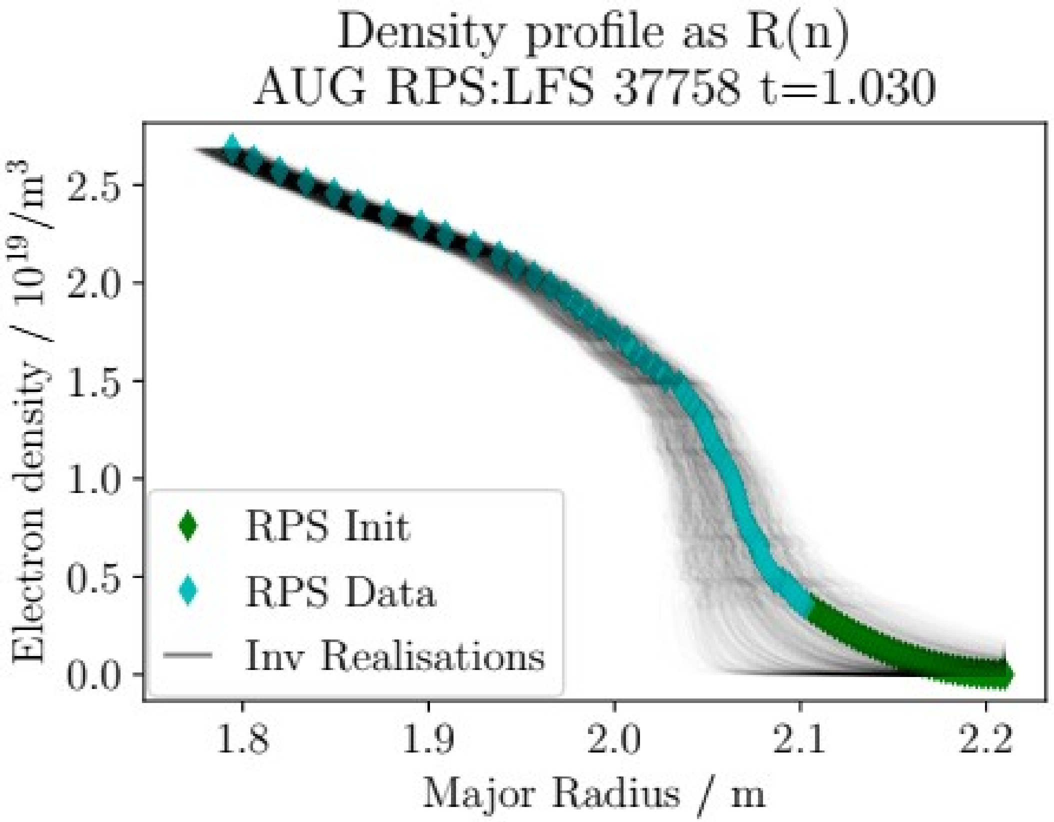


±1 σ = (50±34) percentile

Inversion and Uncertainty Quantification

Evaluate densities on location grid → n(R) with UQ

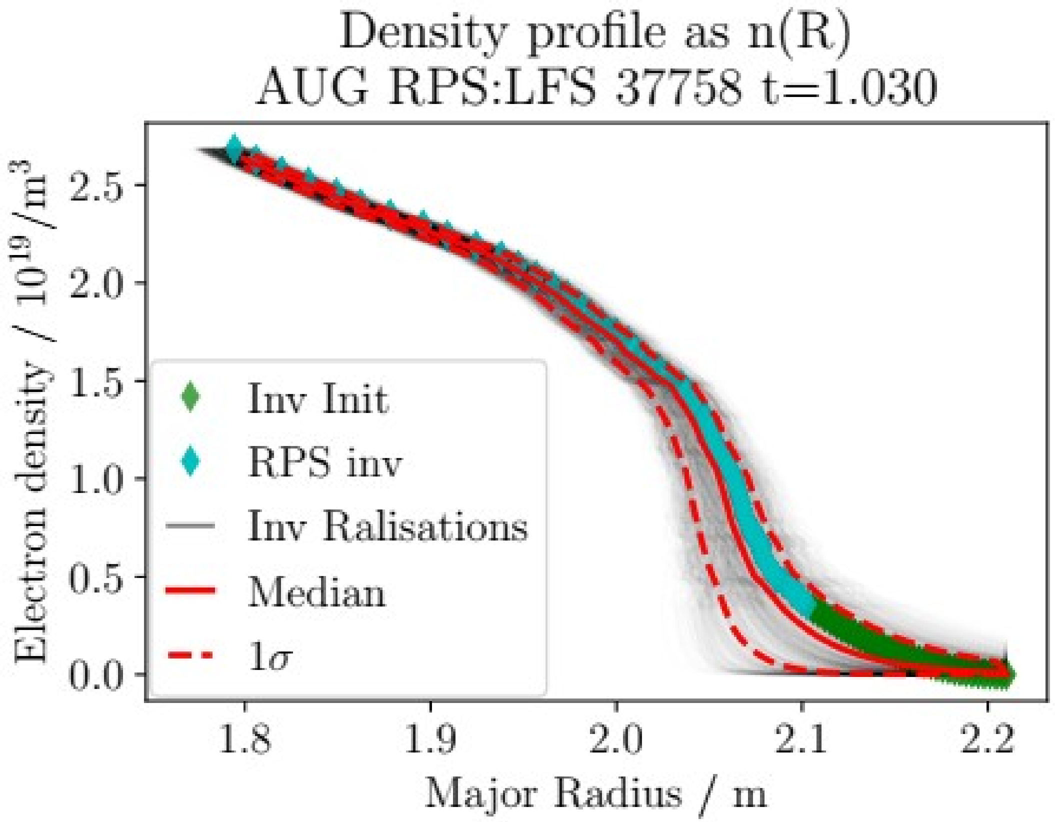


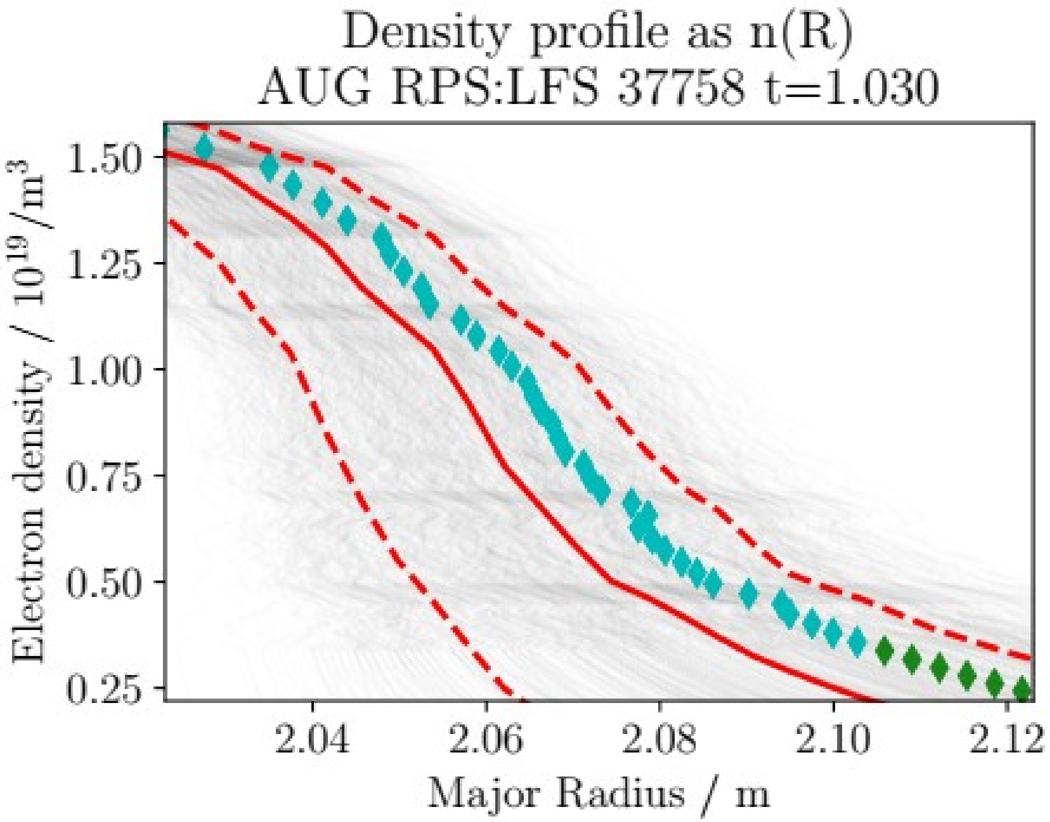


±1 σ = (50±34) percentile

Inversion and Uncertainty Quantification

Median n(R) is function with unique values at desired grid, unlike R(n)

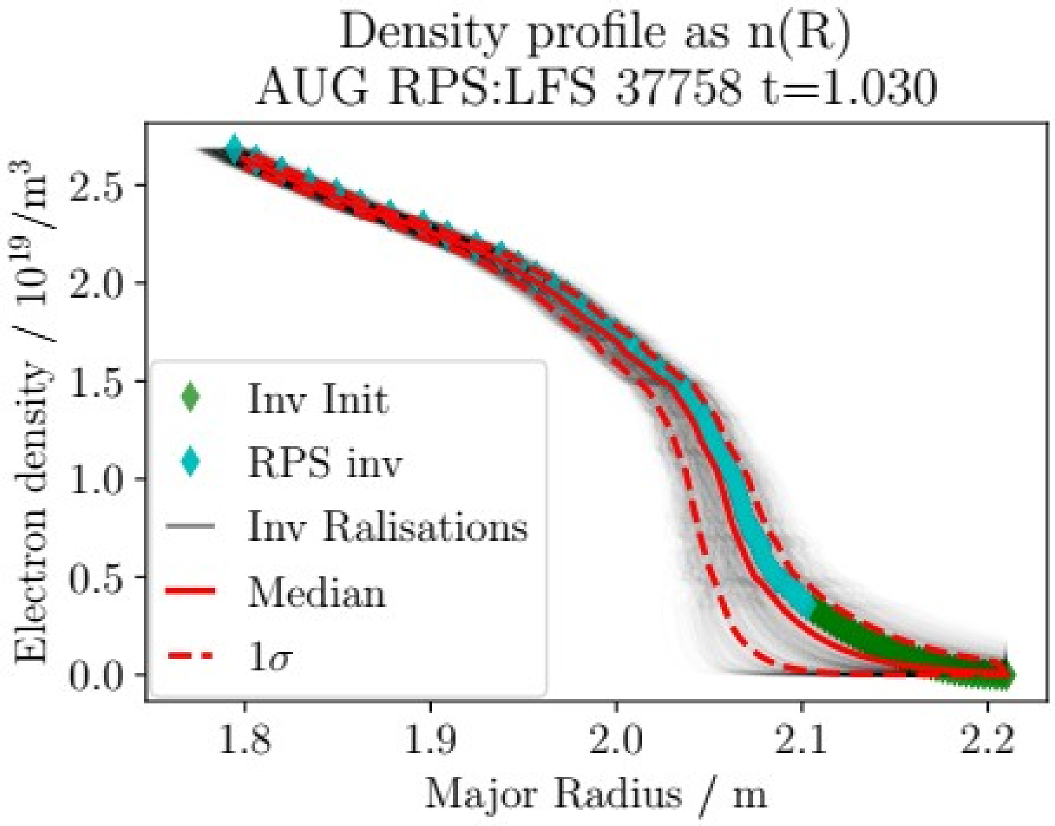


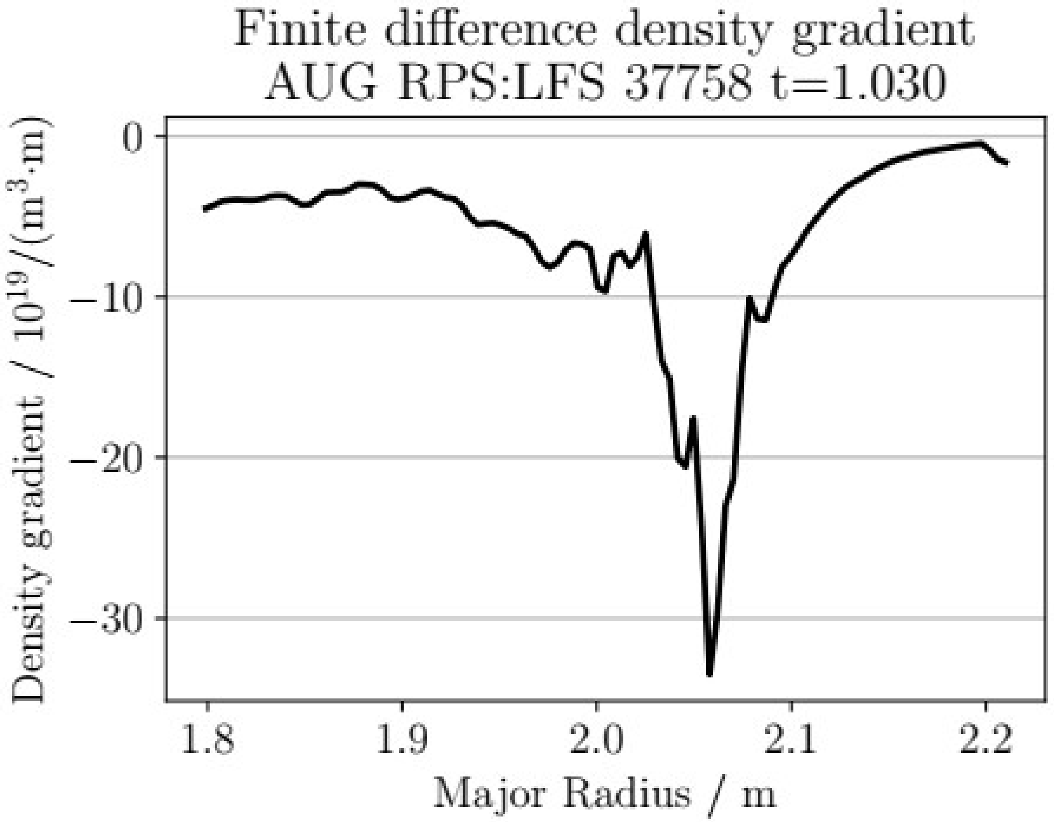


$\pm 1\ \sigma = (50 \pm 34)$ percentile

Inversion and Uncertainty Quantification

Density gradient can be calculated





±1 σ = (50±34) percentile

Inversion and Uncertainty Quantification

Note: n(R) and R(n) essentially the same