Density profile measurements from the developed FMCW Reflectometry system for Aditya-U tokamak.

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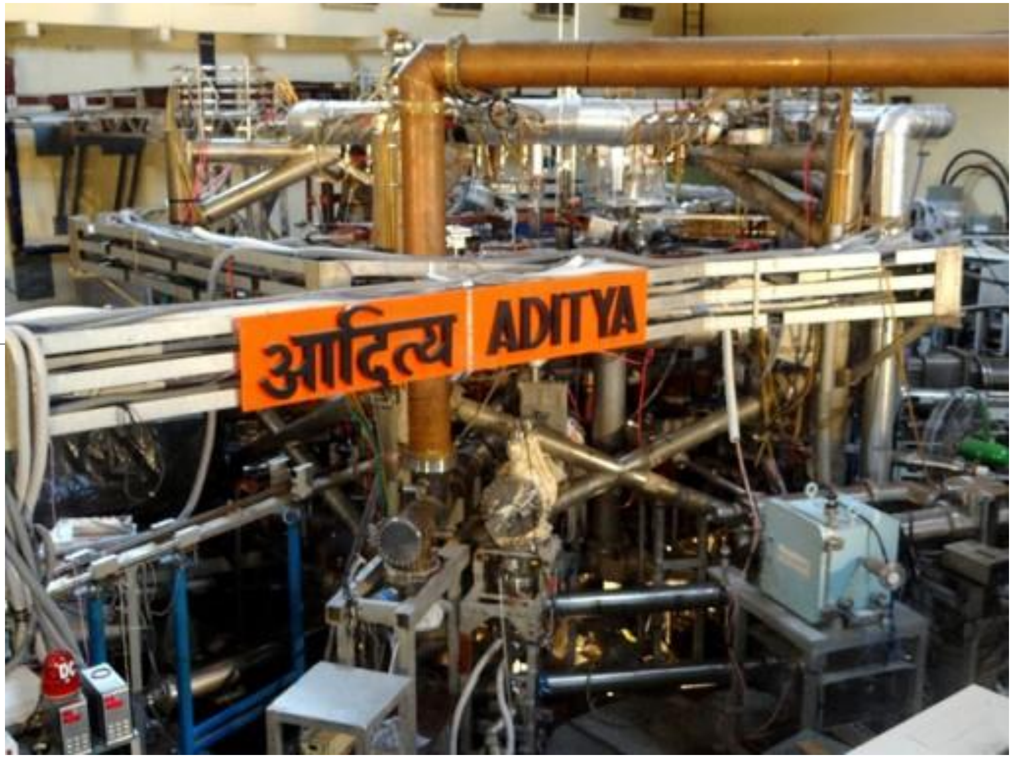
Overview

1. Aditya-U Tokamak
2. Reflectometry System on Aditya-U
3. Signal processing
4. Aditya-U profile measurement results
5. Edge Turbulence simulations using CHM equation
6. Conclusion and Future work

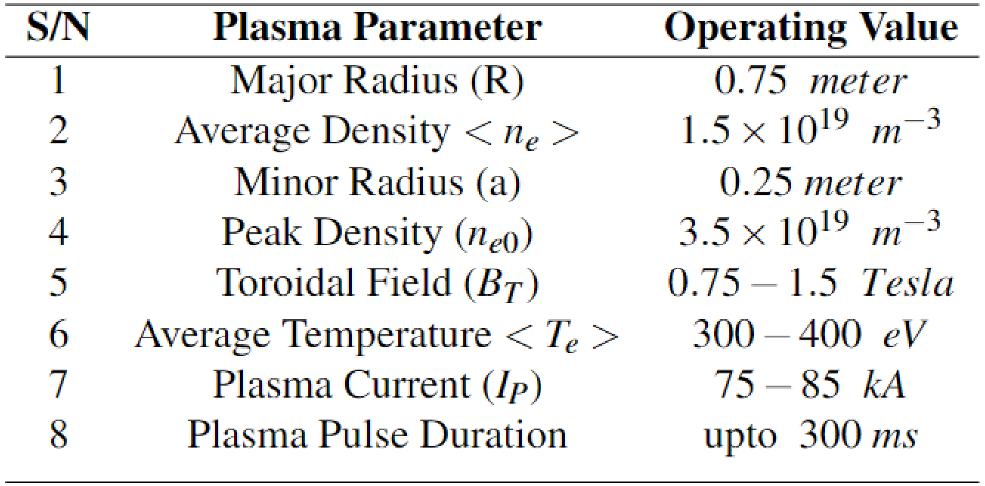
Goals for the Diagnostic

- Edge pedestal region controls the global plasma confinement as large gradients lead to various instabilities where particles and energy are lost to the SOL region.
- Measurements of the pedestal radially and in time are important to detect H-mode and study plasma transport.
- Design and develop a FMCW reflectometer and its associated subsystems.
- Measure plasma density profiles for Aditya-U tokamak
- Indigenous development of Reflectometry on Aditya-U opens up a path for future tokamaks including for fluctuation measurements.

Aditya-Upgrade Tokamak



- Upgradation involved changing the square shaped vacuum vessel to circular shaped one.
- This allowed installation of additional coils to enable formation of shaped plasmas.
- Typically densities upto $n_e = 3 \times 10^{19}\ m^{-3}$with a duration of 350 ms are being achieved.
- FMCW Reflectometry to measure plasma density profiles was established.
- Two channels are operational in K-Band (26-40 GHz) and Ka-Band (18-28 GHz), looking at the horizontal mid-plane.



S/N	Plasma Parameter	Operating Value
1	Major Radius (R)	0.75 *meter*
2	Average Density $< n_e >$	$1.5 \times 10^{19}\ m^{-3}$
3	Minor Radius (a)	0.25 *meter*
4	Peak Density (n_{e0})	$3.5 \times 10^{19}\ m^{-3}$
5	Toroidal Field (B_T)	$0.75 - 1.5$ *Tesla*
6	Average Temperature $< T_e >$	$300 - 400$ *eV*
7	Plasma Current (I_P)	$75 - 85$ *kA*
8	Plasma Pulse Duration	upto 300 *ms*

Operating Frequencies

Assuming a density profile with shaping parameters a and b,

$$n_e(r) = 0.1n_{e0} + 0.9n_{e0}\left[1-\left(\frac{r}{r_{max}}\right)^a\right]^b$$

Reflectometer can measure ~ 80% of Aditya-U plasma

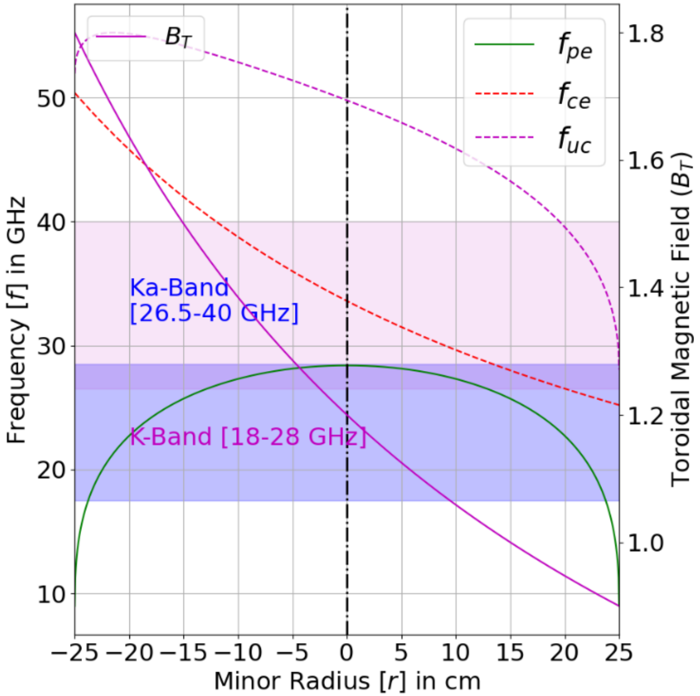
Thickness of reflection / Error in radial location is

For $\rho = \frac{r}{r_{max}} = 0.2$

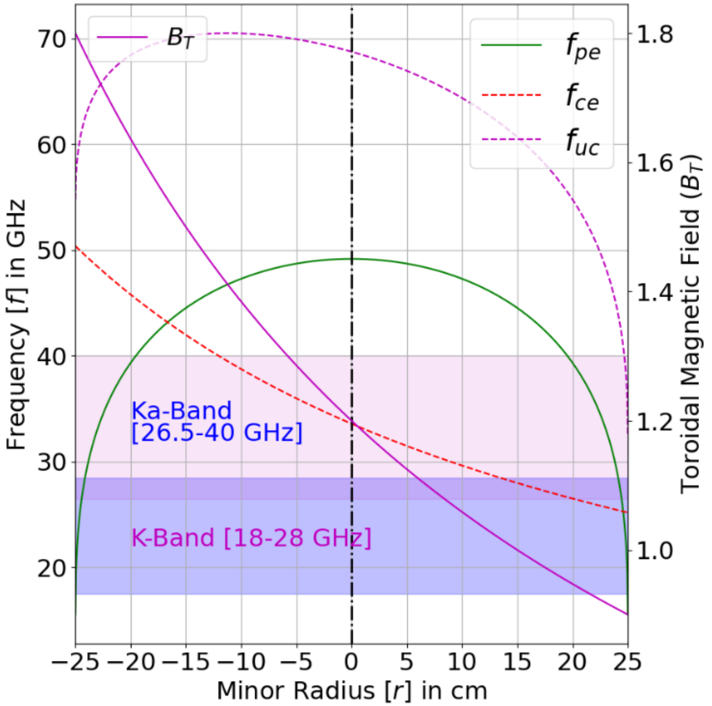
ΔZ = 12.6 mm @ 18 GHz

ΔZ =   9.6 mm @ 40 GHz

$\mathrm{B_T}$ = 1.2 T, $\mathrm{n_e} = 1 \times 10^{19}\ m^{-3}$



$\mathrm{B_T}$ = 1.2 T, $\mathrm{n_e} = 3 \times 10^{19}\ m^{-3}$



Conditions for Reflection in Tokamak Plasma

Plasma is assumed to be,

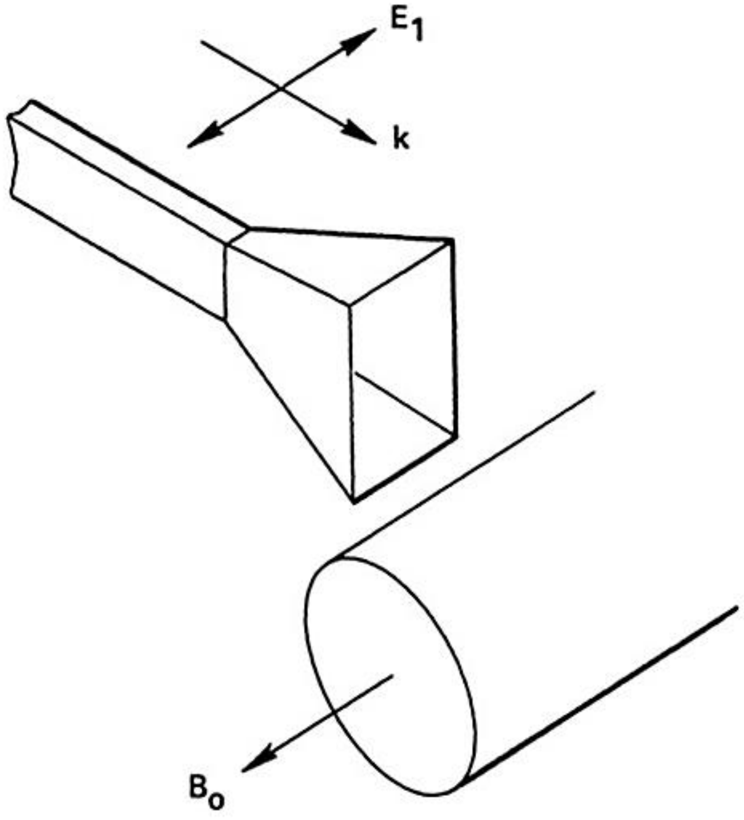
- Cold ($v_{thermal} = 0$)
- Small amplitude waves
- Unbounded and Homogeneous
- Magnetic field $B_0 = B(r)$ only.

If $\vec{k} \perp B_0$ *and* $\vec{k} \perp \vec{E}$ then,

O-Mode: $\vec{E} || B_0$

X-Mode: $\vec{E} \perp B_0$

O-Mode of Propagation

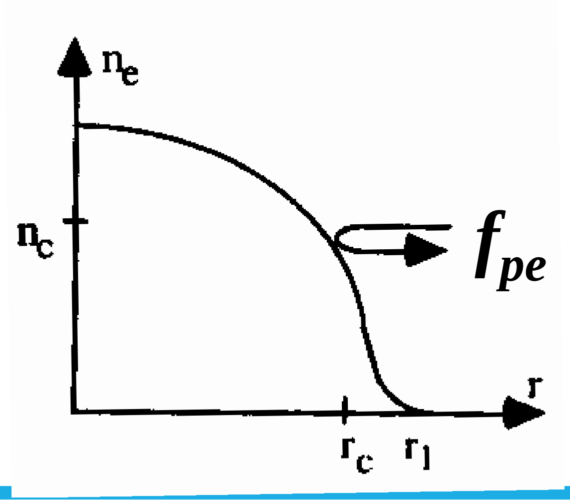


O-Mode $\eta_O^2 = \left[1 - \frac{f_{pe}^2}{f^2}\right]$

X-Mode $\eta_X^2 = \left[\left(1 - \frac{f_{pe}^2}{f^2}\right)\left(\frac{f^2 - f_{pe}^2}{f^2 - f_{pe}^2 - f_{ce}^2}\right)\right]$

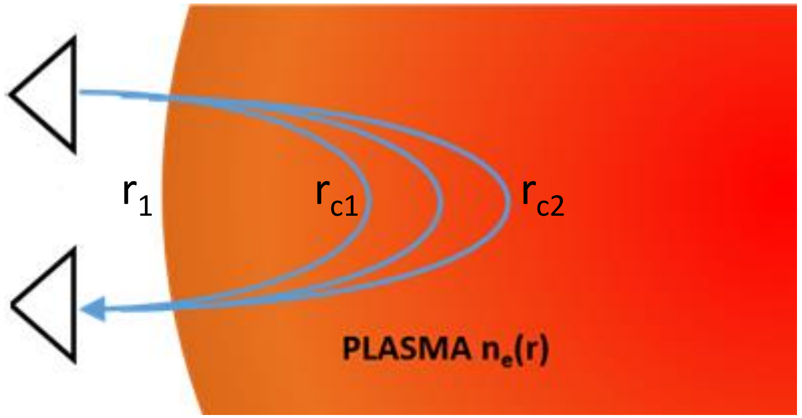
Reflection occurs when $\eta \rightarrow 0$

$f = f_{pe}$ $n_e = f_{pe}^2\, 4\, \frac{\pi m_e \epsilon_0}{e^2}$



Geometrical Optics Approximation and its Validity

A phase shift is introduced in the reflected EM wave due to variation in the refractive index of the plasma along the path of the wave and its subsequent reflection.



$$\phi(f) = \frac{4\pi f}{c}\int_{r(f)}^{r_0} \eta(r,f)dr - \frac{\pi}{2}$$

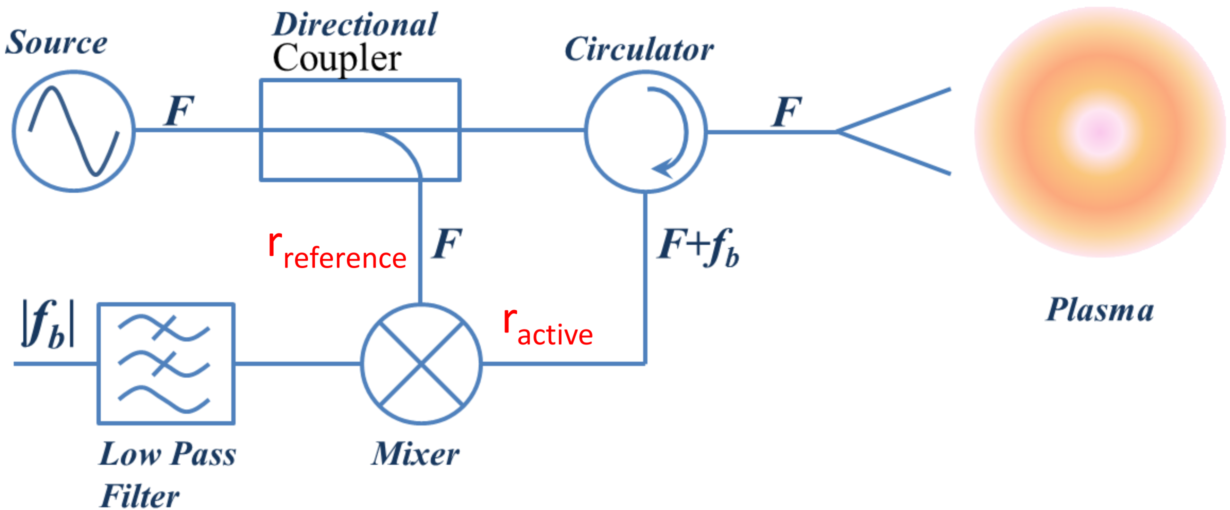
Valid only when the dielectric constant $\varepsilon_r(r)$ varies slowly over the wavelength λ of the probing wave

$$\lambda(r)\frac{d\varepsilon_r(r)}{dr} \ll \varepsilon_r(r)$$

In terms of the density gradient scale length L_{ne}

$$G = \frac{\lambda(r)}{L_{n_e}} \ll 1 \qquad L_{n_e} = \frac{1}{n_e}\left(\frac{dn_e}{dr}\right)^{-1}$$

Measurement of the phase shift introduced in the reflected wave from the plasma is the primary goal of the diagnostic. Density is then calculated from the known probing frequency ***f***.



$$r_{target} = antennae\ mouth\ to\ plasma\ cut-off\ (r_c)$$
$$r_{sys} = dispersion\ inside\ microwave\ circuit$$

$$\phi = k(r)r$$

$$r_a = \frac{\phi_a}{2\pi}\lambda \qquad r_{ref} = \frac{\phi_{ref}}{2\pi}\lambda$$

$$\phi = k(r)\,(r_a - r_{ref})$$

$$d\phi = (r_a - r_{ref}) \cdot dk(r) + k(r) \cdot d(r_a - r_{ref})$$

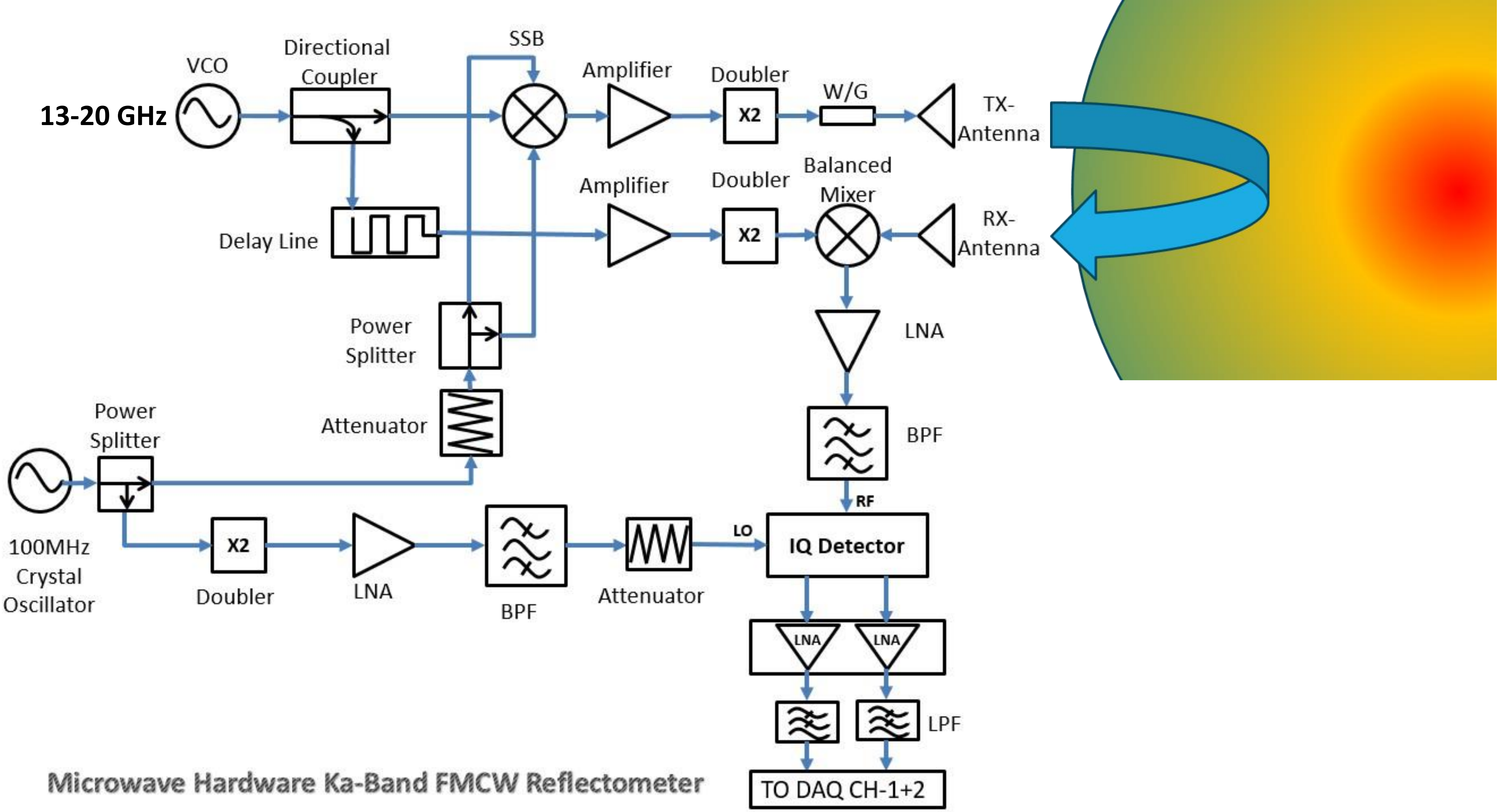
$$\frac{d\phi}{dt} = \frac{2\pi}{c}\left[(r_a - r_{ref})\frac{df}{dt} + f\,\frac{d(r_a - r_{ref})}{dt}\right]$$

$$r_a = r_{sys} + r_{target}$$

$$r = (r_{sys} + r_{target}) - r_{ref}$$

$$2\pi f_b = \frac{d\phi}{dt} \qquad \tau_g = f_b\frac{df}{dt}$$

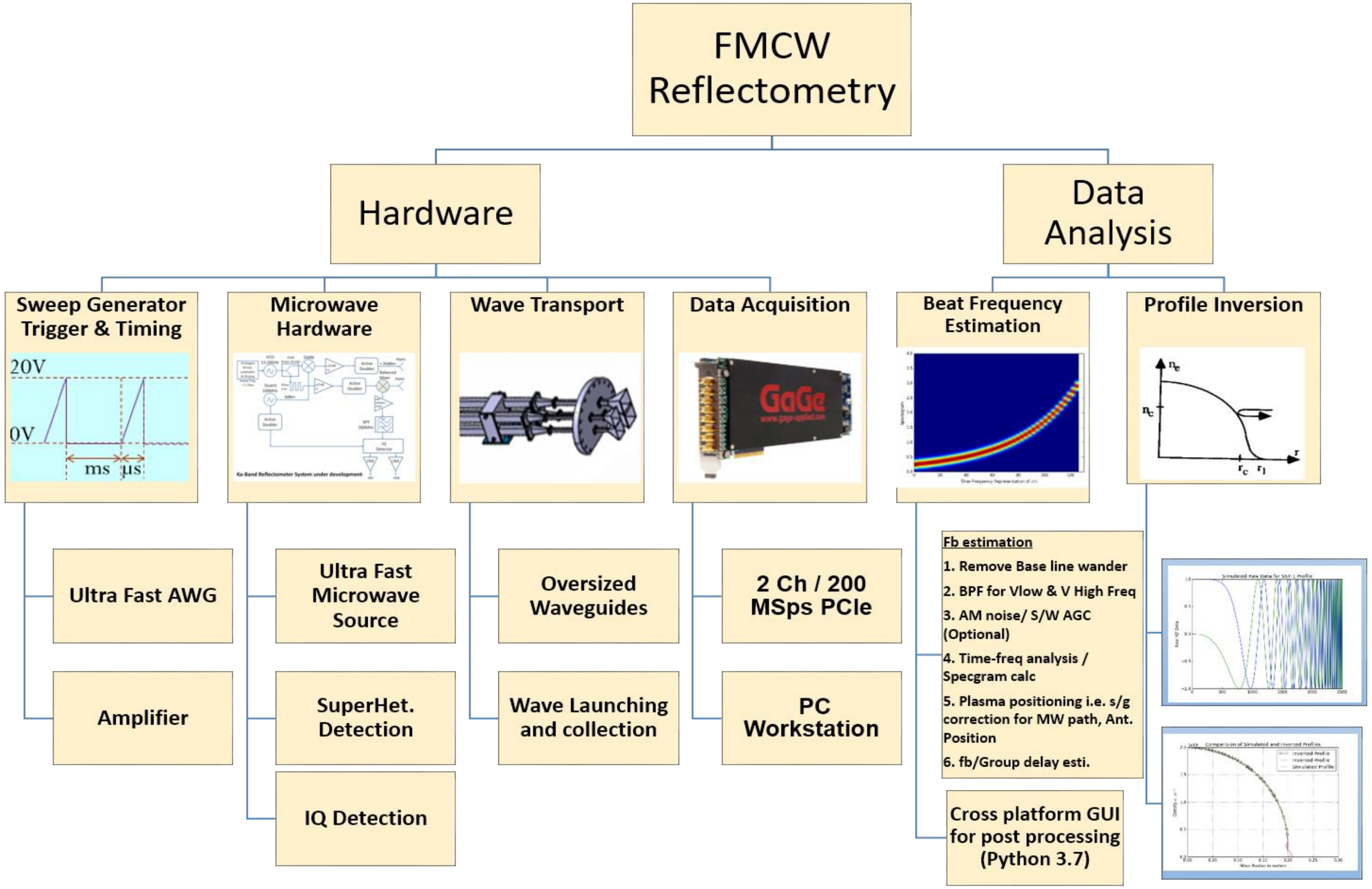
$$f_b = \frac{r_{target}}{c}\frac{df}{dt} + \underbrace{\frac{(r_{sys} - r_{ref})}{c}\frac{df}{dt}}_{\text{constant offset}}$$



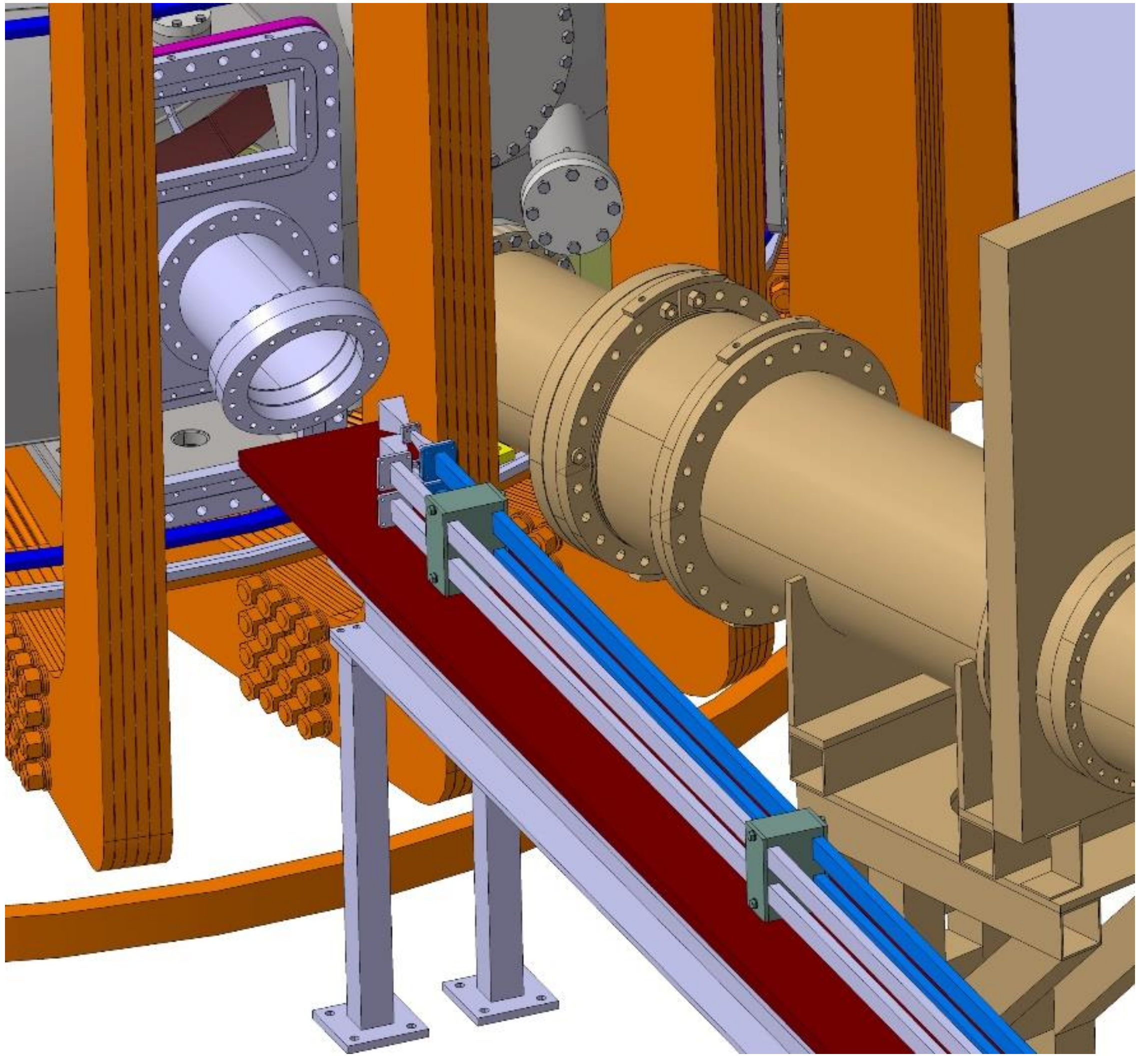
Microwave Hardware Ka-Band FMCW Reflectometer

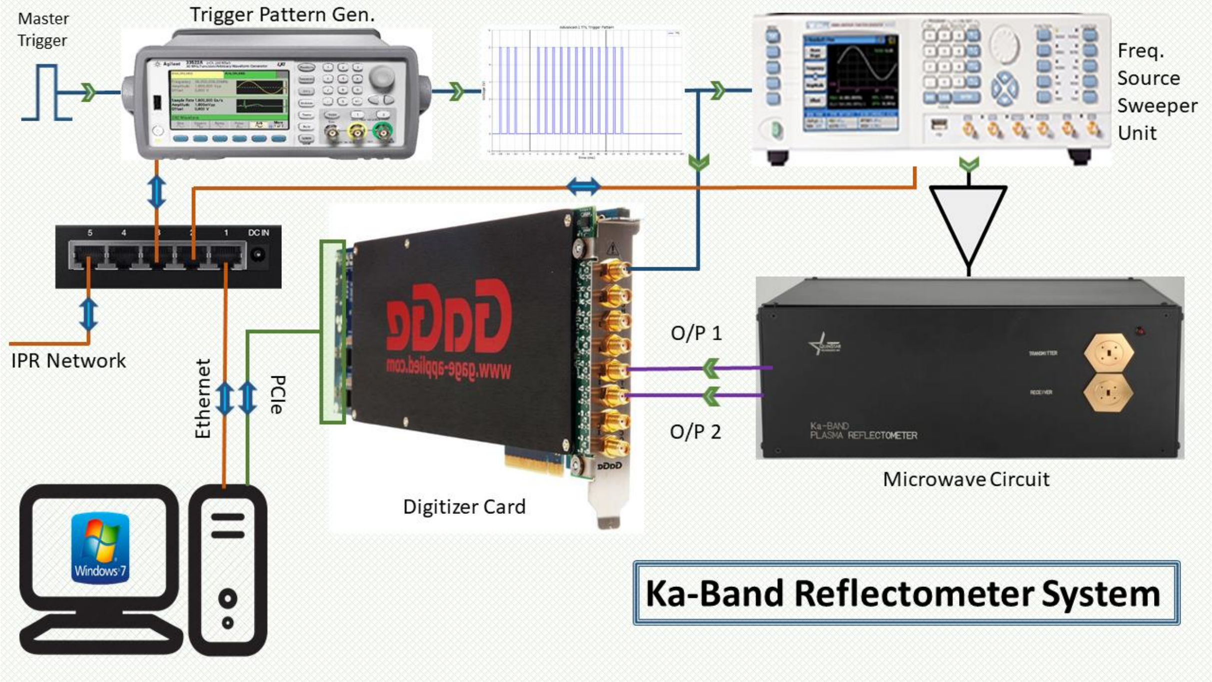
JJU Buch and S. K. Pathak. IOP J Inst., 18, pp. P07022, 2023.

Reflectometry System Development for Aditya-U

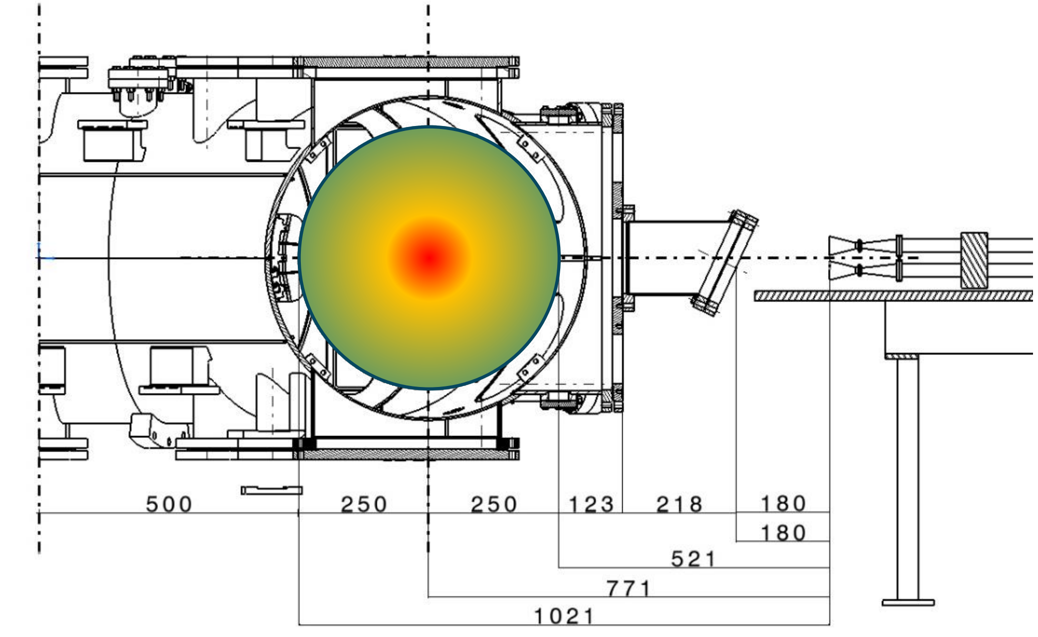


- FMCW Reflectometry
  - Hardware
    - Sweep Generator Trigger & Timing
      - Ultra Fast AWG
      - Amplifier
    - Microwave Hardware
      - Ultra Fast Microwave Source
      - SuperHet. Detection
      - IQ Detection
    - Wave Transport
      - Oversized Waveguides
      - Wave Launching and collection
    - Data Acquisition
      - 2 Ch / 200 MSps PCIe
      - PC Workstation
  - Data Analysis
    - Beat Frequency Estimation
      - Fb estimation
        1. Remove Base line wander
        2. BPF for Vlow & V High Freq
        3. AM noise/ S/W AGC (Optional)
        4. Time-freq analysis / Specgram calc
        5. Plasma positioning i.e. s/g correction for MW path, Ant. Position
        6. fb/Group delay esti.
      - Cross platform GUI for post processing (Python 3.7)
    - Profile Inversion





Ka-Band Reflectometer System



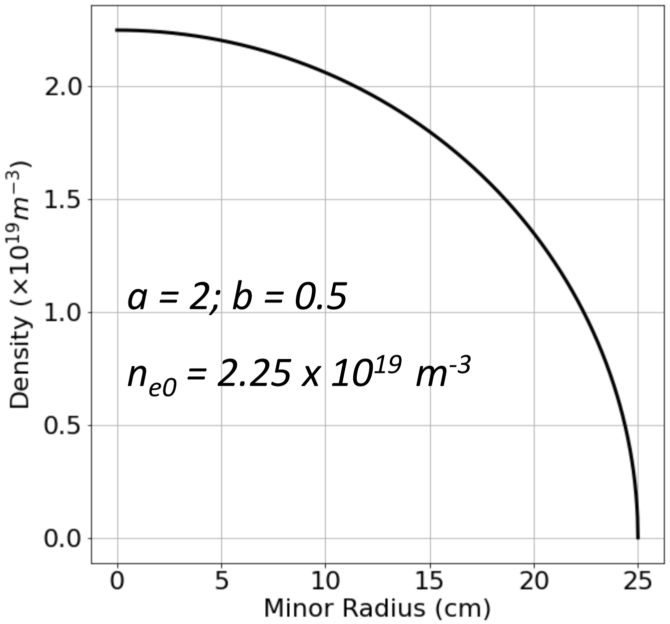
Simulating reflectometer output:

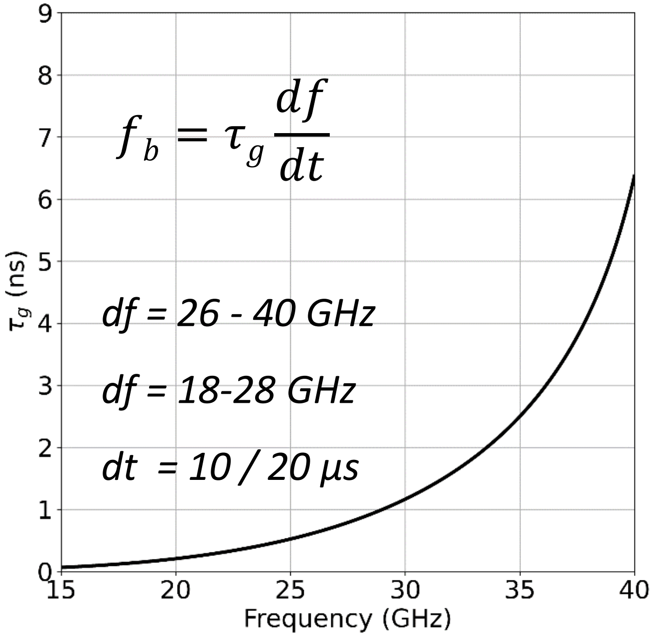
$$n_e(r) = n_{e0}\left[\left(1-\frac{r}{r_{max}}\right)^a\right]^b$$

$$\eta_O = \left(1-\frac{\omega_{pe}^2}{\omega^2}\right)^{1/2}$$

$$\phi = -\frac{2\pi f}{c}\int_F^{F_c}\eta_O dr$$

$$\tau_g = \frac{1}{2\pi}\frac{d\phi}{df}$$





We expect a ***f_b* < 20 MHz** for dt >= 5 *µs for both systems.*

- DAQ with a sampling rate of 200 MSps, 12 bit vertical resolution and 4 GS memory depth
- 10 points / fringe for 5µs sweep
- Triggers 4 sweeps before V_{Loop} and 45 during plasma operation. Trigger resolution is < 5 ns.
- Complete system can be remotely controlled via Ethernet

JJU Buch etal IEEE Microwaves, Antennas, and Propagation, pp. 654–659, 2022.

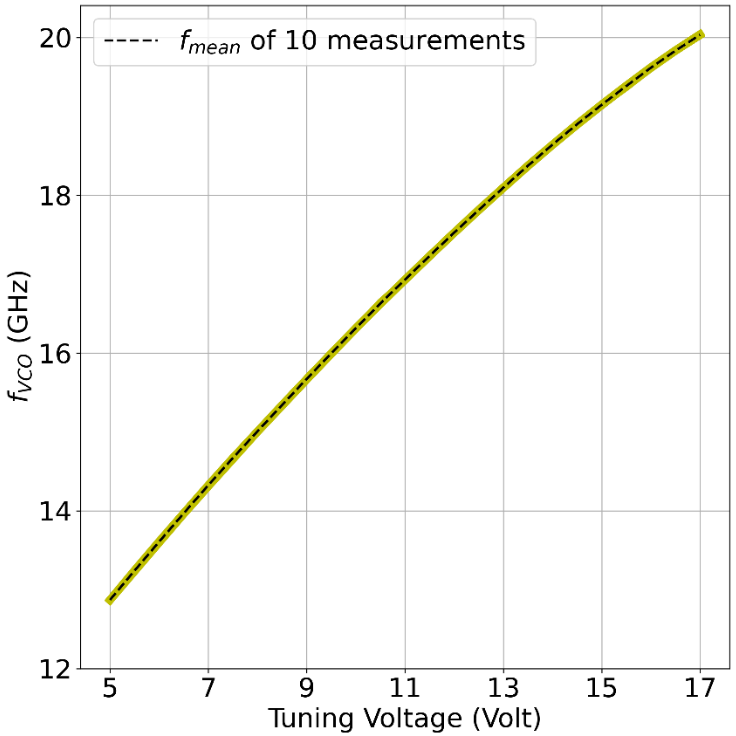
Dispersion due to non-ideal Frequency source

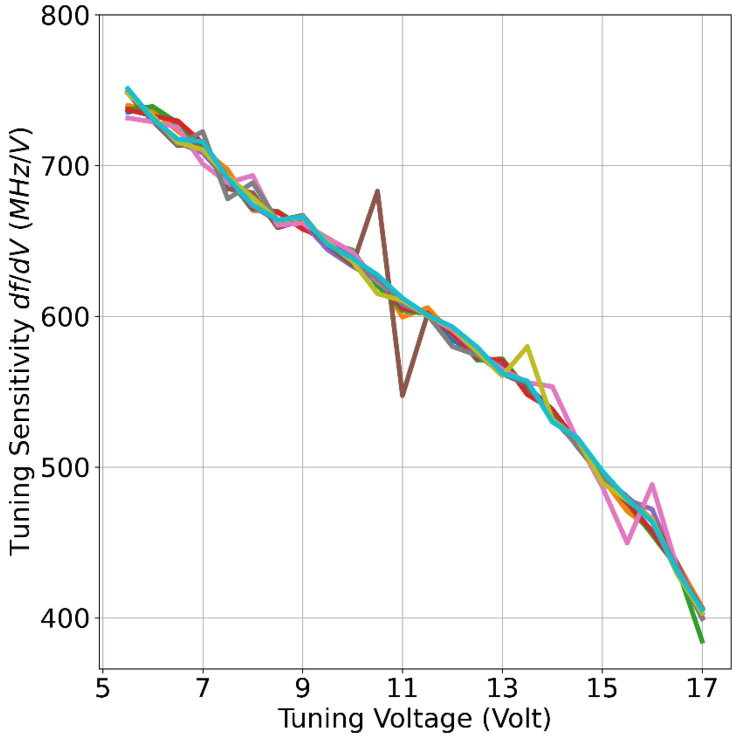
To measure non-linear $\tau_{g,}$ $\frac{df}{dt}$ should be linear

$$f_b = \frac{2d}{c}\frac{df}{dt} = \tau_g \frac{df}{dV}\frac{dV}{dt}$$

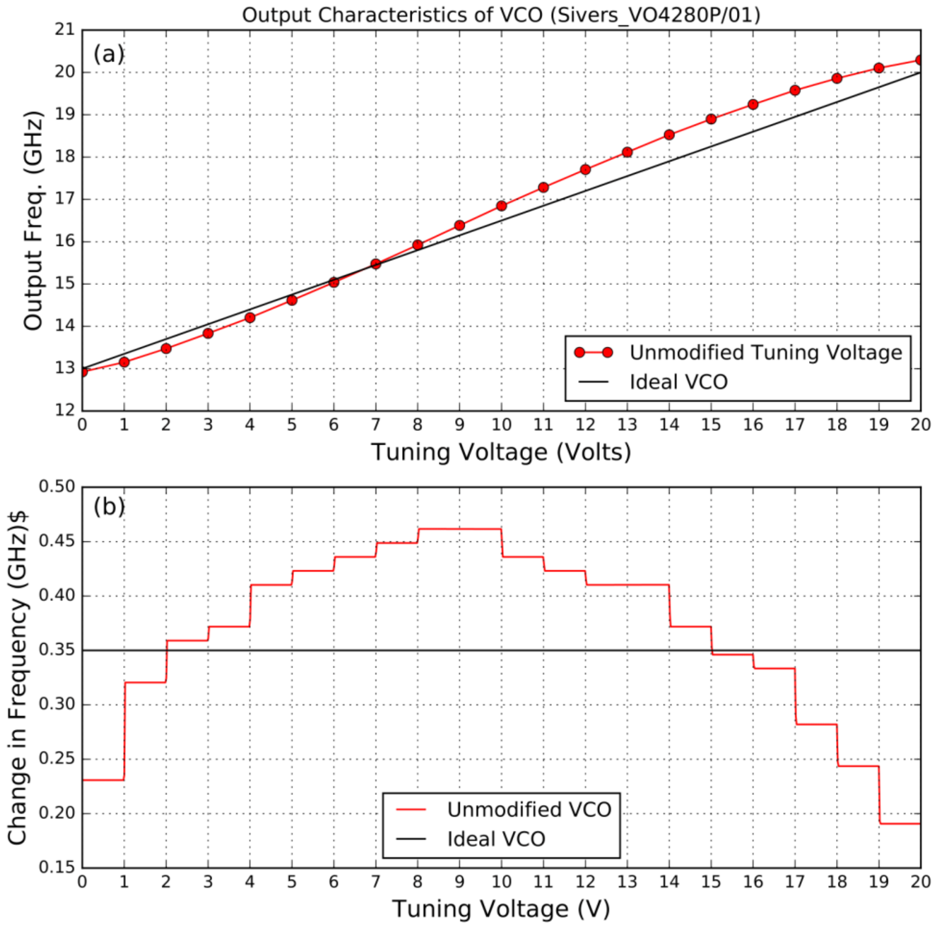
For real VCO source $\frac{df}{dt}$ is non-linear.

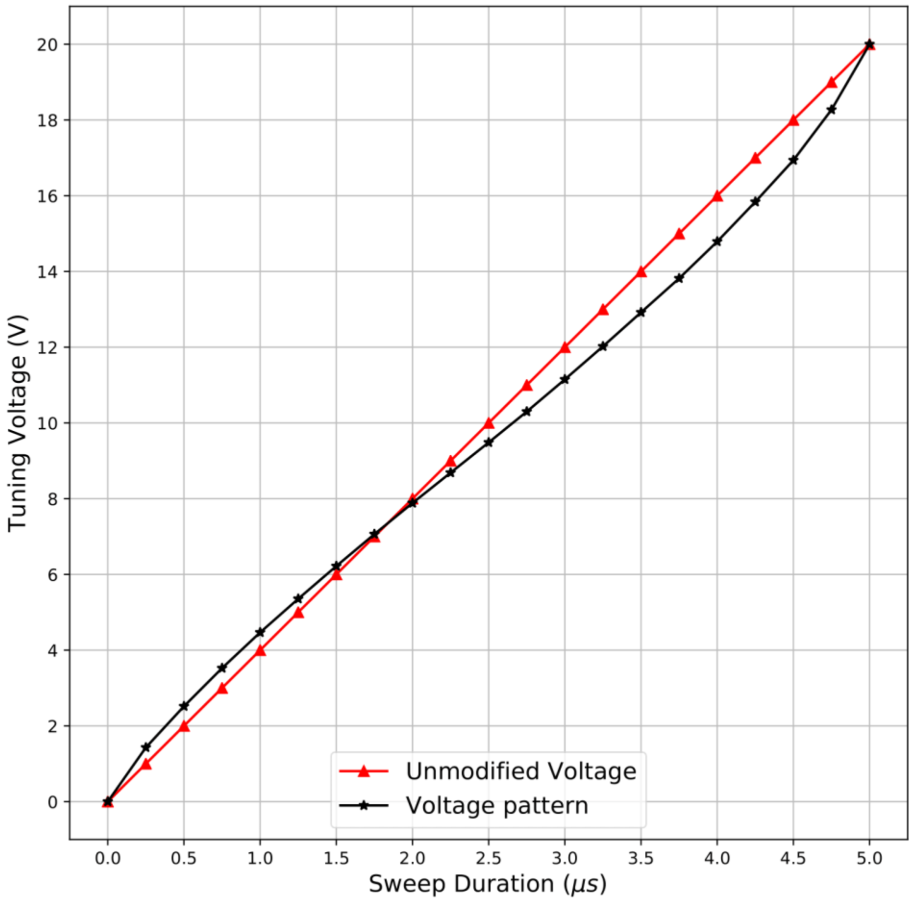
Solution used is to make $\frac{dV}{dt}$ non-linear so that $\frac{df}{dV}\frac{dV}{dt}$ becomes linear



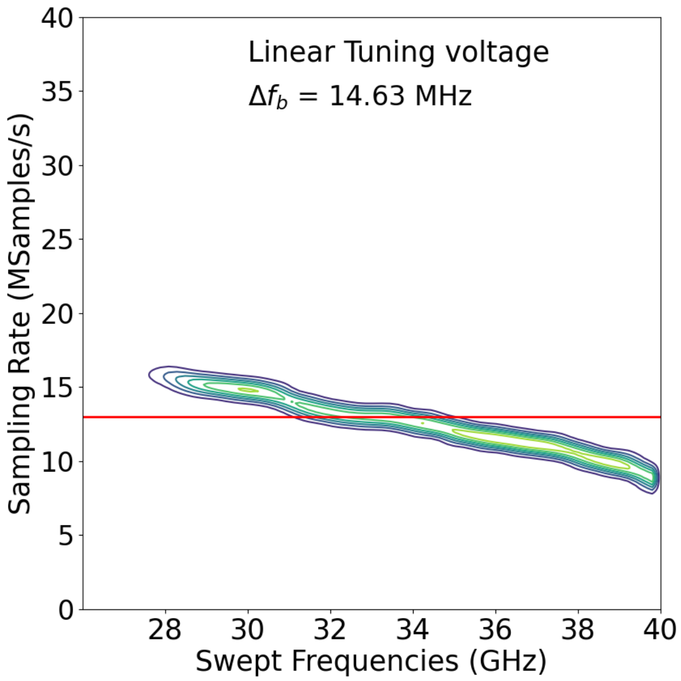


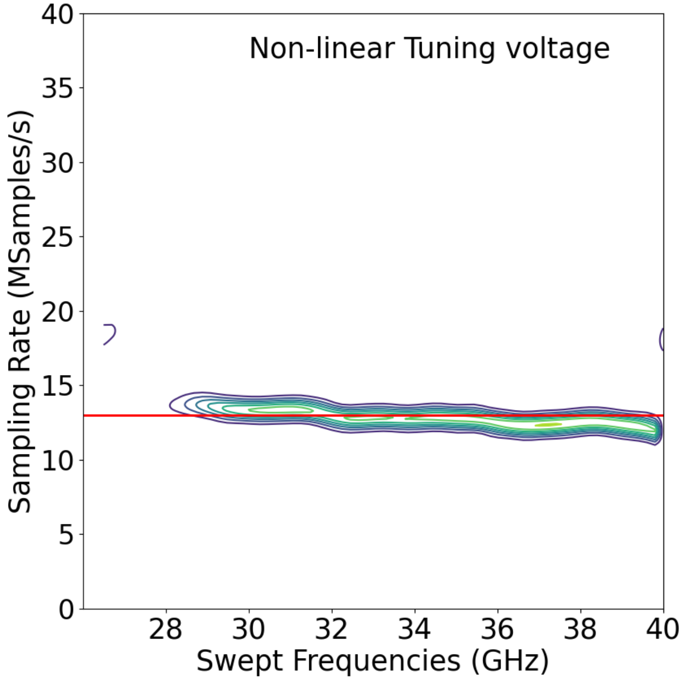
Correcting Frequency Response of VCO

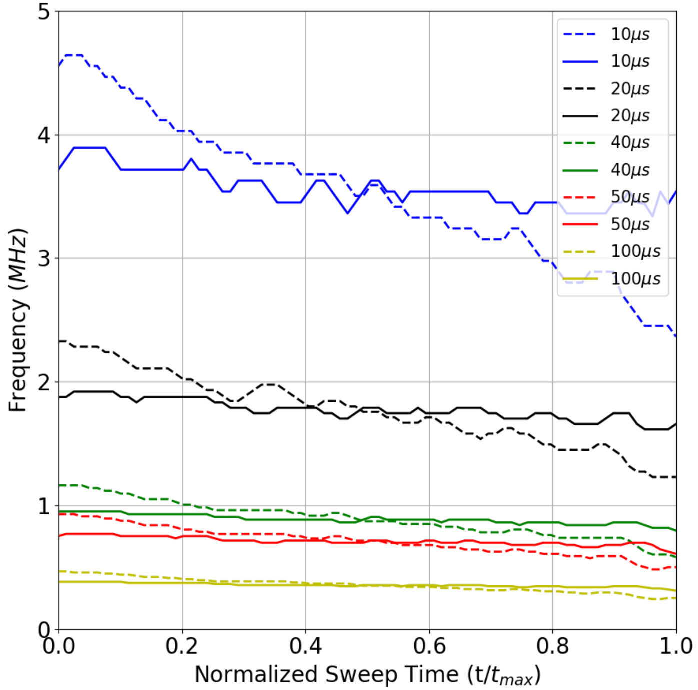




Ka-Band Reflectometer System Response:







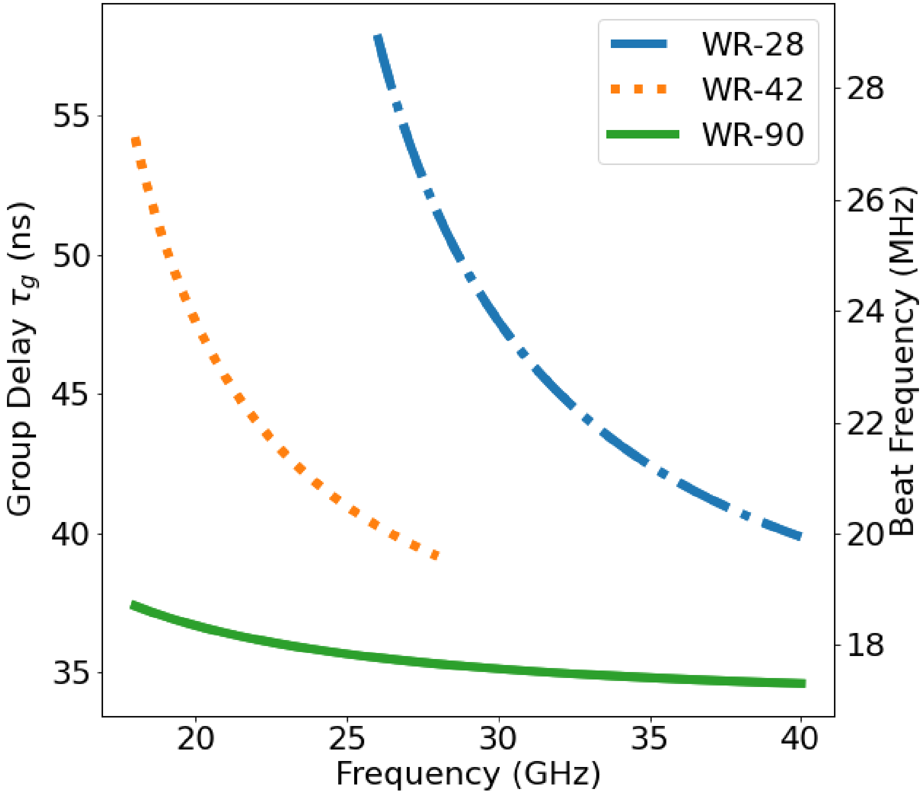
- Source characteristics are measured statically but measured dynamically
- So some non-linearities will remain.

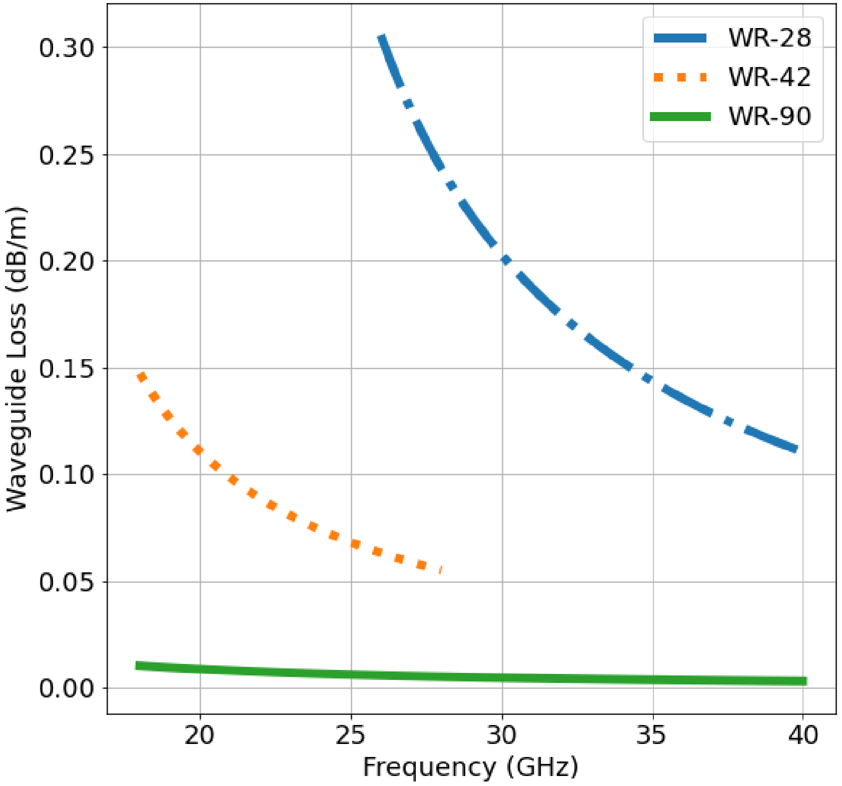
Sweep Times	**Slopes for**	
	linear V	**non-linear V**
$dt(\mu s)$	dV	dV'
10	20.31	4.13
20	4.96	1.24
40	1.23	0.29
50	0.78	0.20
100	0.20	0.05

$$f_b = \tau_g \frac{df}{dt} = \frac{2d}{c}\frac{df}{dt}$$

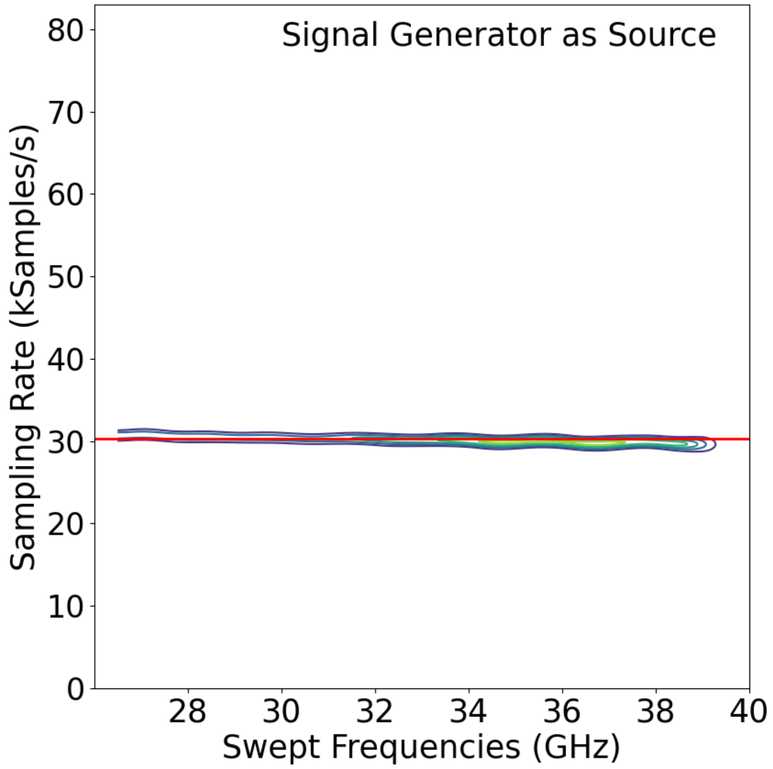
Dispersion due to Waveguide

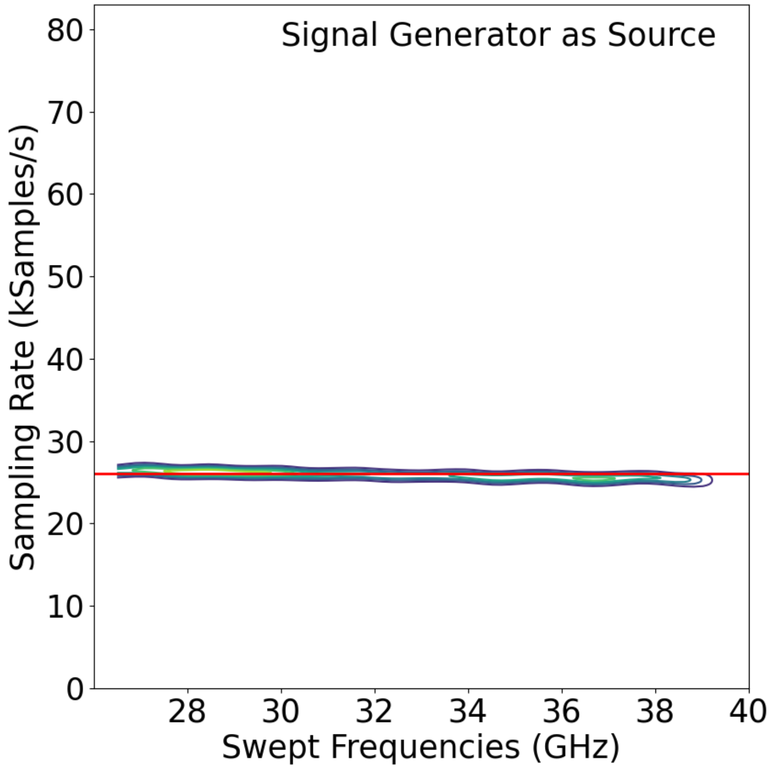
- A large group delay of 15 ns for K-Band and 18ns for Ka-Band reduces to 2.5 ns on using WR-90 waveguides.



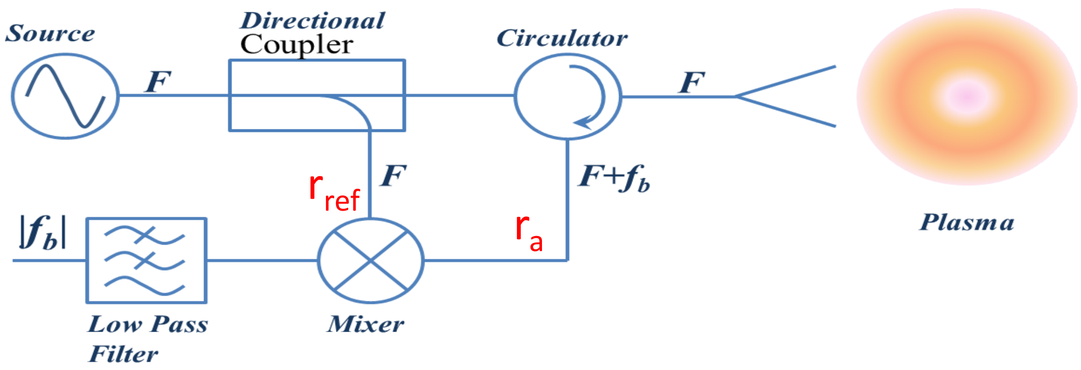


Linearized System Characteristics





Phase Offset Correction



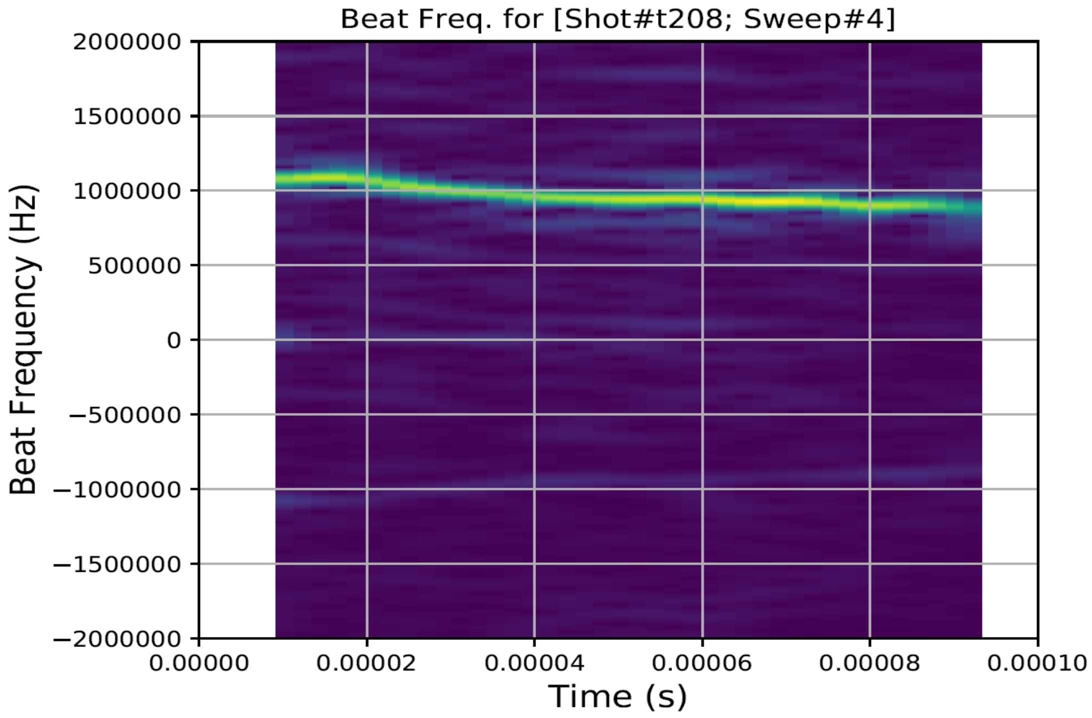
Recall,

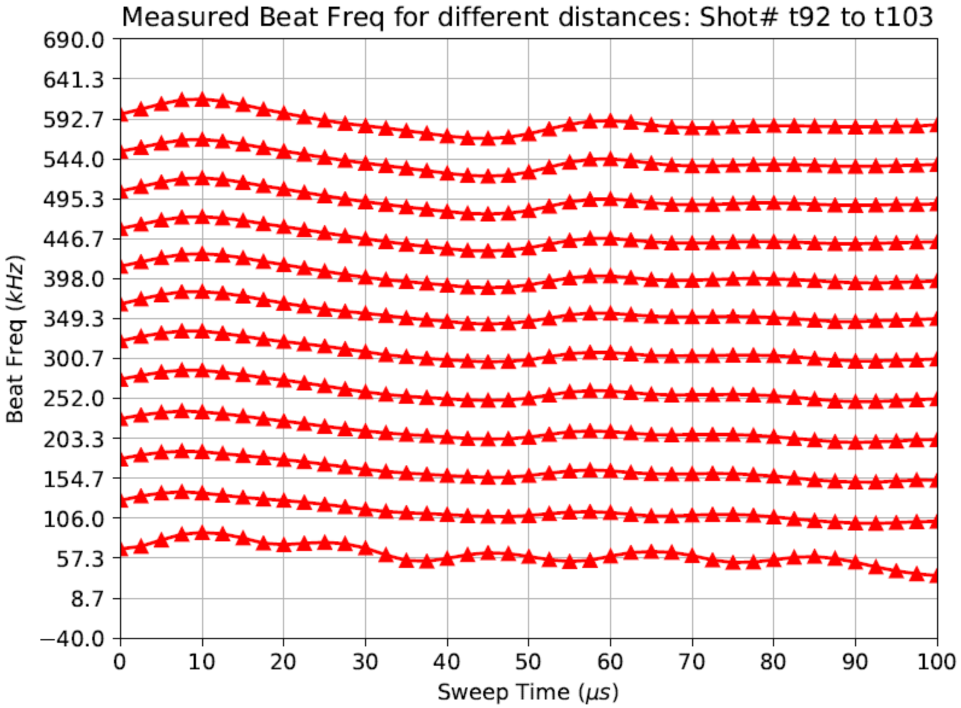
$$\frac{d\phi}{dt} = f_b = \frac{r_{target}}{c}\frac{df}{dt} + \underbrace{\frac{(r_{sys} - r_{ref})}{c}\frac{df}{dt}}_{\text{constant offset}}$$

Equation of St. line: $y = mx + c$

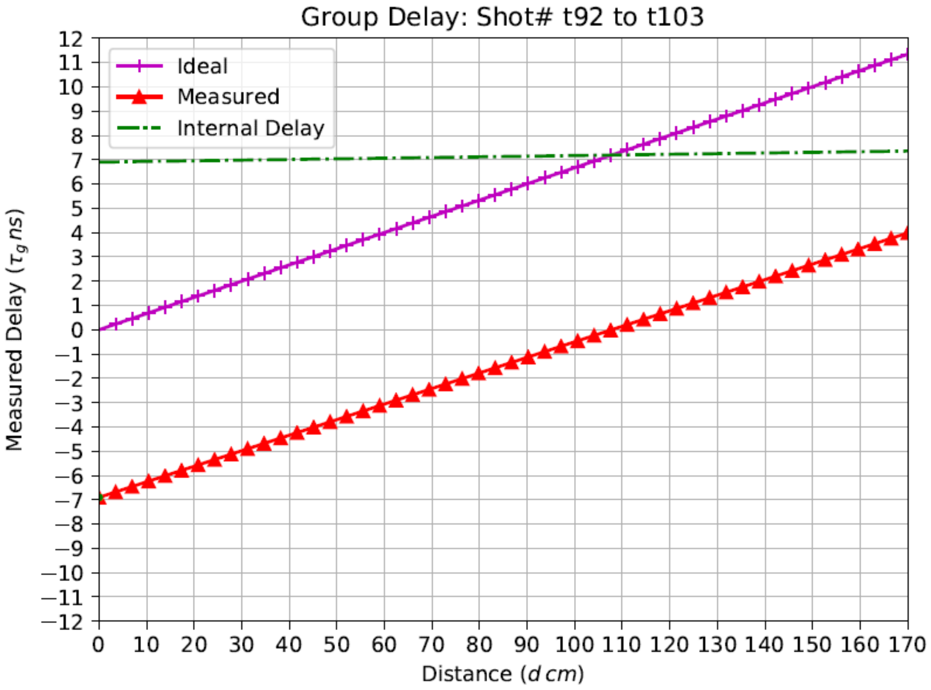
$$x = r_{target} \qquad y = f_b \qquad m = \frac{1}{c}\frac{df}{dt} \qquad c = \frac{(r_{sys} - r_{ref})}{c}\frac{df}{dt}$$

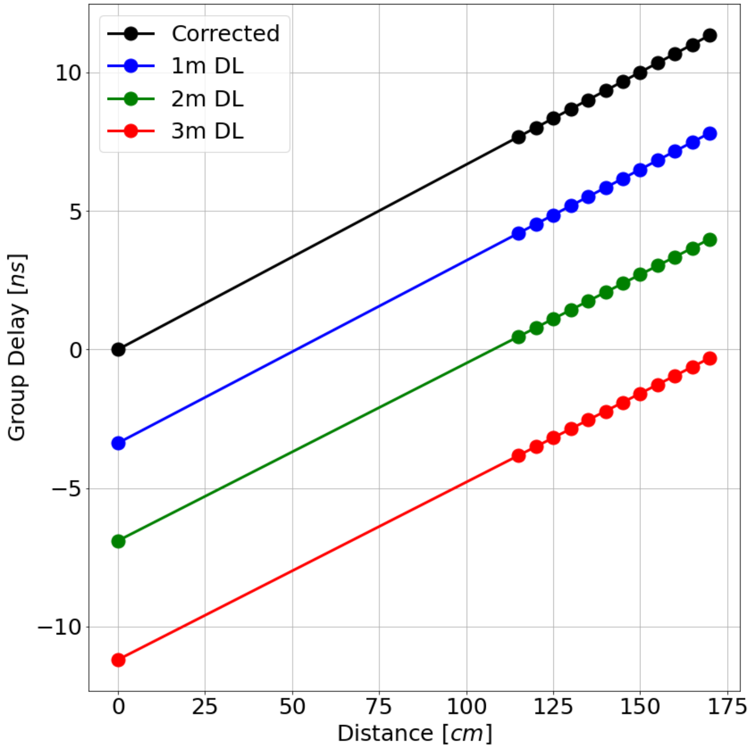
Laboratory Results.

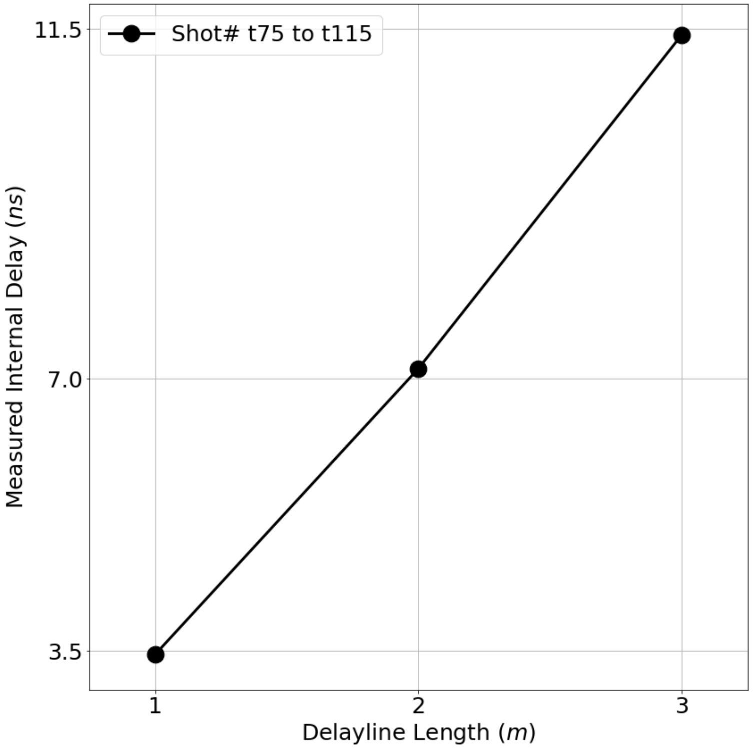




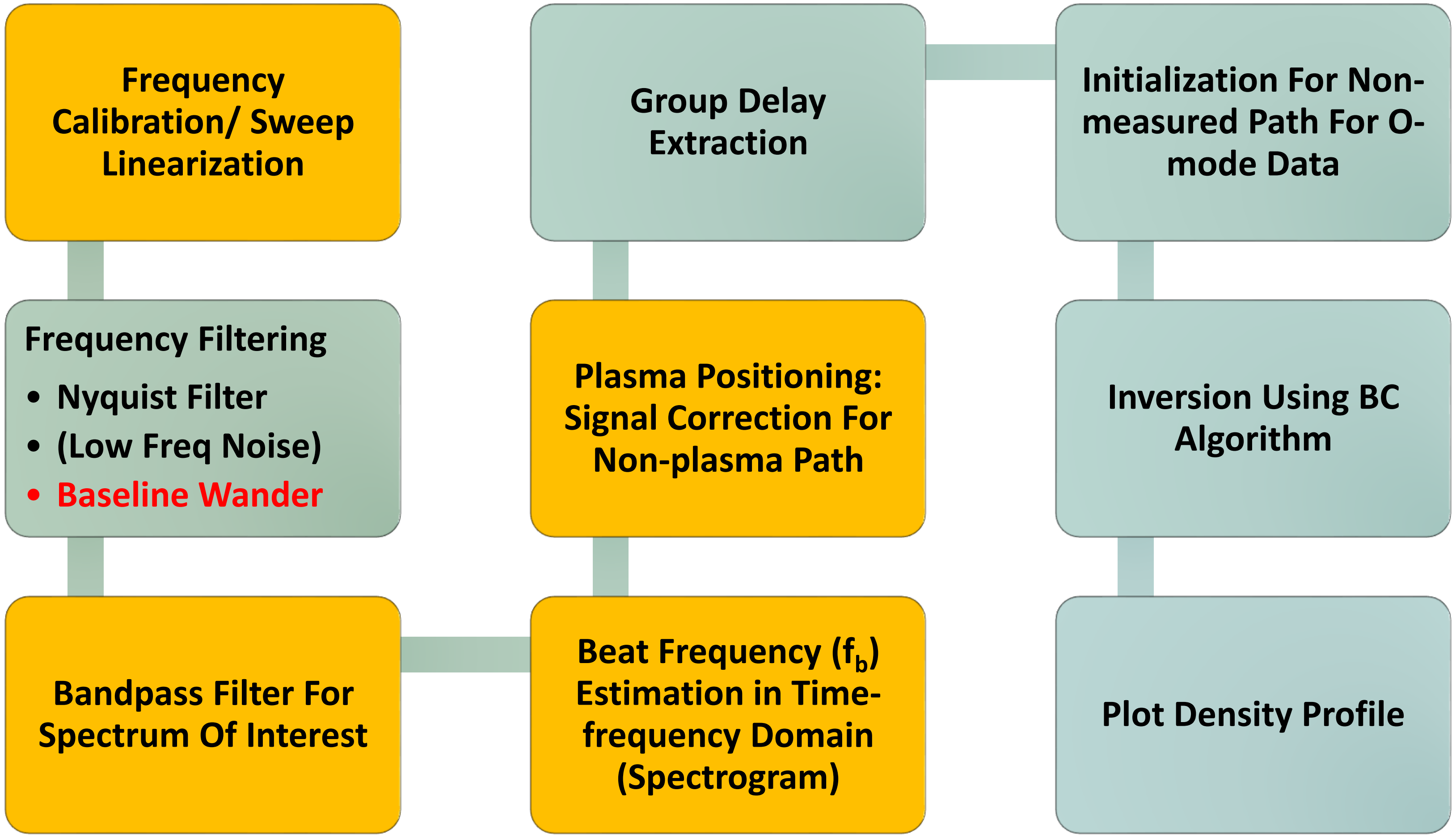
Verification of measured delay







Signal processing output signal



Frequency Calibration/ Sweep Linearization

Frequency Filtering

- **Nyquist Filter**
- **(Low Freq Noise)**
- **Baseline Wander**

Bandpass Filter For Spectrum Of Interest

Beat Frequency (f_b) Estimation in Time-frequency Domain (Spectrogram)

Plasma Positioning: Signal Correction For Non-plasma Path

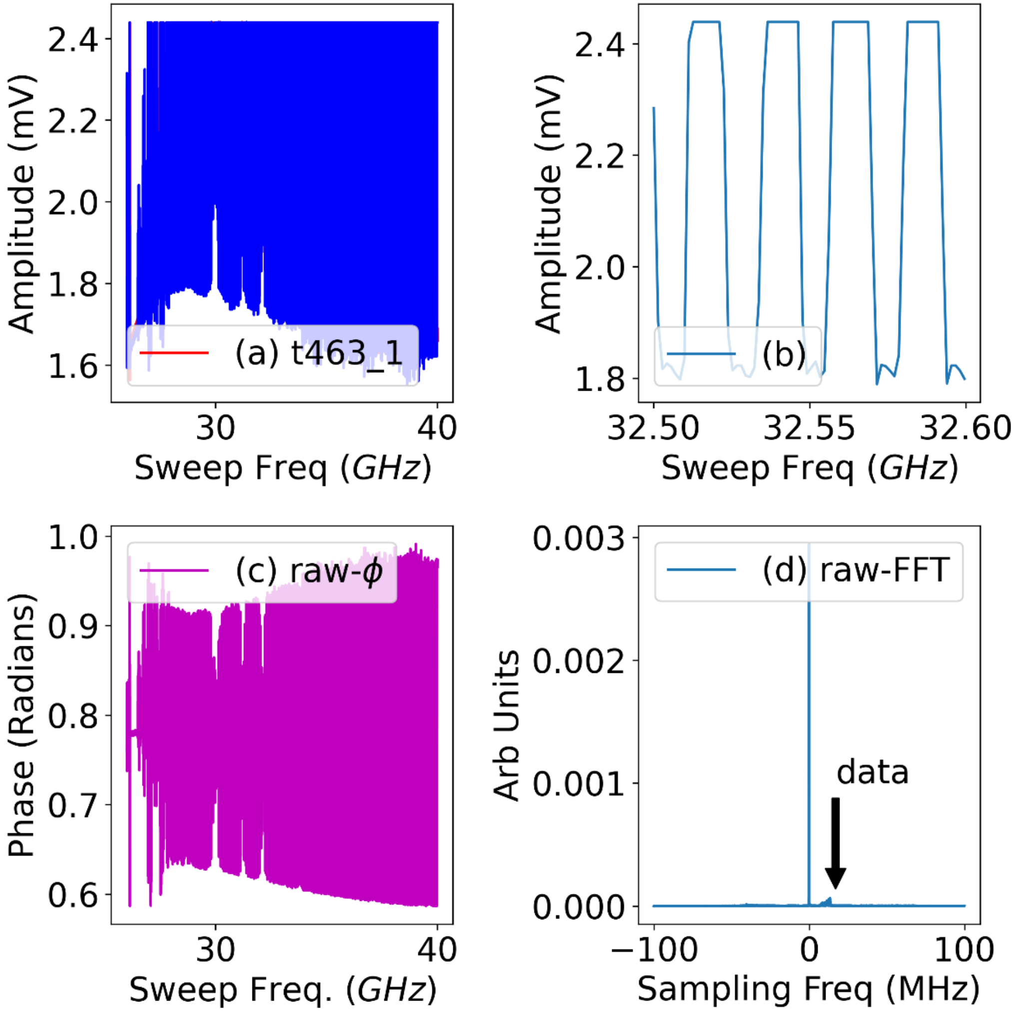
Group Delay Extraction

Initialization For Non-measured Path For O-mode Data

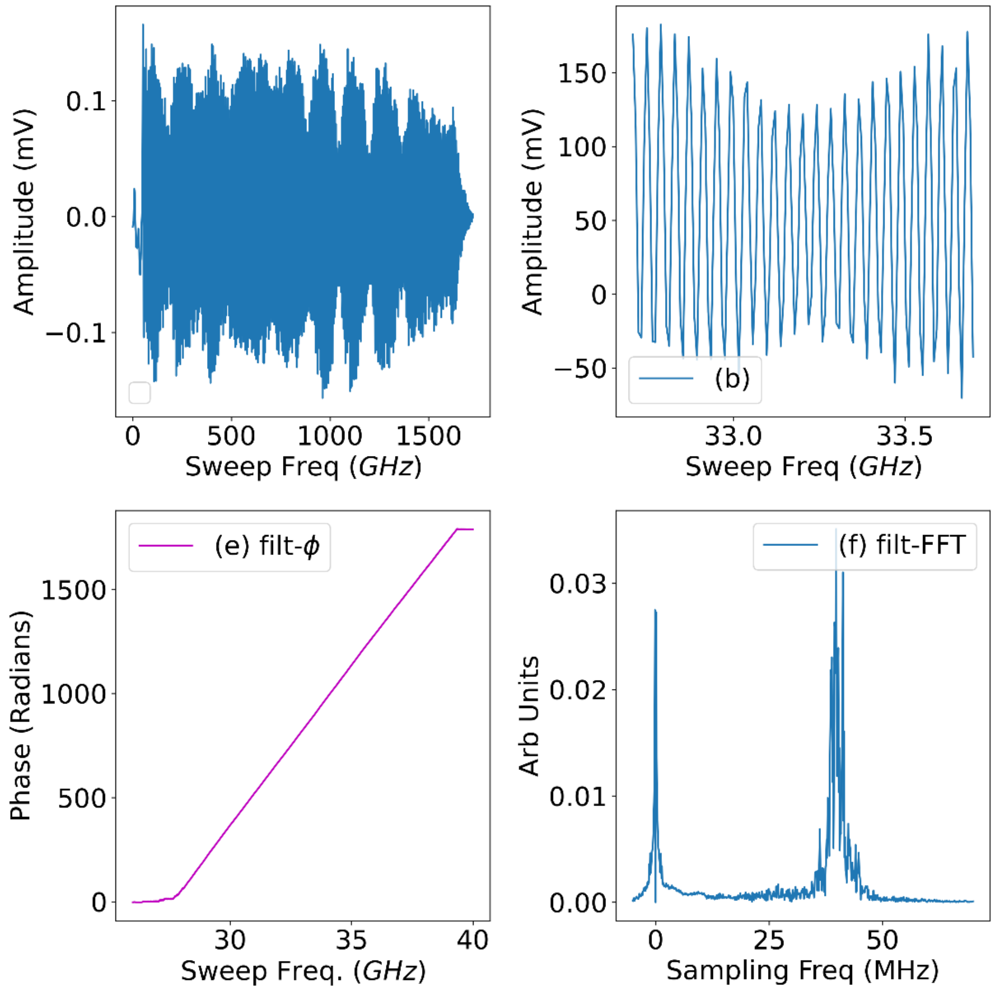
Inversion Using BC Algorithm

Plot Density Profile

RAW DATA t463_1

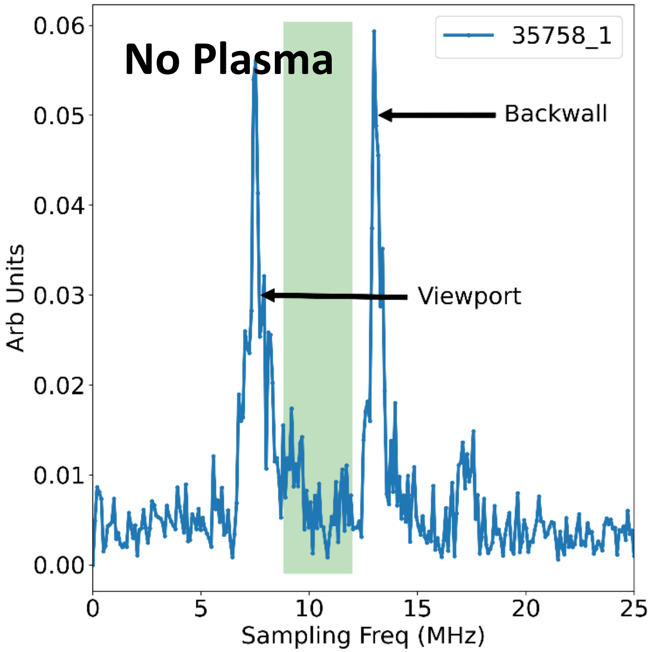


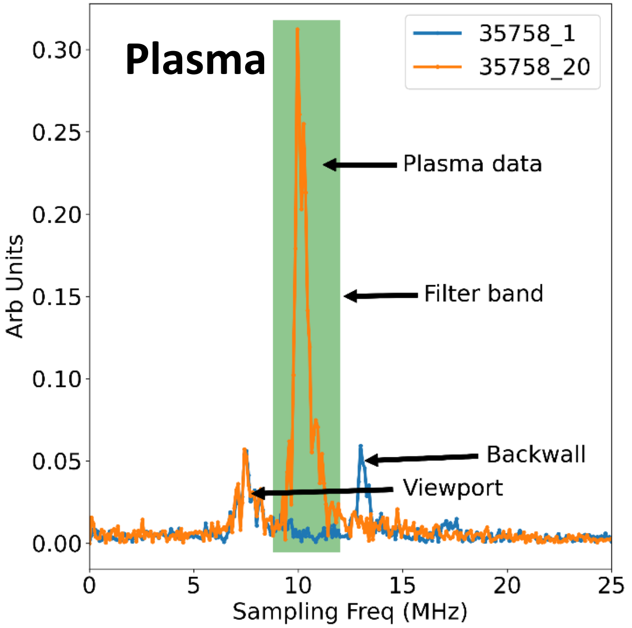
Baseline Wander Removed: t601_1

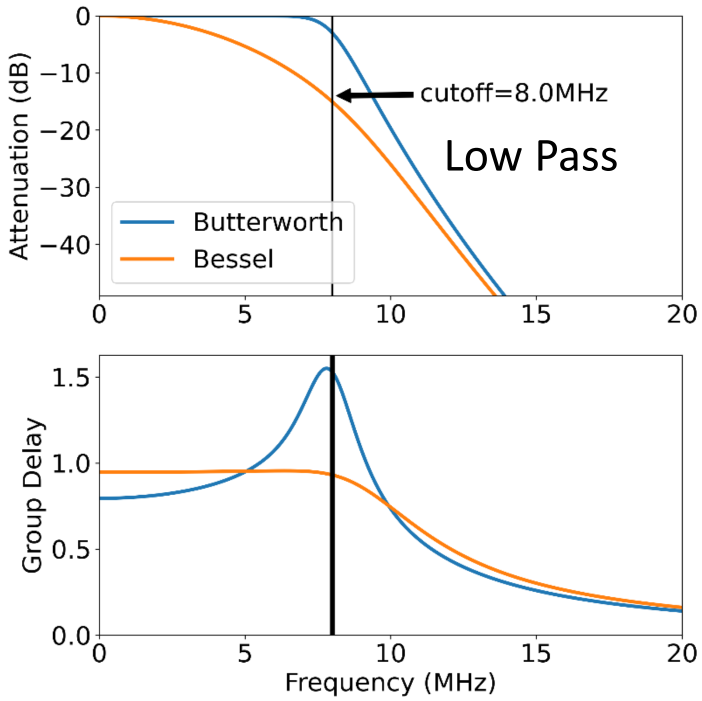


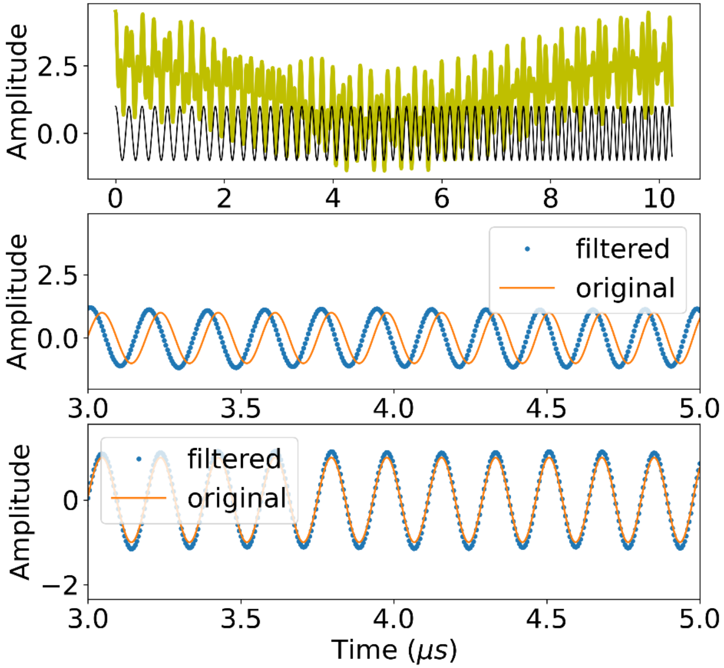
- Baseline offset is seen
- Raw signal saturates
- No phase reconstruction can be done.
- Noise masks the actual data

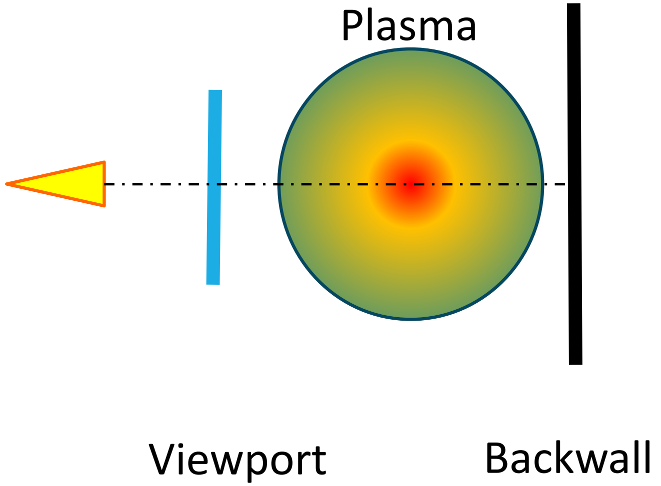
- Another Shot with with baseline wander removed
- Phase reconstruction can be done.

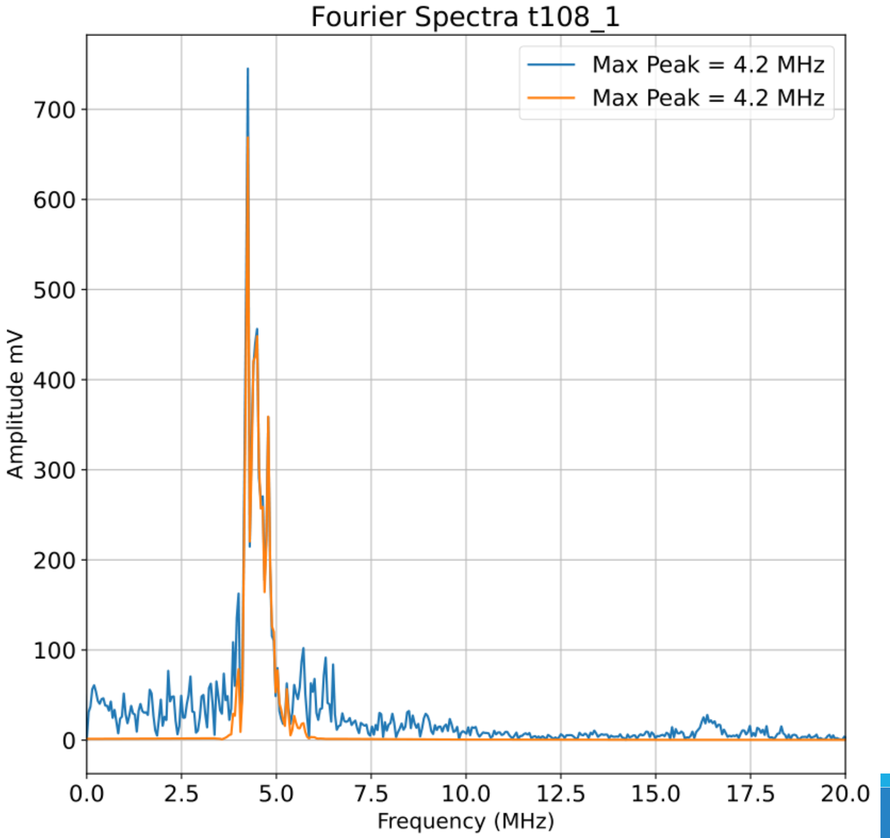

No Plasma
35758_1
Backwall
Viewport
Arb Units
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0
5
10
15
20
25
Sampling Freq (MHz)


Plasma
35758_1
35758_20
Plasma data
Filter band
Backwall
Viewport
Arb Units
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0
5
10
15
20
25
Sampling Freq (MHz)


cutoff=8.0MHz
Low Pass
Butterworth
Bessel
Attenuation (dB)
0
−10
−20
−30
−40
Group Delay
0.0
0.5
1.0
1.5
0
5
10
15
20
Frequency (MHz)

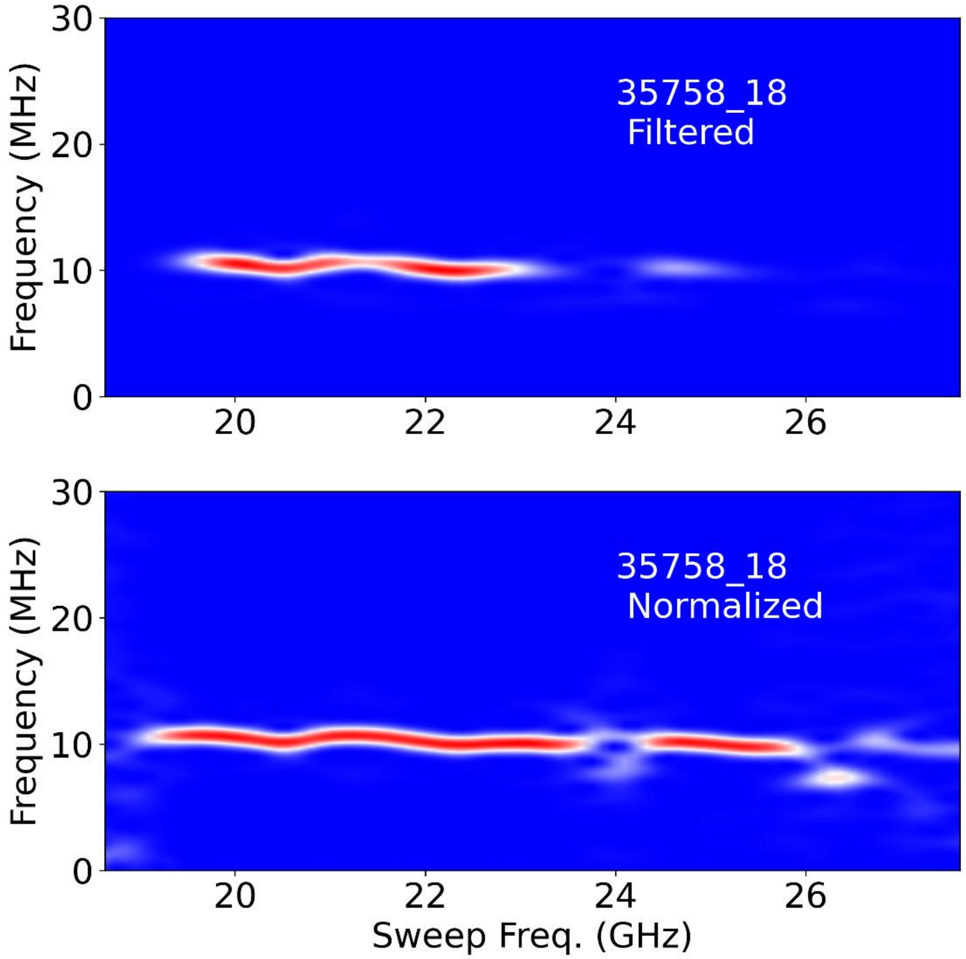

Amplitude
0.0
2.5
0
2
4
6
8
10
filtered
original
3.0
3.5
4.0
4.5
5.0
−2
0
Time (μs)

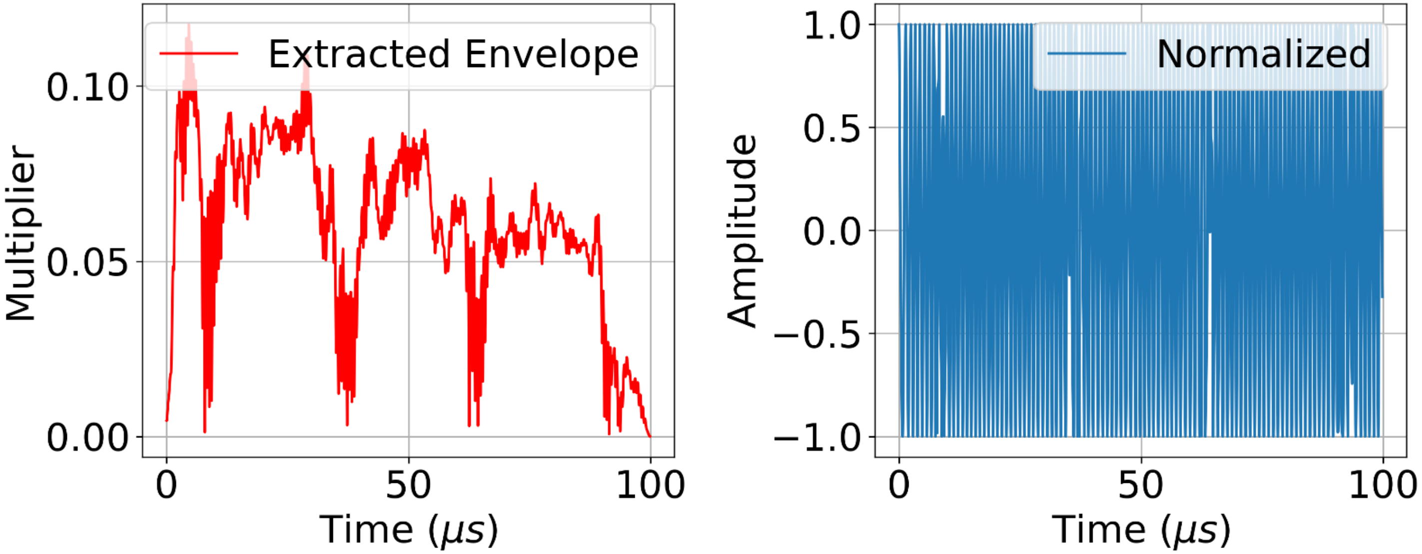

Plasma
Viewport
Backwall


Fourier Spectra t108_1
Max Peak = 4.2 MHz
Max Peak = 4.2 MHz
Amplitude mV
0
100
200
300
400
500
600
700
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
Frequency (MHz)

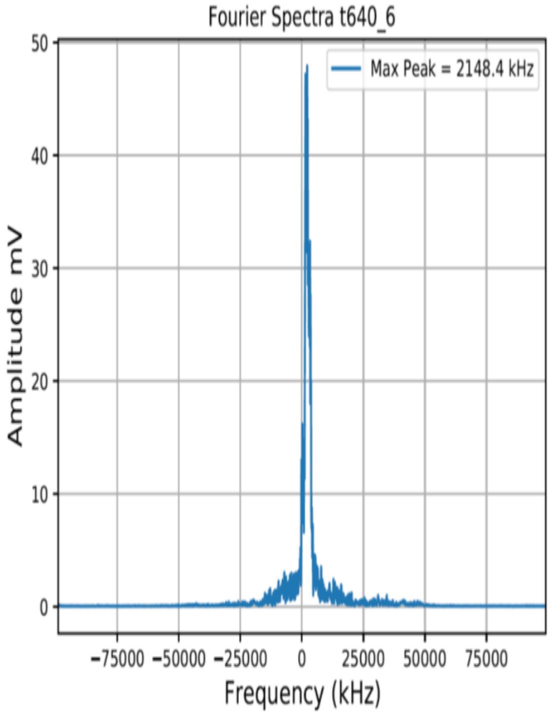
Software Automated Gain Control (AGC):

1. Peak detection after filtering data.
2. Cubic spline interpolation of peaks achieving a smooth envelope curve.
3. Calculate multiplication factor or variable gain curve of envelope curve.
4. Multiply the variable gain curve with filtered data.

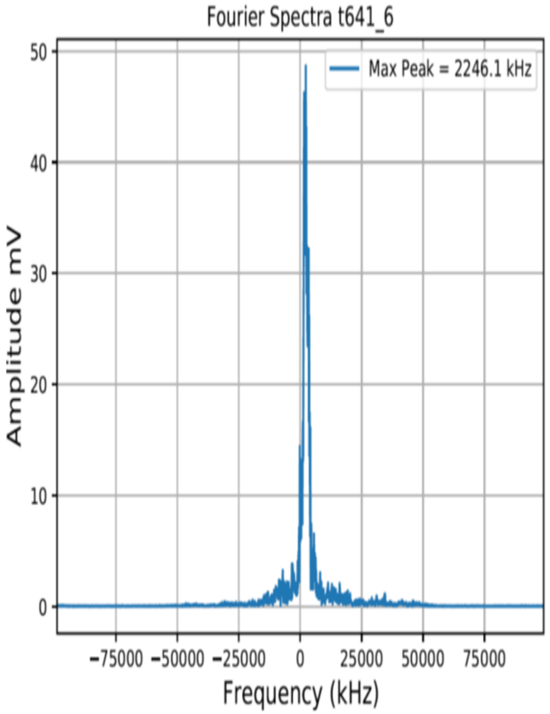




Resolution of measured distance



$f_{peak} = 2246.1\ kHz$



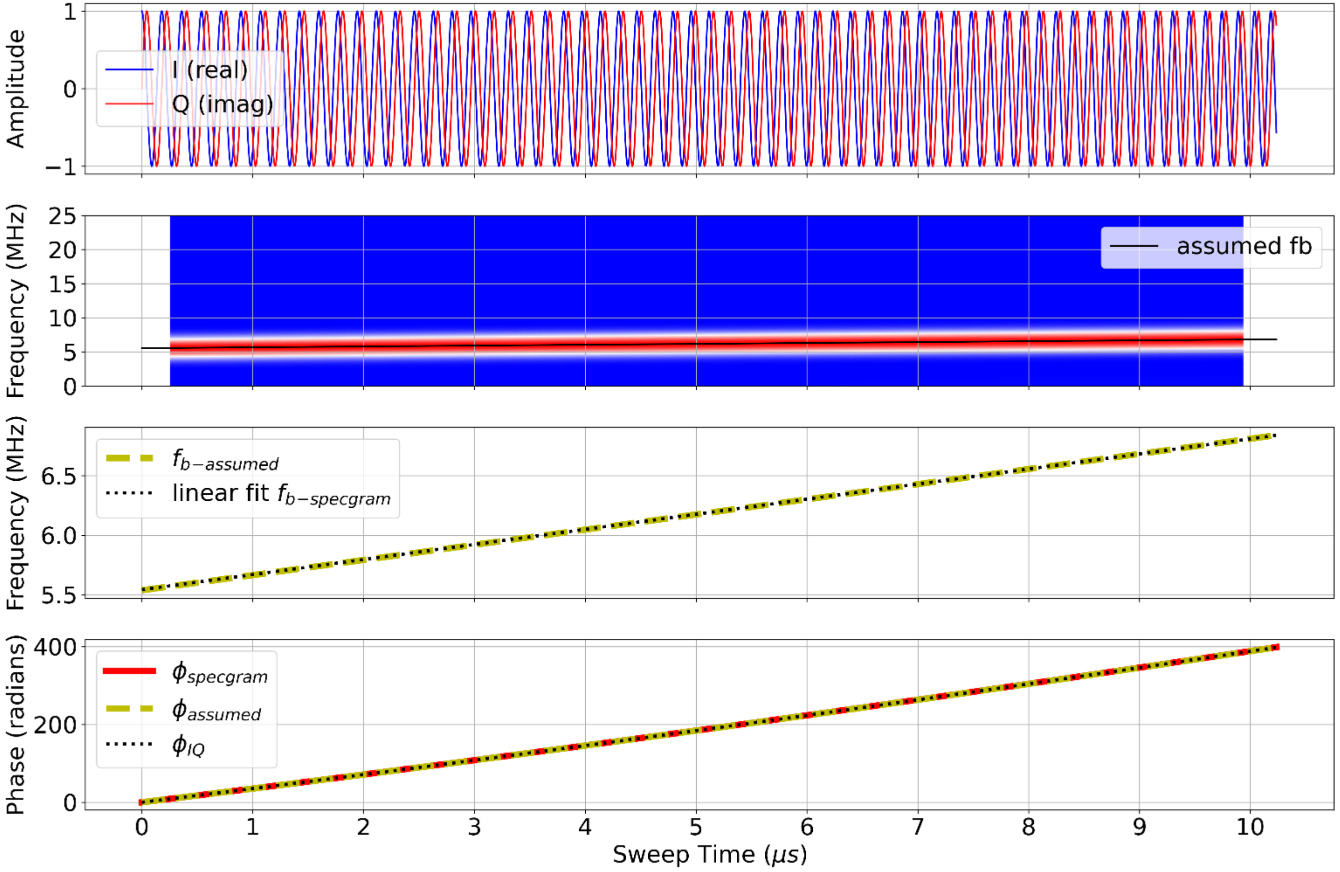
$f_{peak} = 2148.4\ kHz$

$\boldsymbol{df = (40 - 26) GHz}$ $\boldsymbol{dt = 10\ \mu s}$ $\boldsymbol{d = 1\ cm}$

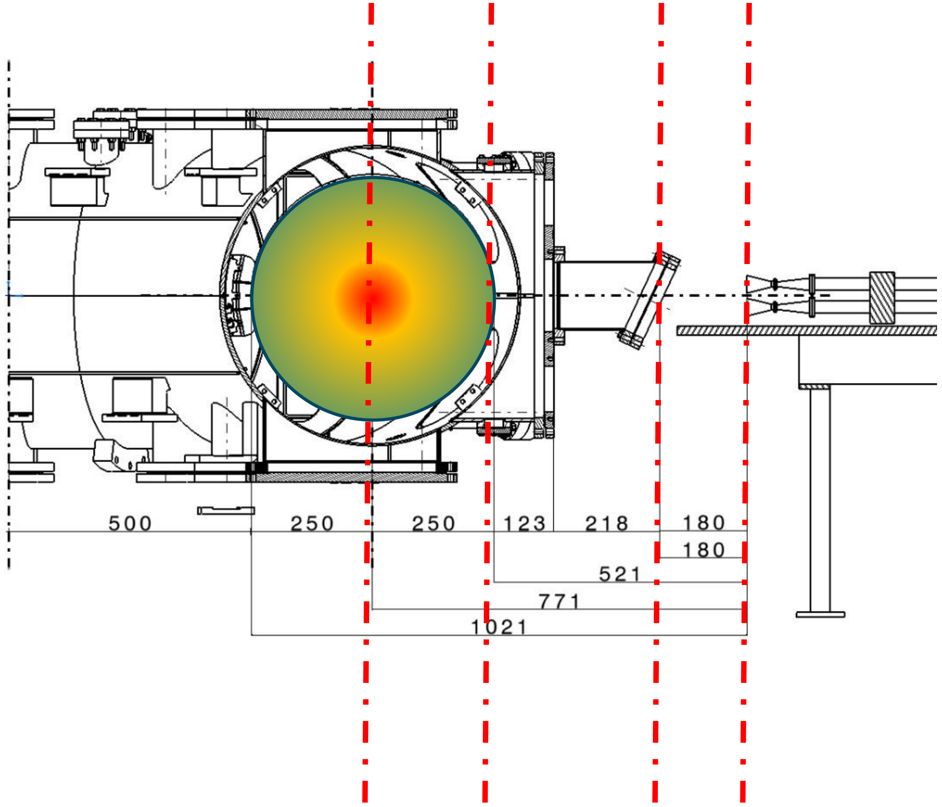
Test Shot	$\boldsymbol{f_b}$ $\boldsymbol{(kHz)}$	$\boldsymbol{\Delta f_b}$	**Δd**
t640	2148.4	-	
t641	2246.1	97.7	1.0468 cm
t642	2343.8	97.7	1.0468 cm
t643	2441.4	97.6	1.0457 cm
t644	2539.1	97.7	1.0468 cm

Extracting Phase from Power Spectral Density

1. Frequency of the signal is estimated using peak detection for the power spectral density in spectrogram.
2. Phase measured is then calculated using $2\pi f_b = \frac{d\phi}{dt}$ or via $\phi = 2\pi(f_{b+i} - f_b)(t_{i+1} - t_i)$
3. To verify the algorithm developed, phase from specgram and the simulated phase were compared with accuracy of **> 10^{-13}** was obtained.



Isolating ϕ_p from plasma



$$\phi = 2\frac{2\pi f}{c}\int_{r_c}^{r_r} N(r)dr - \frac{\pi}{2}$$

Measured ϕ for plasma shot

$$\phi = \phi_{wg} + \phi_{air} + \phi_{vac} + \phi_p$$

Measured ϕ_0 without plasma

$$\phi_0 = \phi_{wg} + \phi_{air}$$

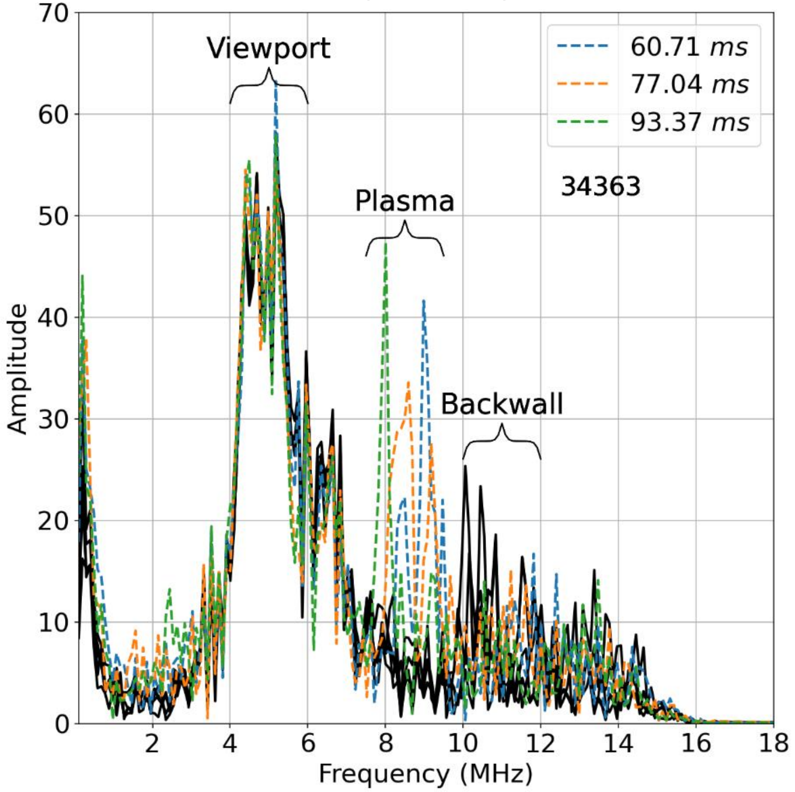
ϕ_p due to plasma

$$\phi_p = \phi - \phi_0 - \phi_{vac}$$

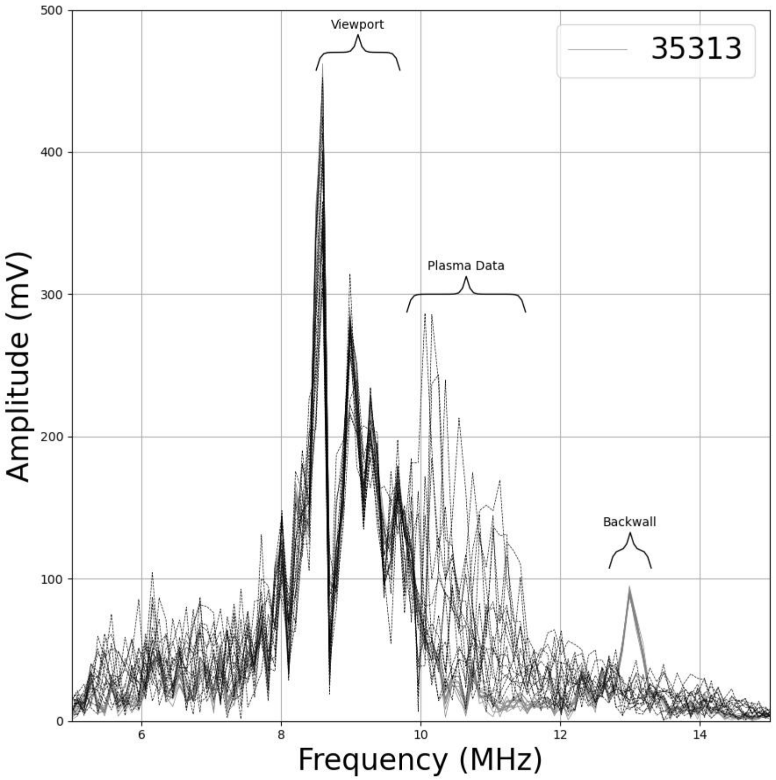
$$\phi_p = \phi - \phi_0 - 2\frac{2\pi f}{c}(r_{limiter} - r_{vp})$$

First Results from Aditya-U

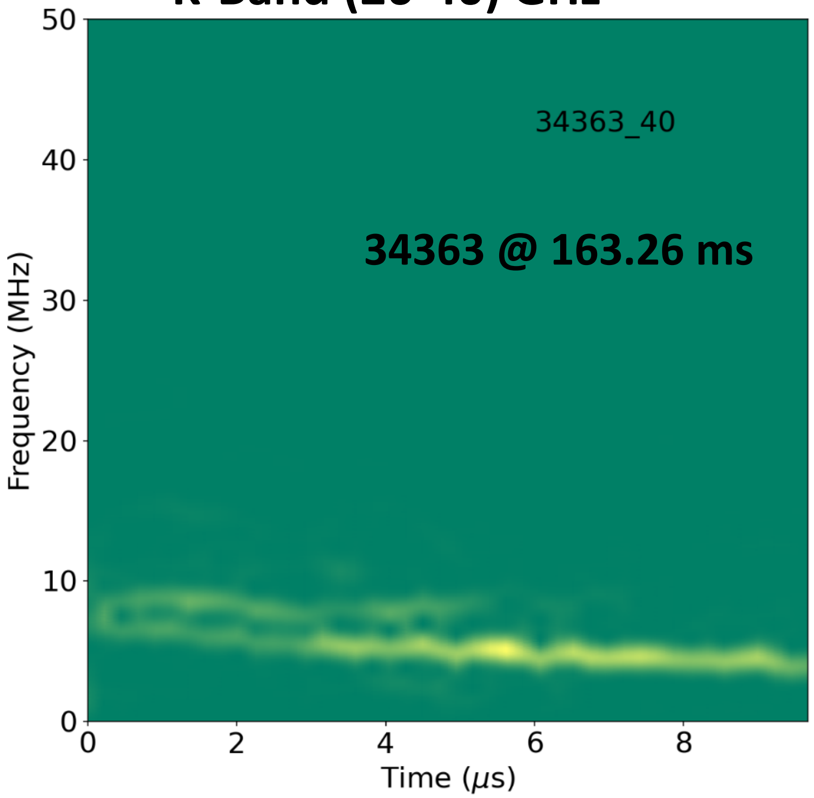
Ka-Band (26-40) GHz



K-Band (18-28) GHz



K-Band (26-40) GHz



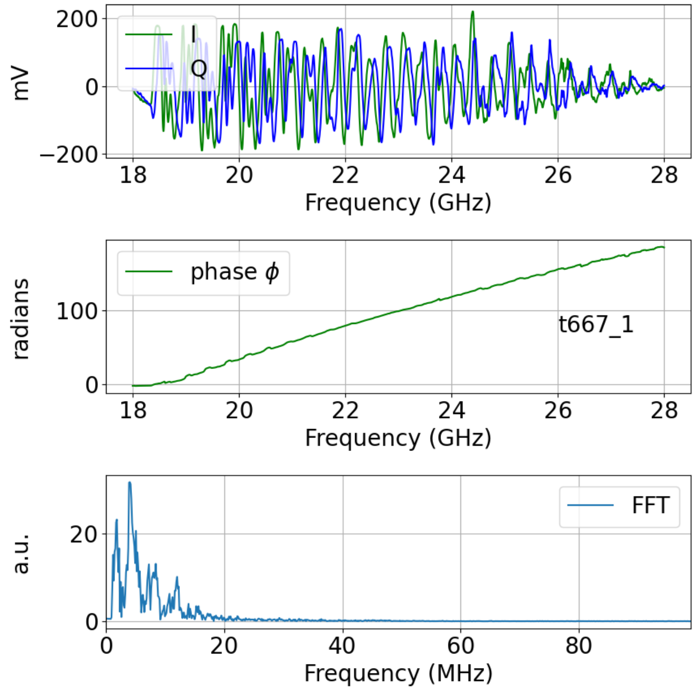
Mitigation of Viewport Reflections

Two solutions are possible:

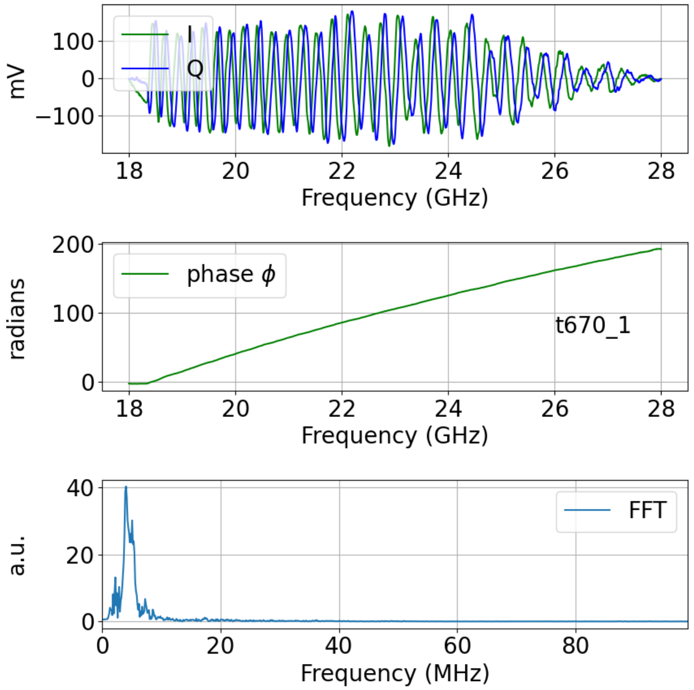
1. Placing antennae inside Vacuum Vessel
2. Placing viewport at an angle

Lab results on tilting viewport at angles 0, 10, 20, 30, 40, 50 deg.

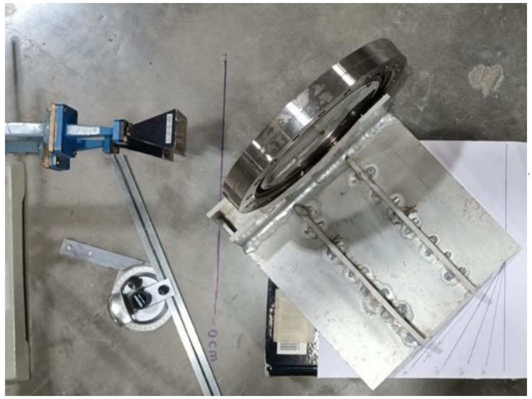
$\boldsymbol{Tilt = 0°}$



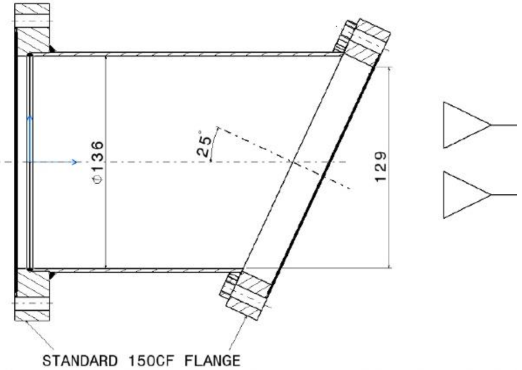
$\boldsymbol{Tilt = 20°}$



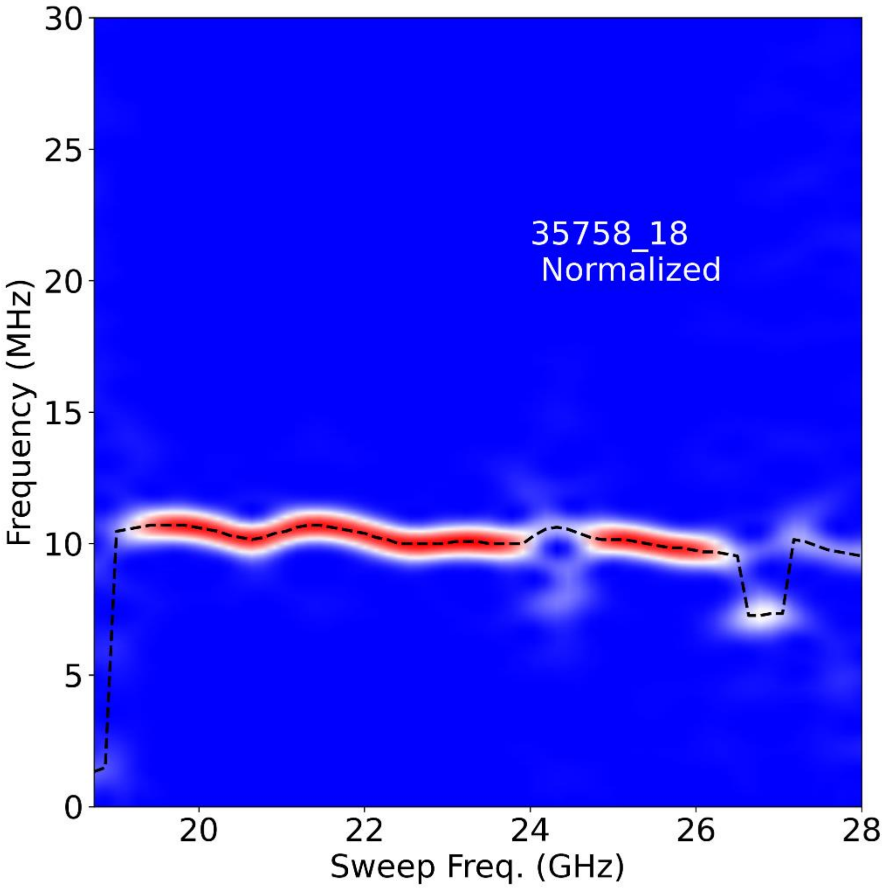
Lab setup

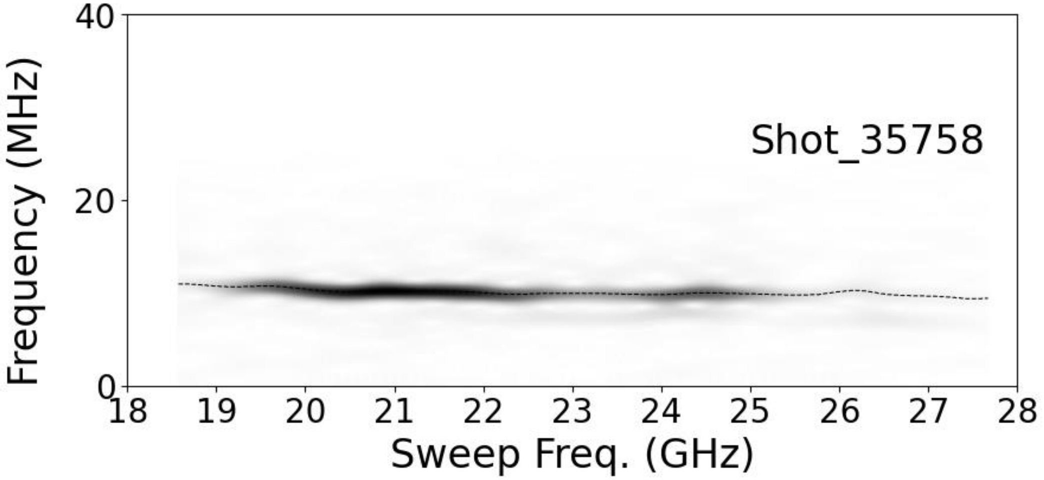


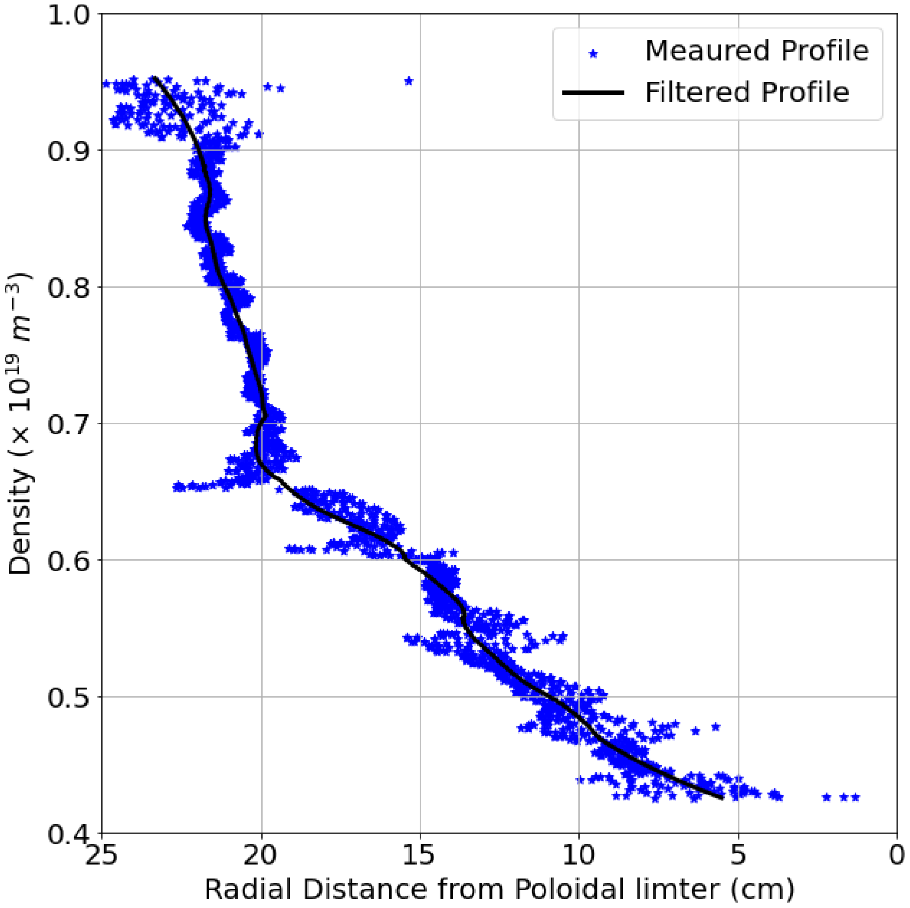
Designed “*I* “ structure

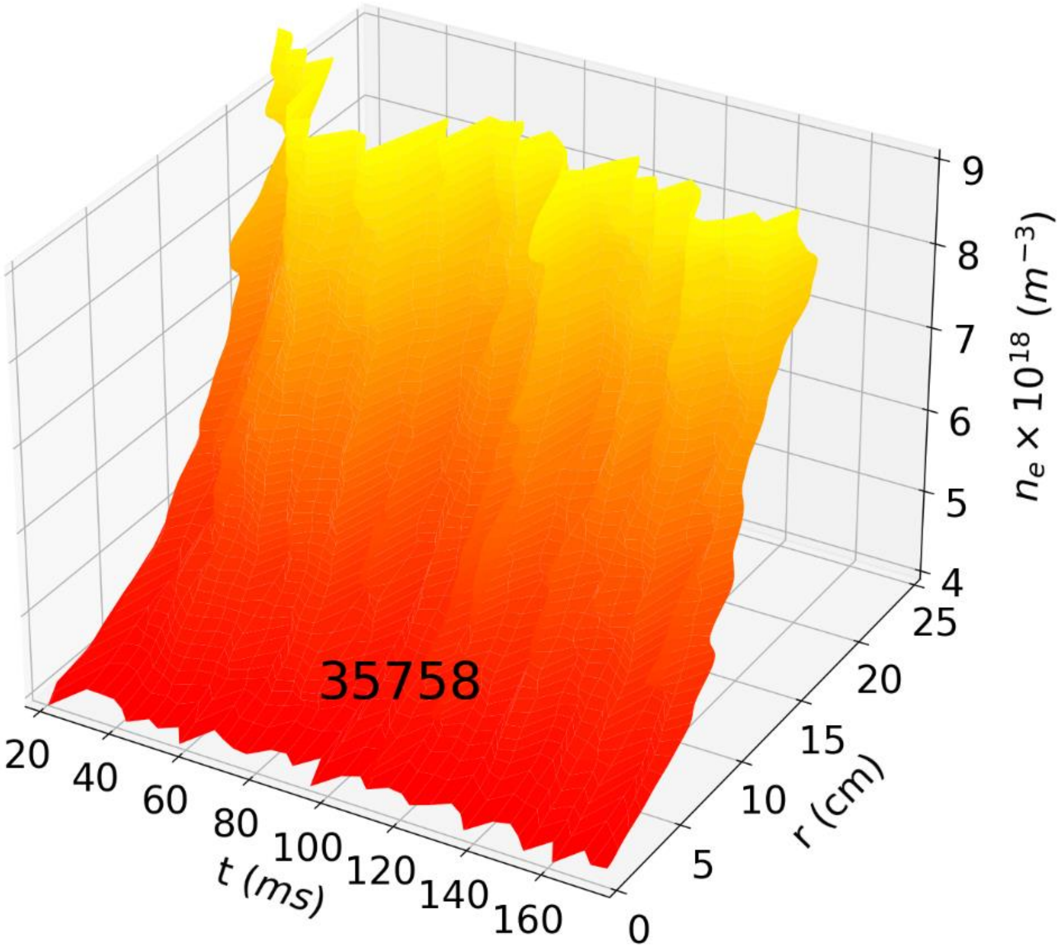


First Profile Measurements: Aditya-U



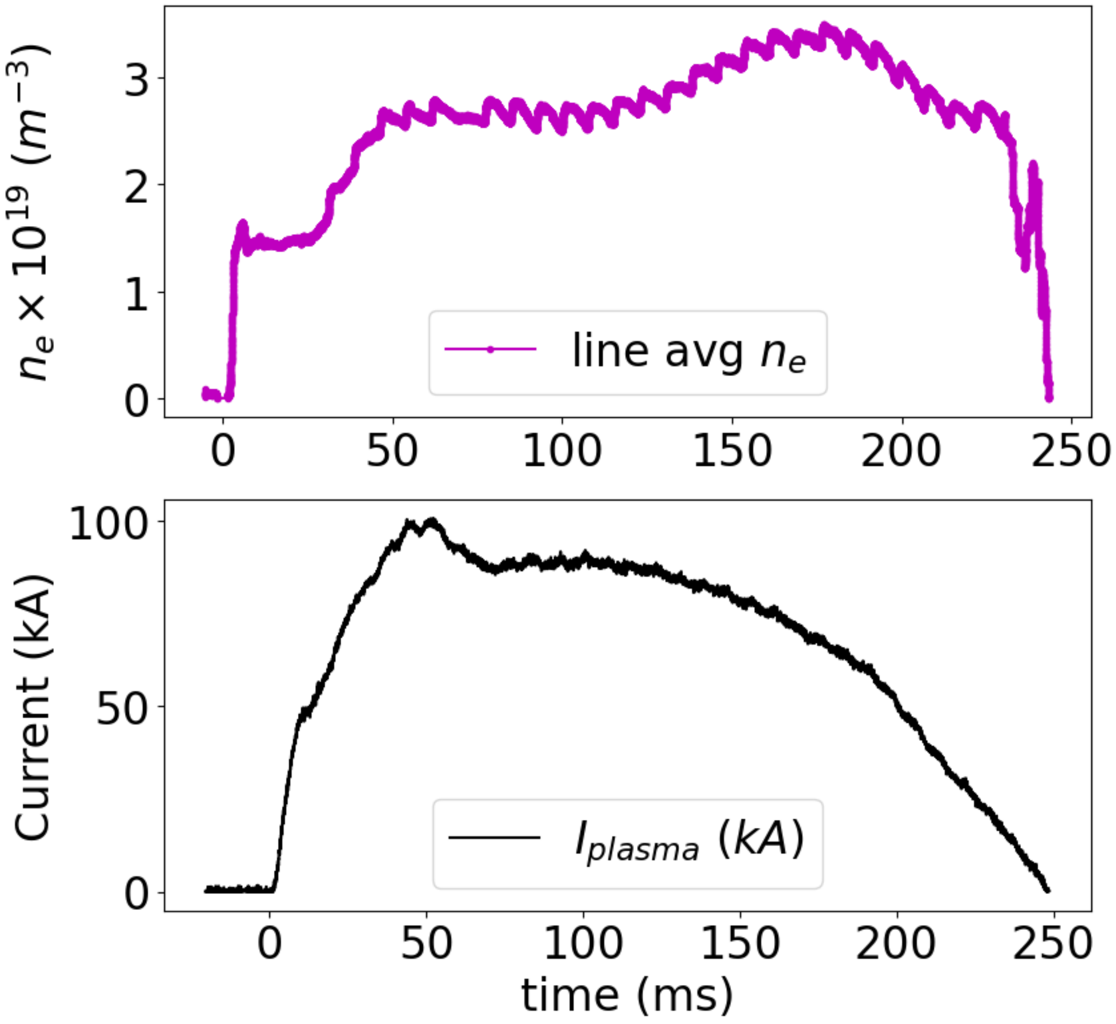






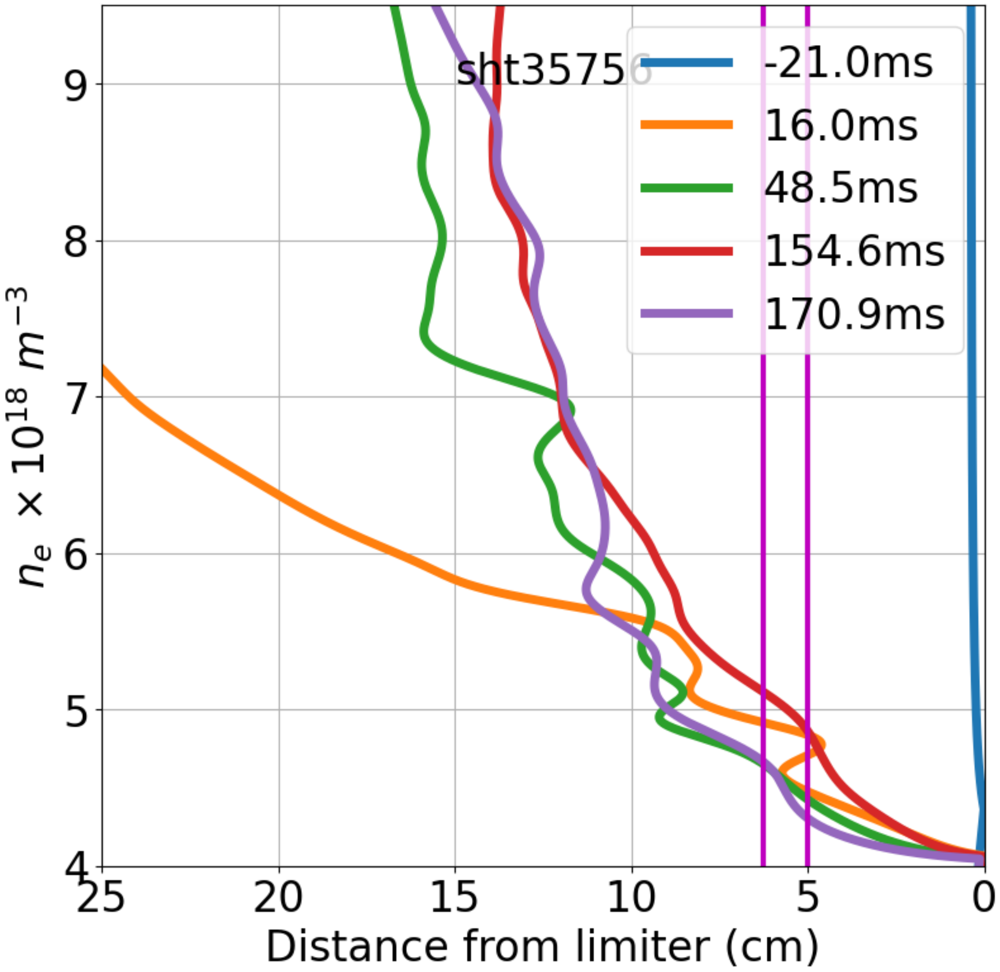
Buch, J. J. U., et al. "O-mode reflectometry on Aditya-U." *Fusion Engineering and Design* 193 (2023): 113746.

Aditya-U Shot # 35756



Time starts from $V_{loop.}$

$t_{vloop} = 0$



Summary of Experimental Work:

1) A density profile FMCW reflectometer operating in O-mode was designed for Aditya-U tokamak
2) Two channels in 18-28 GHz and 26-40 GHz frequency ranges have been designed, installed and operated on Aditya-U tokamak.
3) System linearisation, offset correction and reflection mitigation from the viewport for the Ka-Band system were presented.
4) A signal processing code to remove noise and analyze the baseband signal was developed from scratch. It includes multiple stages of filtering with change in phase, automated gain correction algorithm development was discussed. Phase due to plasma is isolated from the measured phase.
5) Density profiles measured show expected qualitative behavior with changes in plasma current and line averaged density measured by microwave interferometer diagnostic.

Edge Turbulence Simulations

1. To understand diffusion and energy loss across $B_0(r)$ due to cross-field drift dynamics of particles, simulations are required to understand the experimental results obtained.
2. Transport co-efficients estimated by neo-classical theory are an order magnitude lower than those observed experimentally as no strong turbulence is included in the theory.
3. Mazzucato observed broadband turbulence in ATC tokamak confirming micro instabilities were the cause of the anomalous transport.
4. Reflectometry Technique can also measure density fluctuations $\delta n_e/n_e$ as a function of radial position r.
5. A simple pseudo 2D model which takes in to account the strong turbulence observed is explored which can qualitatively estimate $\delta n_e(r)/n_e$ for the measured density profiles.
6. Charney Hasegawa Mima (CHM) model takes the density profile $n_e(r,t)$ as an input and provides turbulence spectra.

Assumptions for CHM equation:

- $\vec{\boldsymbol{B}} = \boldsymbol{B_0}\,\hat{\boldsymbol{z}}$ *only, and* $\boldsymbol{n_e} = \boldsymbol{n_e}(\boldsymbol{x})$, $\widehat{\boldsymbol{x}}$ is the radial direction and $\hat{y}$ is the poloidal.
- System is electrostatic, $\vec{\boldsymbol{B}} \neq \vec{\boldsymbol{B}}(t) \Rightarrow \vec{E} = -\vec{\nabla}\phi$
- Density response to $\vec{\boldsymbol{B}}$ is adiabatic. The response of electrons to the potential determines the phase shift between the density and potential profiles. The special case where the phase shift is zero gives $\delta ne/ne = e\varphi_k/Te$ and is called the adiabatic response
- Frequencies of interest $\omega \ll \omega_{ce}$ Scales involved are so large that ion gyrations are averages over one cycle.
- Cold plasma approximation is used. For Aditya tokamak ions have a temperature much lower than the electrons even in the plasma core with the ratio $T_i/T_e \approx 0.35$.

Normalized CHM equation is

$$\left[\frac{\partial}{\partial t} - \left(\vec{\nabla}\phi \times \hat{z}\right) \cdot \vec{\nabla}\right][lnn_0 + (\nabla^2\phi - k^2\phi)] = 0$$

Taking $\Omega = \nabla^2\phi - \mathrm{k}^2\phi$ and adding a viscosity term $-\nu\nabla^2\Omega$

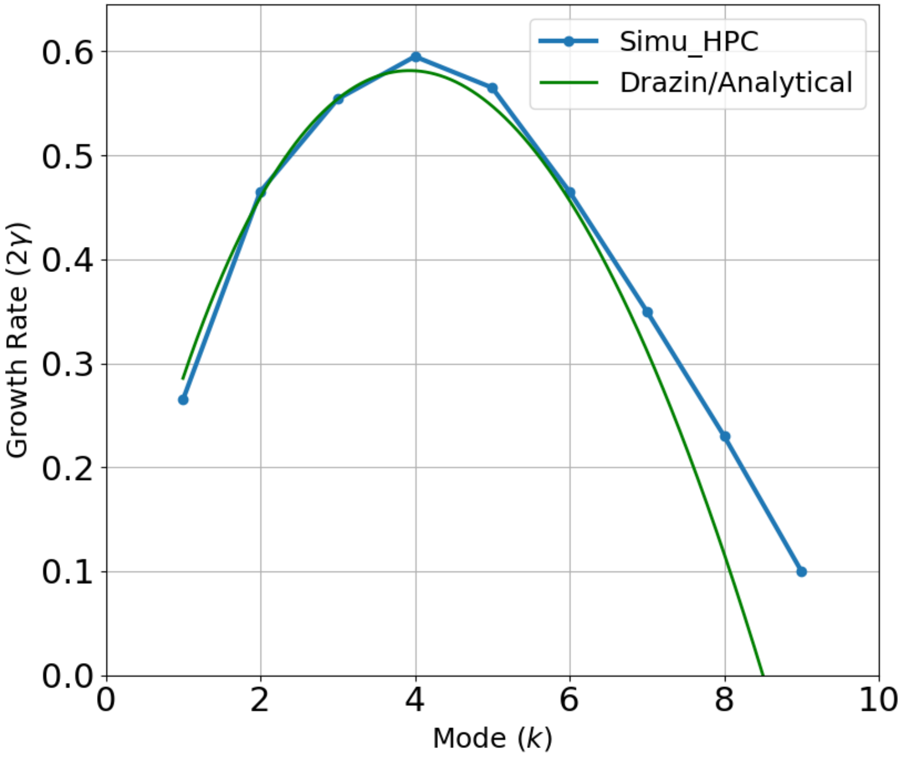
$$\boxed{\frac{\partial\Omega}{\partial t} = ((\nabla\phi \times \hat{z}) \cdot \nabla)\Omega - \nu\nabla^2\Omega + ((\nabla\phi \times \hat{z}) \cdot \nabla)\ln n_0(x)}$$

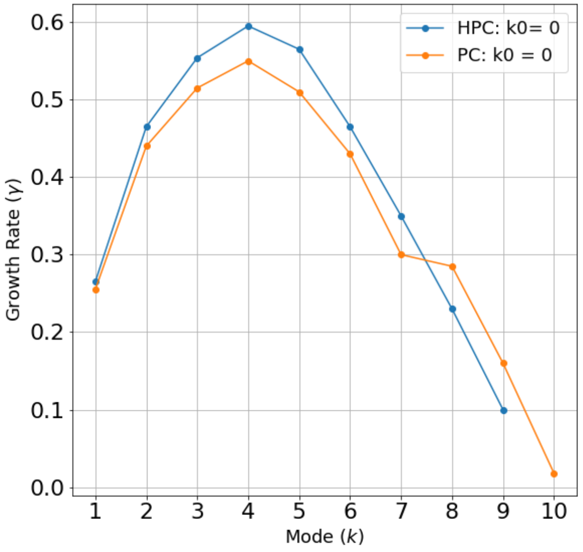
Here $(\boldsymbol{\nabla}\boldsymbol{\phi} \times \hat{\boldsymbol{z}})$ is the generalized drift velocity for a fluid parcel across $\mathbf{B}_0\hat{\boldsymbol{z}}$

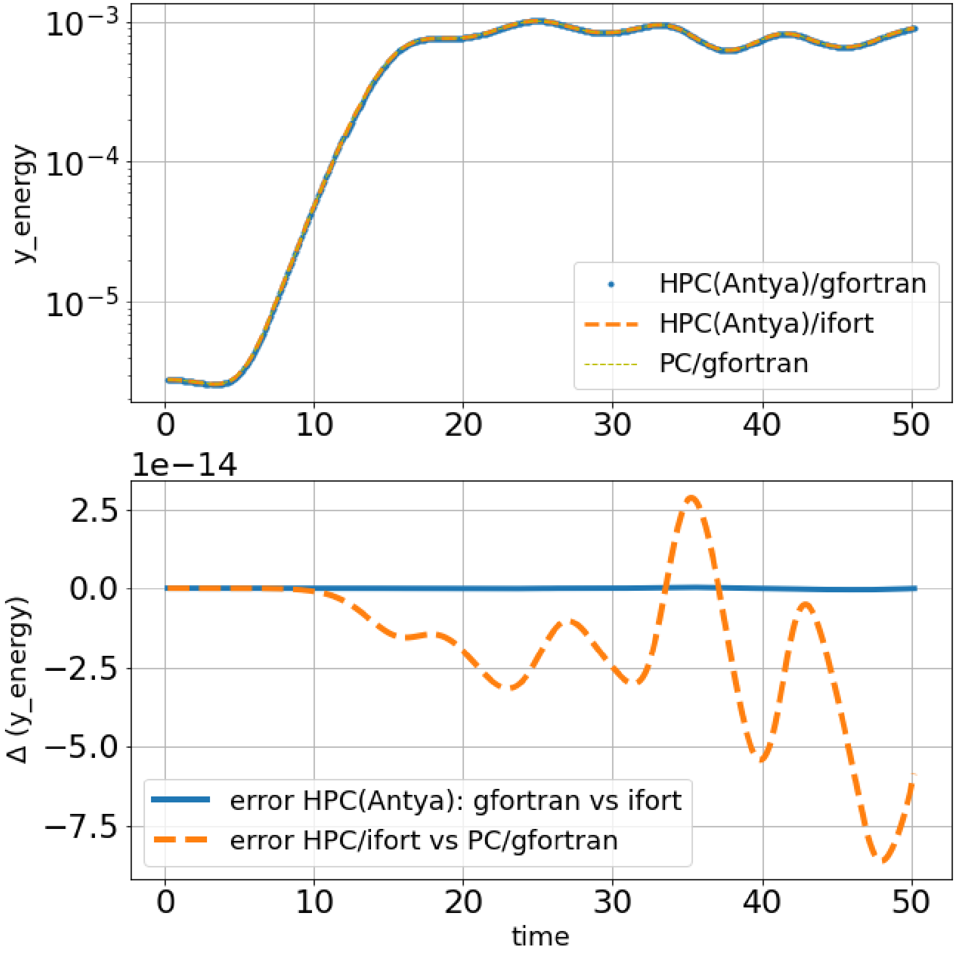
Equation describes evolution of potential ϕ and therefore density in space and time.

It is valid for incompressible ($\overrightarrow{\boldsymbol{\nabla}} \cdot \vec{\boldsymbol{u}} = \mathbf{0}$) flow in the inviscid (viscosity $\boldsymbol{\nu} \rightarrow \mathbf{0}$) limit.

Reduces to Navier-Stokes equation when $\mathbf{k}^2 \rightarrow \mathbf{0}$.



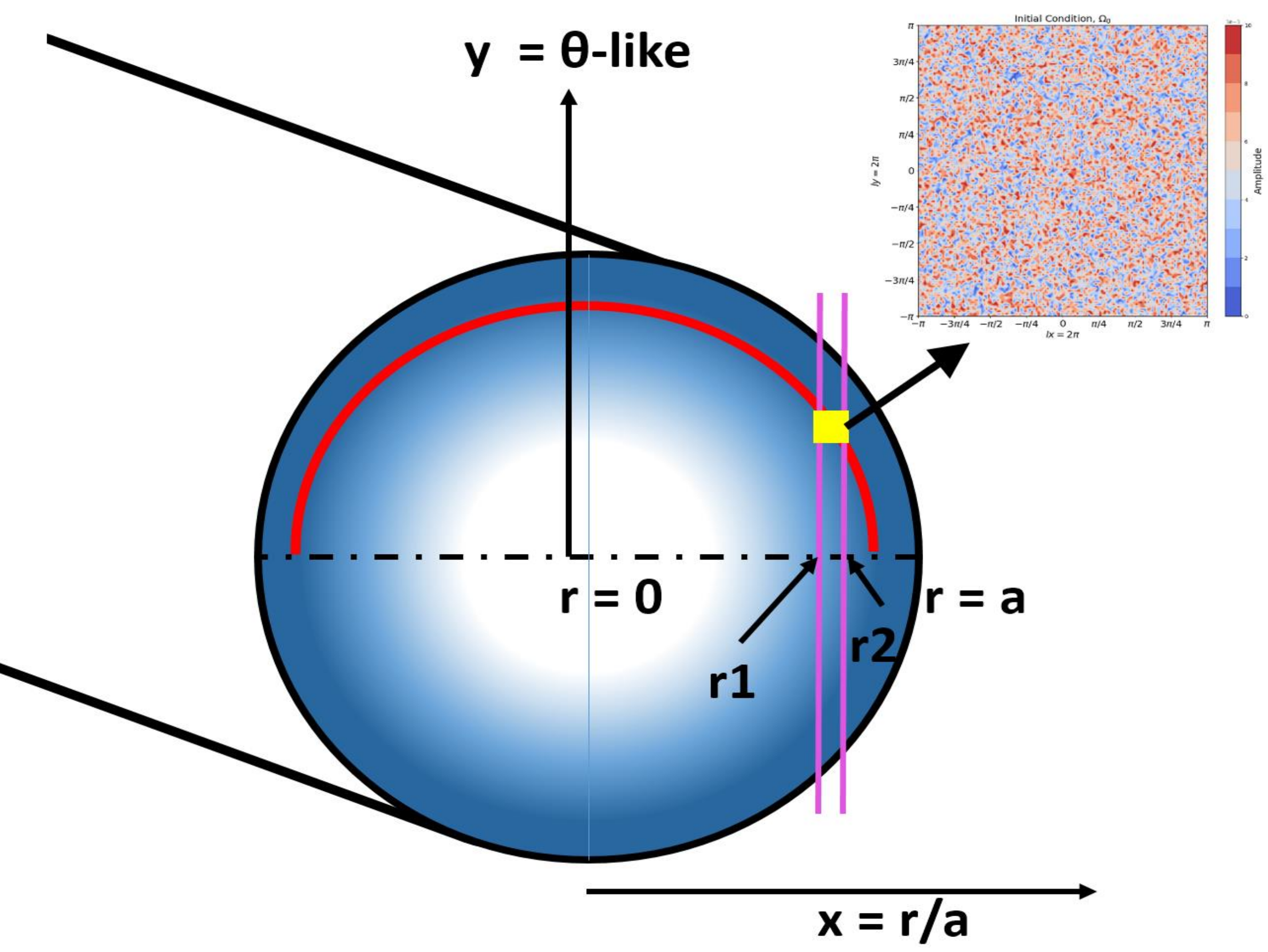


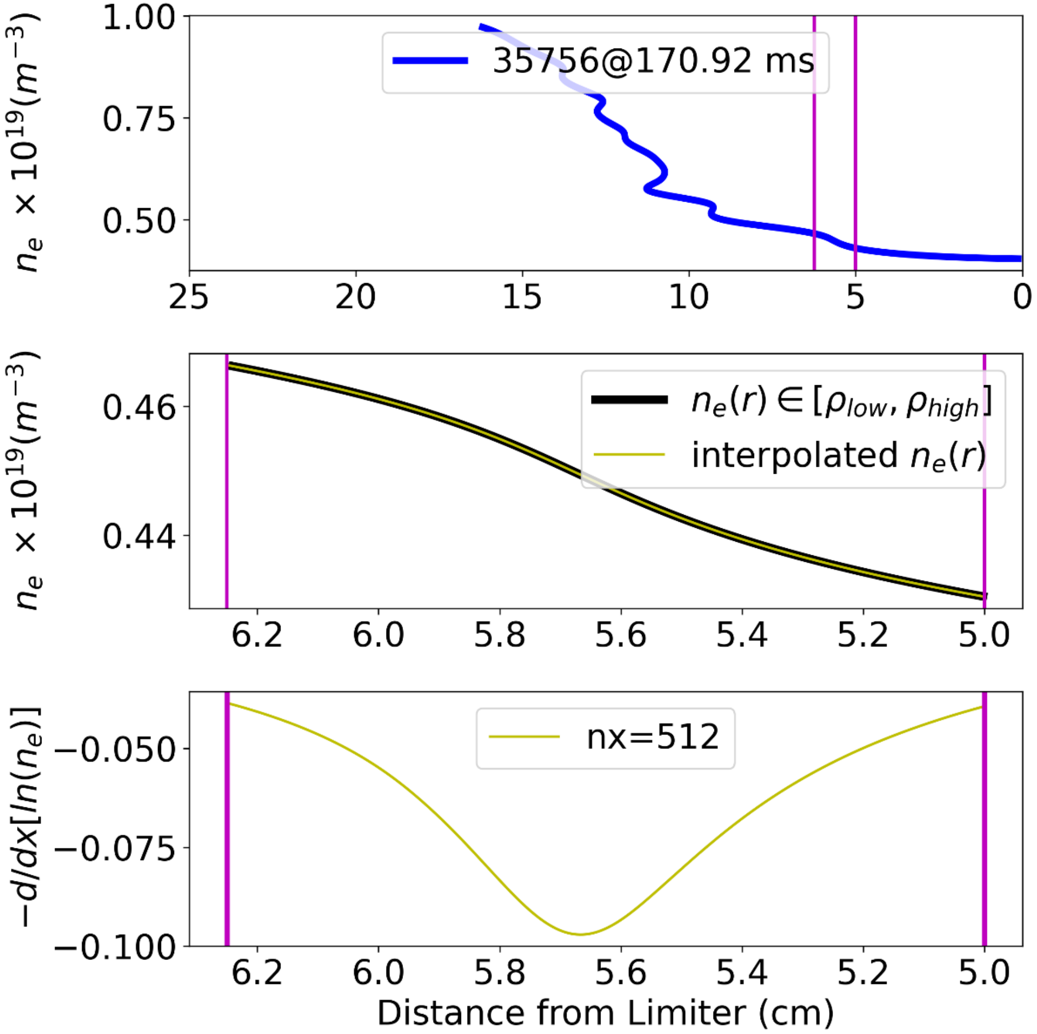


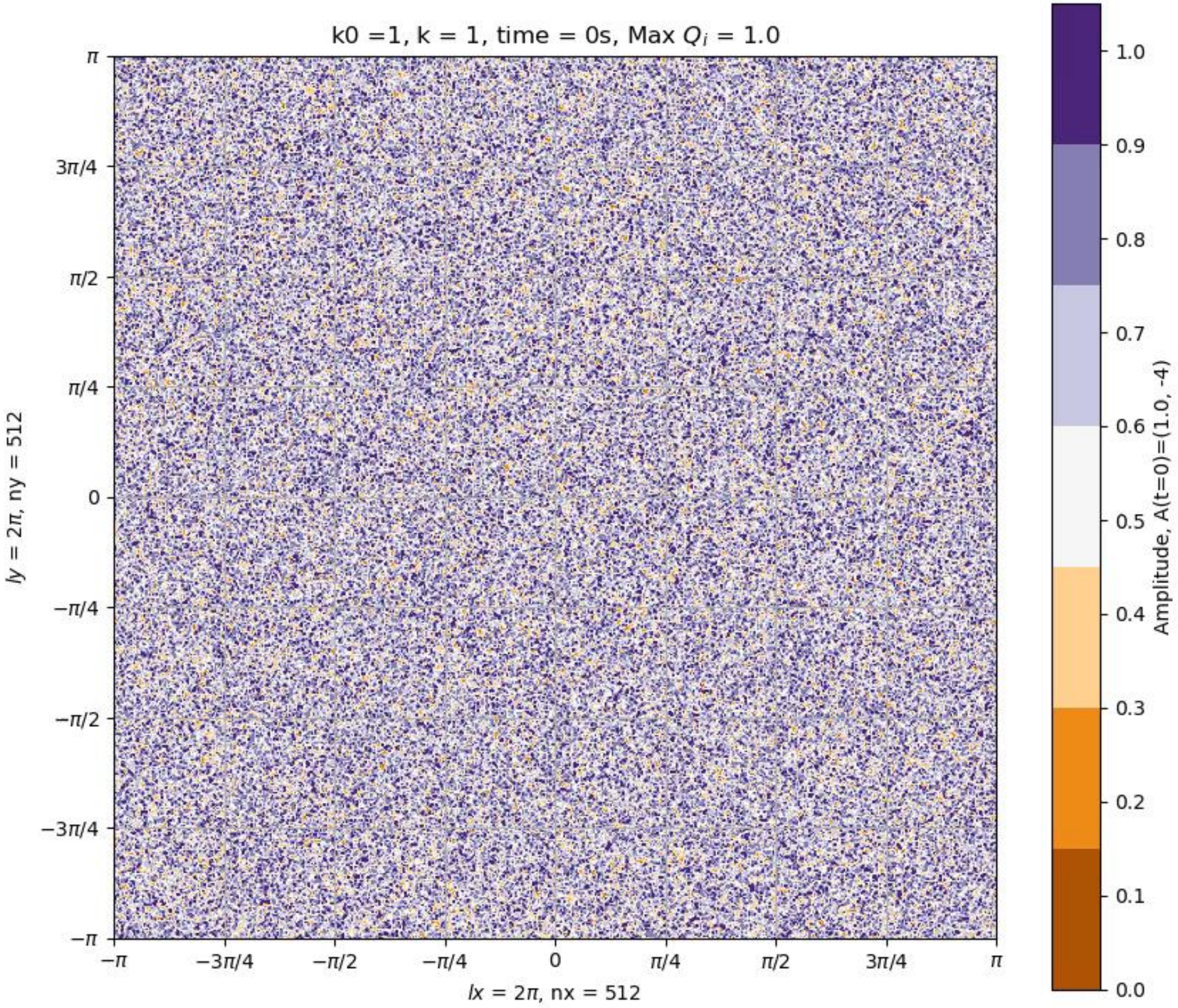
- Code is benchmarked following Drazin[1,2].
- It was parallelized using OpenMP and deployed on HPC Antya.
- Right most figure shows excellent matching between all different versions of the code and for different compilers.

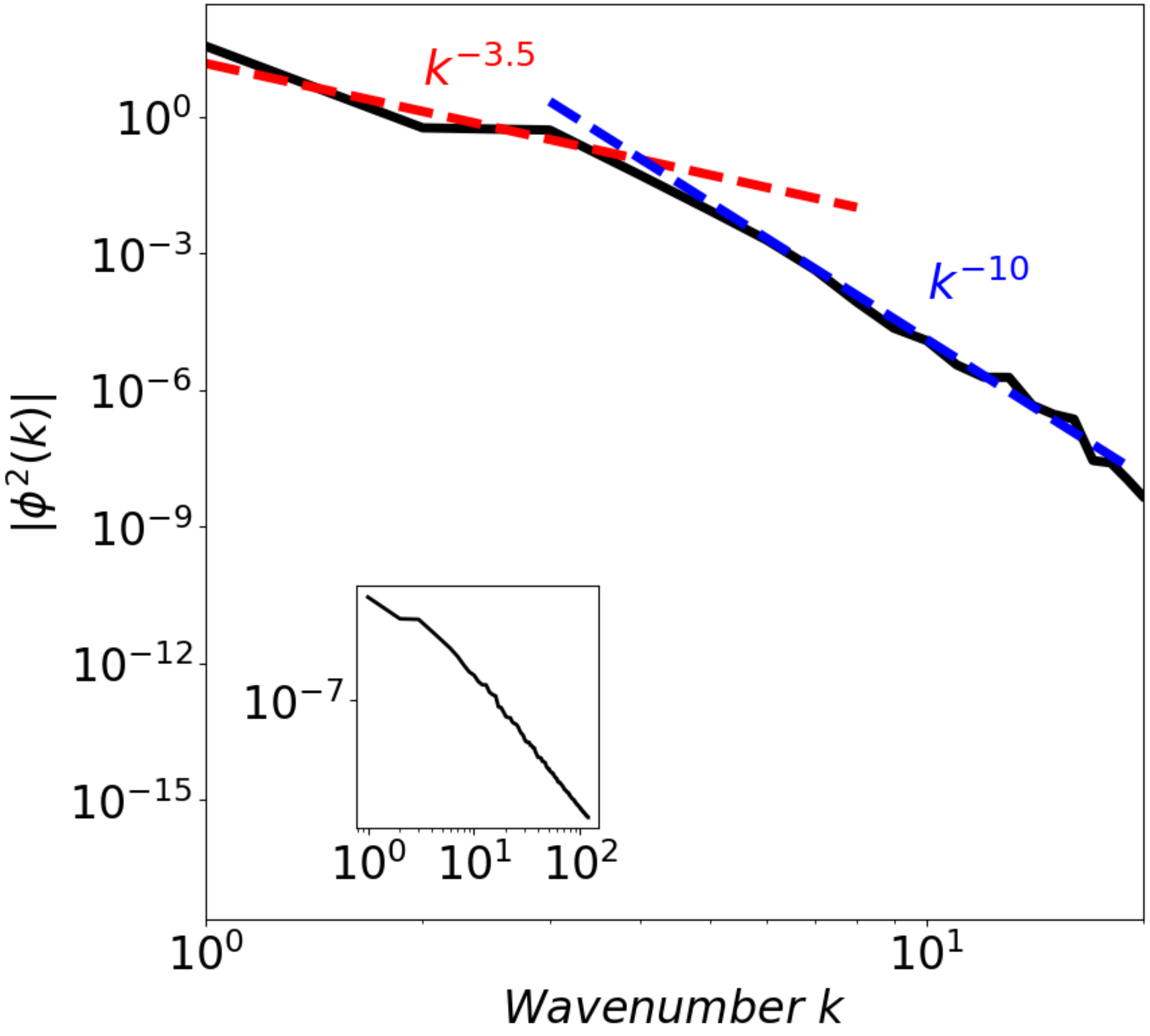
1. Drazin J Fluid Mech. 10, 571-583, 1961
2. Ashwin PRL 104, 215003, 2010.

Application to Aditya-U Shot # 35756 @ 170.92 ms






k0 =1, k = 1, time = 0s, Max Q_i = 1.0
π
3π/4
π/2
π/4
0
−π/4
−π/2
−3π/4
−π
ly = 2π, ny = 512
−π
−3π/4
−π/2
−π/4
0
π/4
π/2
3π/4
π
lx = 2π, nx = 512
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Amplitude, A(t=0)=(1.0, -4)



For collisionless plasma, relation between n_e and Plasma Potential $\phi(k)$,

$$n_e(r) = n_0 e^{\left\{\frac{e\phi}{T_e}\right\}} \sim n_0 + \frac{e\phi}{T_e}$$

Drift velocity is given by

$$\vec{v}_E = \frac{-\vec{\nabla}\phi \times \vec{z}}{B_0}$$

In general,

$$\frac{\delta n_e}{n_0} = \frac{e\phi}{T_e}(1 - i\Delta)$$

If assume phase difference Δ between n_e and ϕ is zero, then,

$$\frac{\delta n_e}{n_0} = -\frac{e\delta\phi}{T_e}$$

$$\frac{\delta n_e}{n_0} \approx \delta\phi(k)$$

Thus in ab instabilities

Qualitatively density fluctuations $\delta n_e/n_0$ follow plasma potential ϕ, but for the phase.

Ref: Weiland J. Collective Modes in Inhomogeneous Plasmas: Kinetic and Advanced Fluid Theory. CRC Press; 1999.

Summary:

- Understanding characteristics of the edge plasma is important both for predicting particle and heat fluxes onto the surfaces and for influencing the behavior of the core plasma.
- Transport of particles and energy and therefore the electrostatic potential ϕ in the edge region requires a quasi 2D description.
- We solve the lowest order description of the turbulent transport which includes the contribution of the inhomogeneity of the density profile described by the CHM equation.
- CHM equation was solved with measured Aditya-U density profile as an input.
- Time evolution of the electrostatic potential is obtained for a small region in the edge of the Aditya-U plasma for assumed density profile.
- An inverse cascade of the eddies is observed along with a drift along the $\theta - like$ direction of the Aditya-U.
- Inertial range of the inverse cascade follows the $|\phi^2(k)| \propto k^{-3}$while it decays as k^{-14}.

Future Work:

1. Extend the frequency range by adding higher frequency channels for a more complete coverage of plasma operation range
2. This will also enable use of X-mode of operation can measure past the point of peak density due to its B_0 dependence.
3. Perform dynamic linearisation of the frequency source for improving spatial resolution.

Thank You !