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Work in Progress: Direct optimization of ion transport in a W7-X-like reactor case

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Neoclassical stellarator optimization has traditionally focused on minimizing $\epsilon_{\mathbf{eff}}$

- $\epsilon_{\mathbf{eff}}$ [1] is the geometric factor that characterizes the amount of radial transport due to particles in the $1/\nu$ regime
  - $E_r/v = 0$ in the $1/\nu$ regime, which is approximately true for electrons → minimizing $\epsilon_{\mathbf{eff}}$ minimizes radial *electron* transport
- Radial ion transport "appears" to be larger than radial electron transport (ions have larger mass → smaller collision frequency → larger diffusion coefficient), but the charge separation results in an inward-pointing ("ion root") electric field to make the transport ambipolar [2]
- The ambipolar electric field aids in confining fuel ions, but the force it exerts on particles is proportional to Z → drives accumulation of high-Z impurities in the plasma

[1]: V. V. Nemov et al., PoP 6, 4622-4632 (1999).
[2]: D. D. M. Ho and R. M. Kulsrud, PoF 30, 442-461 (1987).

Minimizing ion transport directly can create beneficial properties

- Standard expression for the neoclassical particle flux per unit area of species s is

$$\Gamma_s^{neo} = -\mathbf{n}_s \mathbf{L}_{11}^s \left(\frac{n_s'}{n_s} - \frac{q_s E_r}{T_s} + \left(\frac{L_{12}^s}{L_{11}^s} - \frac{3}{2}\right)\frac{T_s'}{T_s}\right), \text{ where}$$

$$L_{ij}^s = \frac{2}{\sqrt{\pi}} \int_0^\infty h_i h_j \sqrt{K_s}\, e^{-K_s} D_{11}^s(K_s)\, dK_s \text{ for } h_1 = 1,\ h_2 = K_s,\ K_s = \frac{m_s v_s^2/2}{T_s} = \left(\frac{v_s}{v_{s,th}}\right)^2, \text{ and}$$

$$D_{11}^s = \frac{\pi}{16} \frac{m_s^2 v_s^3}{\iota R_0 B_0^2 q_s^2} D_{11}^* \propto m_s^{1/2} T_s^{3/2} K_s^{3/2} D_{11}^* \text{ [3]}$$

- Note that $L_{11}^s \propto m_s^{1/2} T_s^{3/2} \int_0^\infty K_s^2 e^{-K_s} D_{11}^* dK_s$, so we would typically expect $L_{11}^e \ll L_{11}^i$

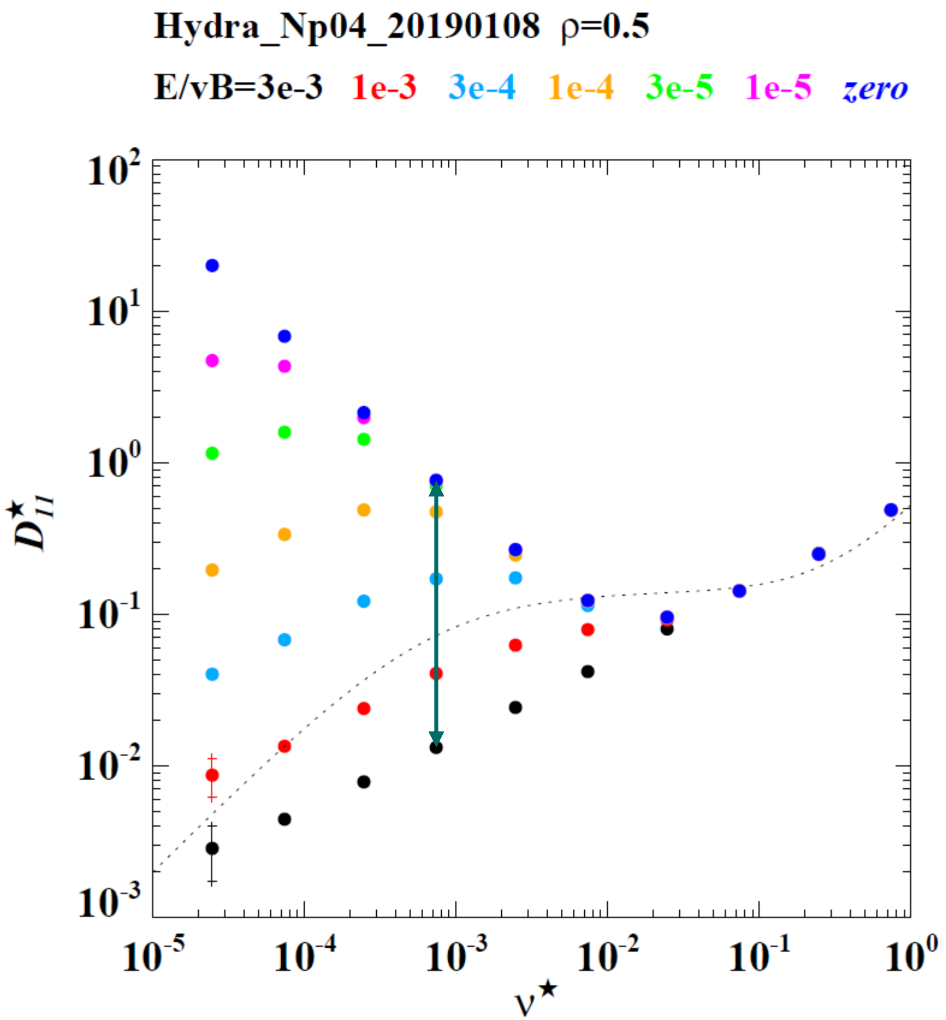
[3]: C. D. Beidler et al., NF 51, 076001 (2011).

Minimizing ion transport directly can create beneficial properties

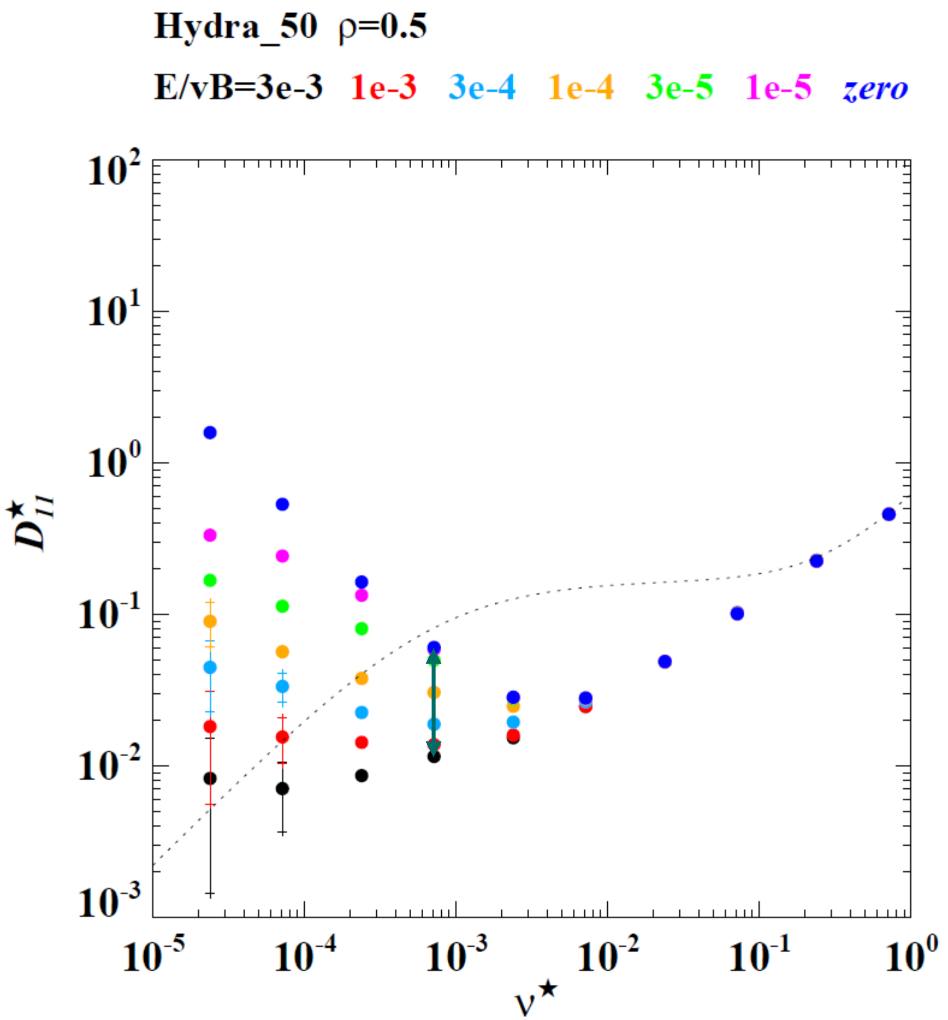
- As mentioned in [4] and [5], increasing the value of the ratio of thermal transport coefficients L_{11}^{e}/L_{11}^{i} can lead to “temperature screening of impurities” and, if increased further, an outward-pointing (“electron root”) ambipolar electric field can result
  - Electron root can aid with fuel confinement [6] (due to large $|E_r|$) and driving out impurities (due to $E_r > 0$)
- L_{11}^{e}/L_{11}^{i} can be increased by increasing the temperature or lowering the density (lowering the collisionality) because of the different collisionality scalings in the $1/\nu$ and $\sqrt{\nu}$ regimes [4]
  - But there are practical limits to this in a reactor
- Can we optimize the magnetic field to increase L_{11}^{e}/L_{11}^{i} rather than relying so heavily on the temperature and density profiles?

[4]: J. L. Velasco et al., NF 57, 016016 (2017).
[5]: C. D. Beidler et al., oral presentation, Simons Collaboration Greifswald Retreat (July 2022).
[6]: H.E. Mynick and W. N. G. Hitchon, NF 23, 1053-1059 (1983).

The magnetic field can substantially affect L_{11}^e/L_{11}^i



“Magic Configuration” (MC)



“Low-ϵ_{eff} Configuration” (LEC)

Figures taken from [5]: C. D. Beidler et al., oral presentation, Simons Collaboration Greifswald Retreat (July 2022).

The magnetic field can substantially affect L_{11}^{e}/L_{11}^{i}

- The MC has a large "spread" between the electron- and ion-relevant D_{11}^{*} values, whereas the LEC has "squashed" them together
- Because $L_{11}^{s} \propto m_s^{1/2} T_s^{3/2} \int_0^{\infty} K_s^2 e^{-K_s} D_{11}^{*} dK_s$, the MC has larger L_{11}^{e}/L_{11}^{i} than the LEC
- Calculations by Håkan and Craig using DKES [7,8] and NTSS [9] suggest the MC with self-consistent, reactor-relevant profiles can have an electron root out to roughly $\rho = \sqrt{s} = 0.4$
  - The LEC and W7-X high-mirror reactor have ion roots when the same calculations done for them

[7]: S. P. Hirshman et al., PoF 29, 2951-2959 (1986).
[8]: W. I. van Rij and S. P. Hirshman, PoF B 1, 563-569 (1989).
[9]: Y. Turkin et al., Fus. Sci. and Tech. 50, 387-394 (2006).

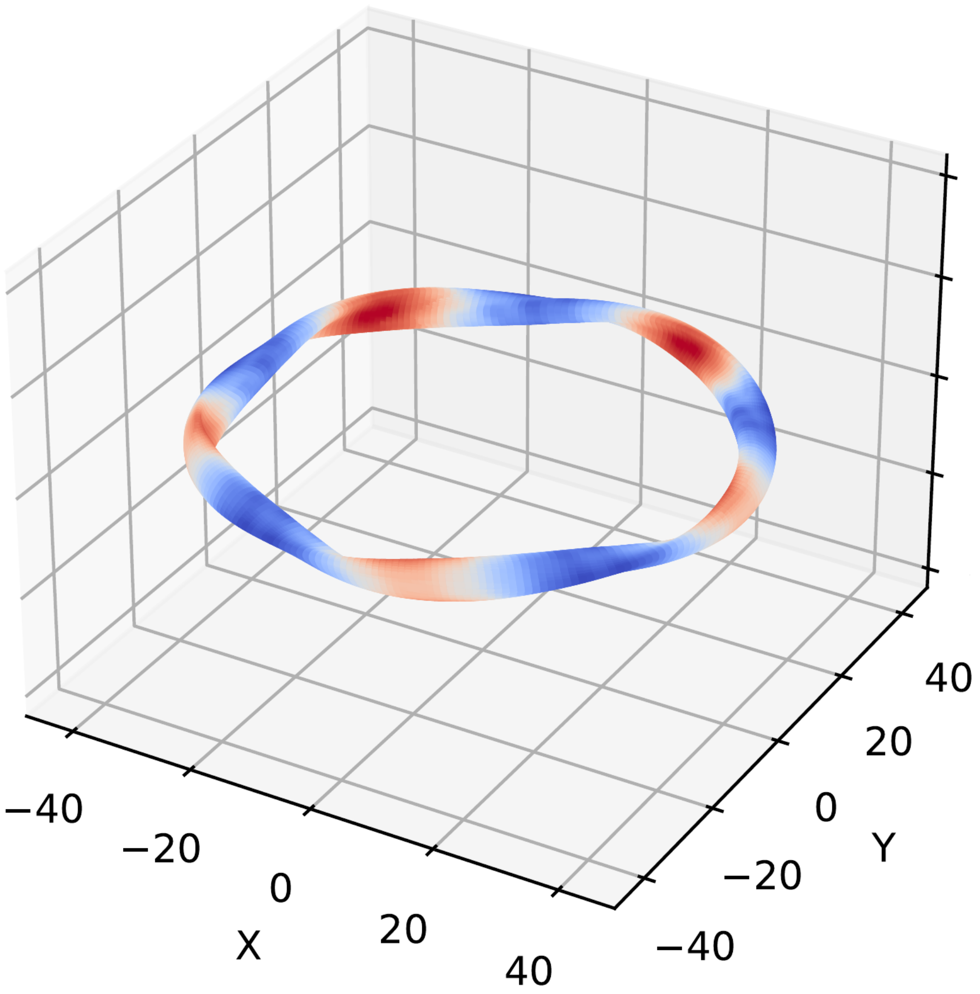
General optimization ideas

- Use STELLOPT with VMEC [10] for finite-β optimizations; change the boundary shape and see how it affects neoclassical transport properties
  - Utilize differential evolution [11] and a Garabedian boundary representation [12] to more reliably find global minima
- Over-optimizing the electron transport creates problems, so target "moderate" ϵ_{eff}, say $1\% - 3\%$
- Minimizing the ion transport is good; choose several pairs of ν/v and E_r/v relevant to ions in reactors, calculate corresponding D_{11}^* values with DKES, and drive them down as far as possible
- Can "polish" the configuration by using SFINCS [13] to directly target thermal transport coefficients
- Postprocess with DKES and NTSS to search for an electron root

[10]: S. P. Hirshman and J. C. Whitson, PoF 26, 3553-3568 (1983).
[11]: R. Storn and K. Price, J. of Global Opt. 1997, 341-359 (1997).
[12]: P. R. Garabedian, Proc. Nat. Acad. Sci. 95, 9732-9737 (1998).
[13]: M. Landreman et al., PoP 21, 042503 (2014).

Optimization starting point: W7-X-like reactor

- **Start from the reactor configuration of [14]**
  - **Based on the W7-X high-mirror configuration**
  - **Major radius ≈ 22 m, aspect ratio ≈ 10.9, on-axis magnetic field ≈ 5.3 T, volume-averaged $\beta \approx 3.5\%$, zero bootstrap current assumed**
- **Increase major radius such that the initial aspect ratio is ≈ 20**



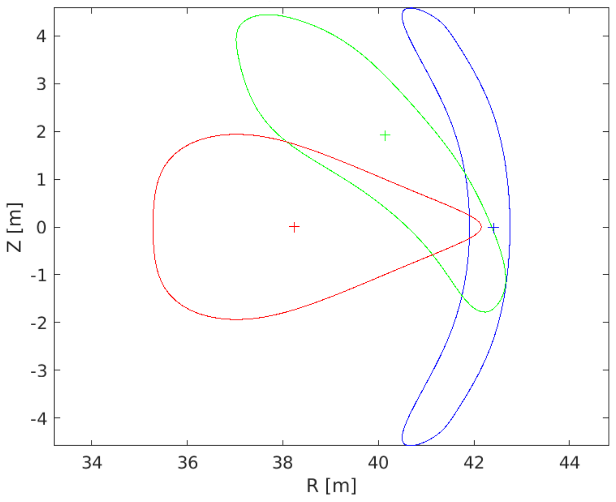
[14]: S. A. Lazerson et al., PPCF 63, 125033 (2021).

Outline of best optimization recipe so far

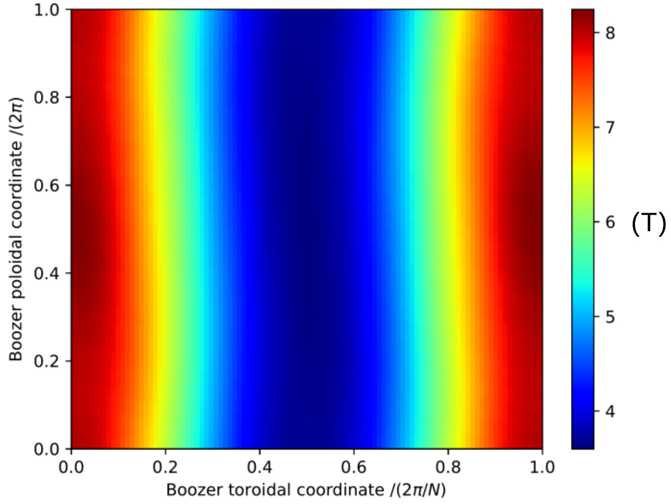
- All optimizations targeted quantities on the $\rho = 0.25, 0.5, 0.75$ flux surfaces
- Step 1: Target QP symmetry
- Step 2:
  - Lower weight on QP symmetry targets to make them of secondary importance
  - Target $\epsilon_{\text{eff}} = 1\%$
  - Minimize 3×3 "grid" of ion-relevant monoenergetic transport coefficients using DKES
    - ν/v in the range 2.5E-6 – 1.0E-4, E_r/v in the range 5.0E-4 – 1.5E-2
- Step 3: Use SFINCS to directly target thermal transport coefficients
  - Set $E_r = 15$ kV/m (somewhat arbitrary)
  - Try to hold L_{11}^e steady and minimize L_{11}^i, L_{31}^e, and L_{31}^i (least-squares objective, all terms weighted equally)
  - Optimization driven by differential evolution implemented in SciPy [15]

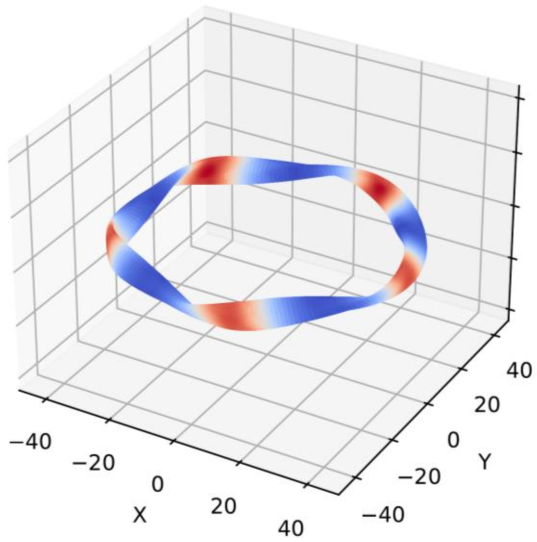
[15]: P. Virtanen et al., Nat. Meth. 17, 261-272 (2020).

Postprocessing an example optimization



$\rho = 0.5$

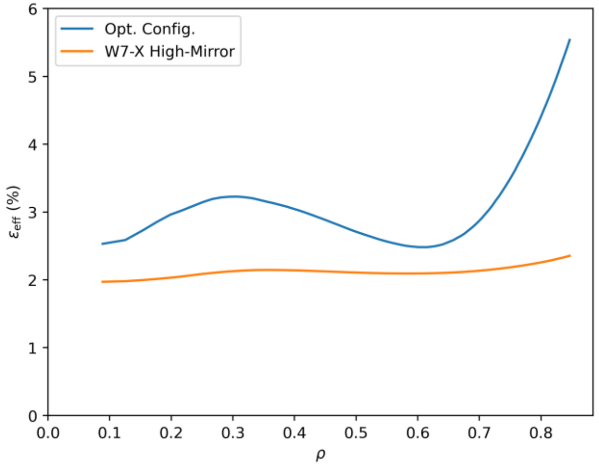


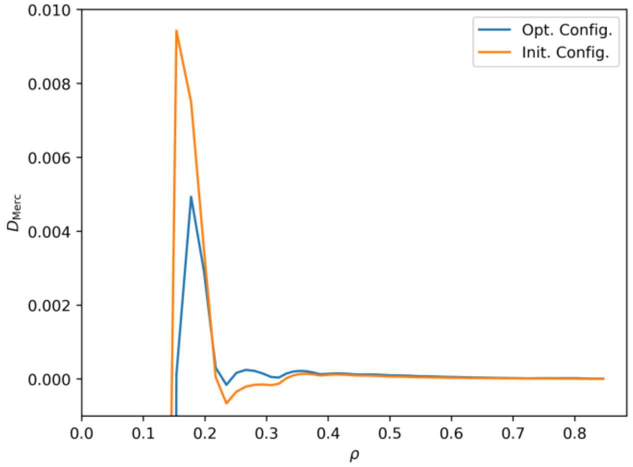


$B_{min} \approx 3.3$ T

$B_{max} \approx 8.1$ T

$\langle\beta\rangle \approx 3.49\%$



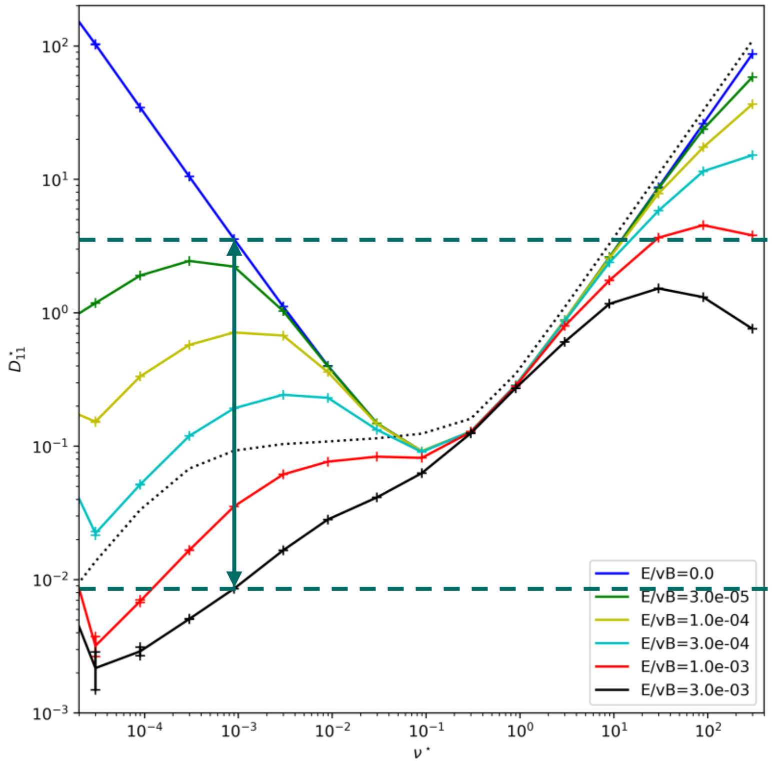


D_{Merc} formulation is from [16] and [17]:

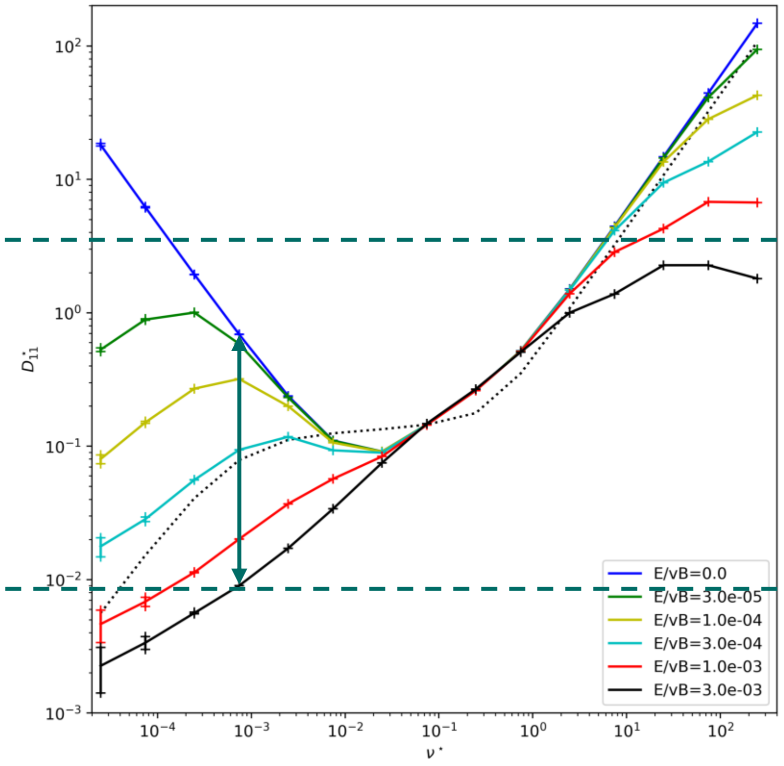
$D_{\mathrm{Merc}} > 0$ indicates stability

[16]: F. Bauer et al., “Magnetohydrodynamic Equilibrium and Stability of Stellarators,” Springer-Verlag (1984).
[17]: K. Ichiguchi et al., NF 33, 481-492 (1993).

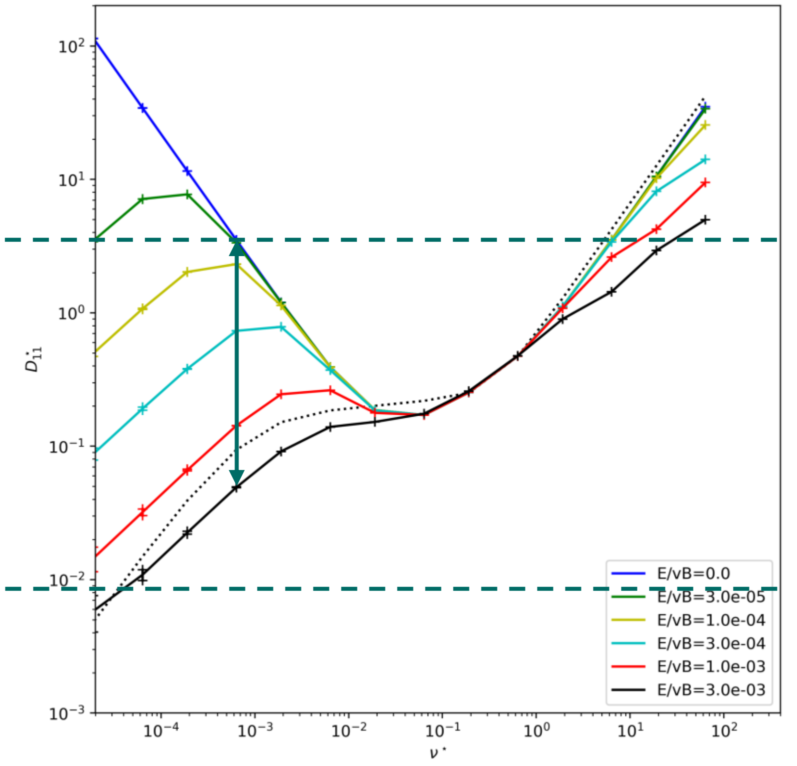
Postprocessing an example optimization



Current Opt., $\rho \approx 0.365$



MC, $\rho \approx 0.365$



W7-X High-Mirror, $\rho \approx 0.365$

Postprocessing an example optimization

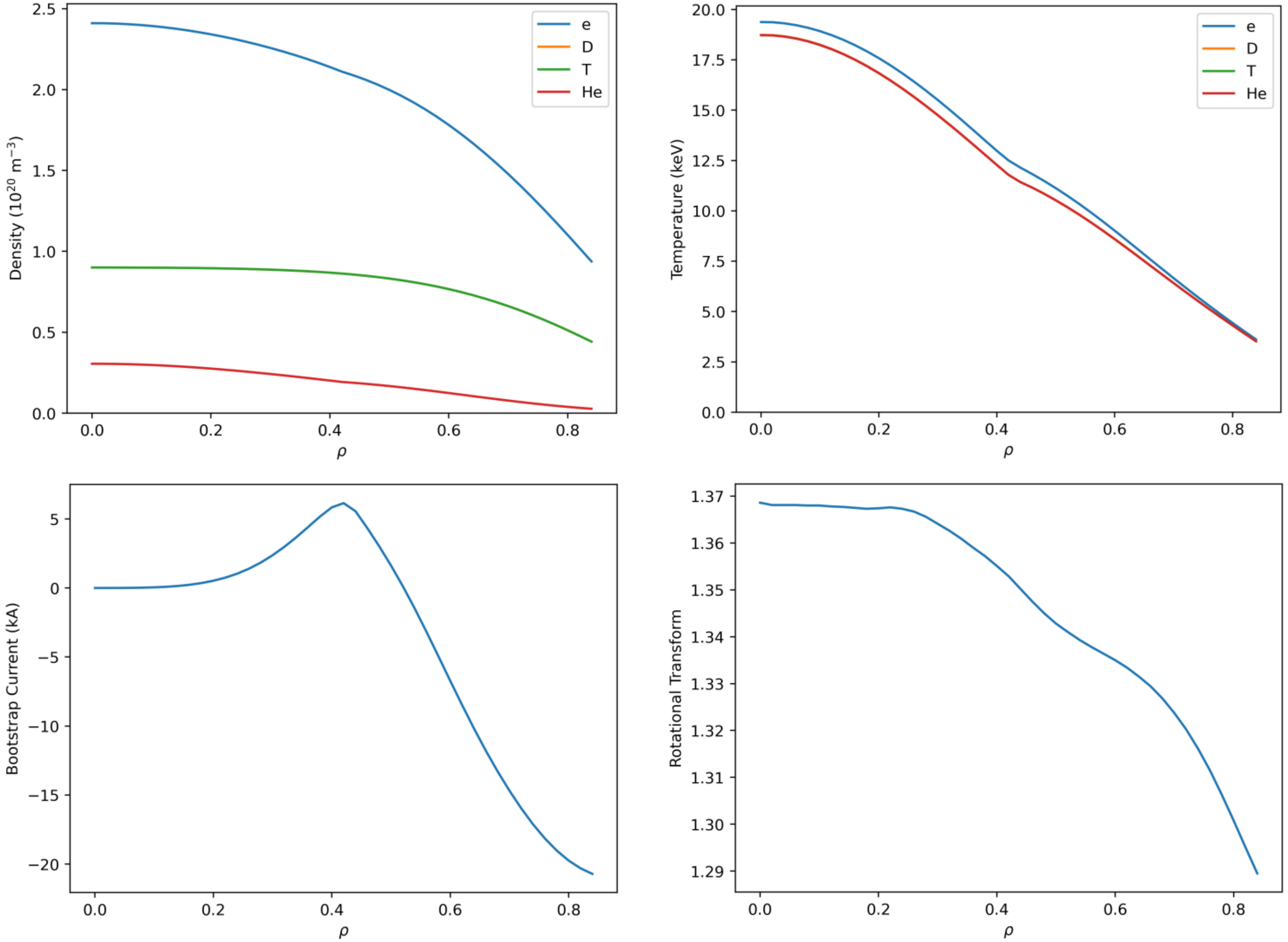
- **Use NTSS for self-consistent transport calculations:**
  - **Scale reactor volume to 1450 m^3 and B_0 to 5 T**
  - **Fix n_D and n_T profiles with core values of $\approx 0.9 \times 10^{20}\ \mathrm{m}^{-3}$; relatively flat in ρ (in the core)**
    - **Use D-T cross section to calculate n_{He} and get n_e from quasineutrality**
  - **Initialize core temperatures at ≈ 15 keV; roughly parabolic in ρ**
  - **Include very simple turbulent transport model (W7-AS fit) [18]:**

$$Q_s^{turb} = -C_1 P^{3/4} T_s' \text{ and } \Gamma_s^{turb} = -C_2 P^{3/4} \frac{n_s'}{n_s}, \text{ where } P = P_\alpha - P_{Br}$$

  - **We assume that startup (density ramping, ECRH heating, etc.) is performed such that an electron-root E_r can be produced in the plasma; this is path-dependent [19,20]**
    - **Exact procedure is beyond the scope of this work – we focus on making the electron root realizable from a neoclassical point of view**
  - **NTSS does not change the magnetic configuration itself**

[18]: H. Ringler et al., PPCF 32, 933-948 (1990).
[19]: D. E. Hastings et al., NF 25, 445-454 (1985).
[20]: D. E. Hastings, PoF 29, 536-243 (1986).

Postprocessing an example optimization



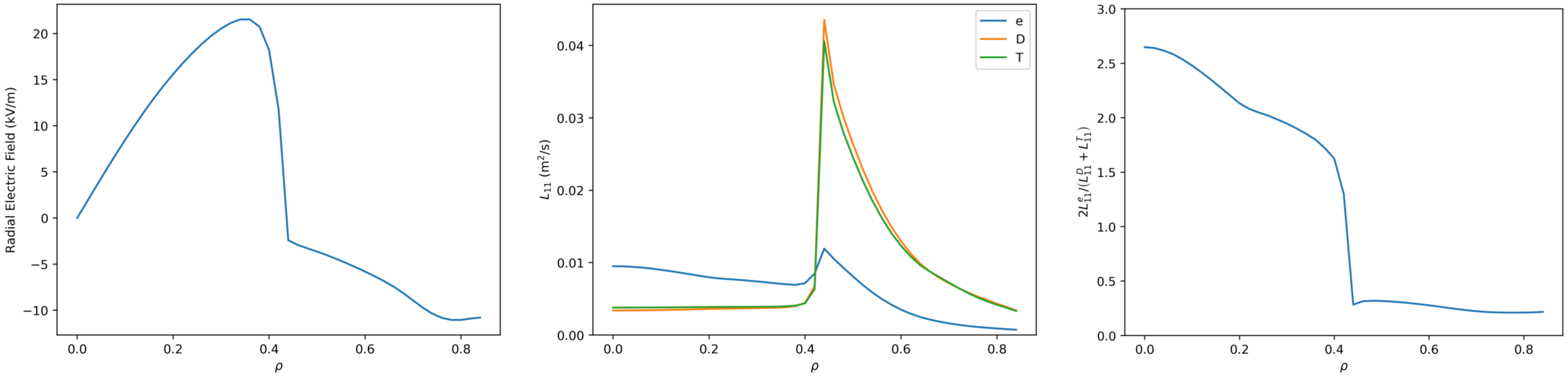
$$< \beta > = 4.93\%$$

$$P_\alpha = 488.88 \text{ MW}$$

$$P_{Br} = 72.64 \text{ MW}$$

$$\tau_E = 2.09 \text{ s} = 1.28\tau_E^{ISS04}$$

Postprocessing an example optimization



Early conclusions

- The magnetic field geometry can be optimized to modify ion and electron transport separately
  - We can use this to raise L_{11}^{e}/L_{11}^{i} and gain beneficial properties by doing so
- So far, these optimizations have been quite slow – computationally intensive

Future directions

- Finding a configuration that is not “troublesome” near the LCFS is essentially mandatory
- Including other objectives, such as aspect ratio, mirror ratio, elongation, shear, magnetic well, etc. is appropriate
- KNOSOS [21] may be a good replacement for DKES in the optimizations – it can calculate various neoclassical quantities (including tangential magnetic drifts) very quickly for low-collisionality plasmas
- Transport solvers that couple to gyrokinetic codes are becoming available and may be a good replacement for NTSS:
  - TANGO + KNOSOS + GENE-3D [22]
  - Trinity3D + KNOSOS + GX (under development)
- Suggestions?

[21]: J. L. Velasco et al., J. Comp. Phys. 418, 109512 (2020).
[22]: A. Bañón Navarro et al., NF 63, 054003 (2023).

Attributions

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- Computations were performed on the Cobra and Raven HPC systems at the Max Planck Computing and Data Facility.

Extra slides below

Other optimized configuration plots