MAX-PLANCK-INSTITUT
FÜR PLASMAPHYSIK

Study of finite Larmor radius (FLR) physics in tokamaks and stellarators with the gyrokinetic turbulence code GENE-X

José Capitán, Philipp Ulbl, Baptiste J. Frei, Frank Jenko

HEPP Introductory Talk

EUROfusion

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SPAIN

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Bachelor → Physics

Master → thesis in **CIEMAT** doing simulations with `stella` in **W7-X high mirror** and **CIEMAT-QI** configs.

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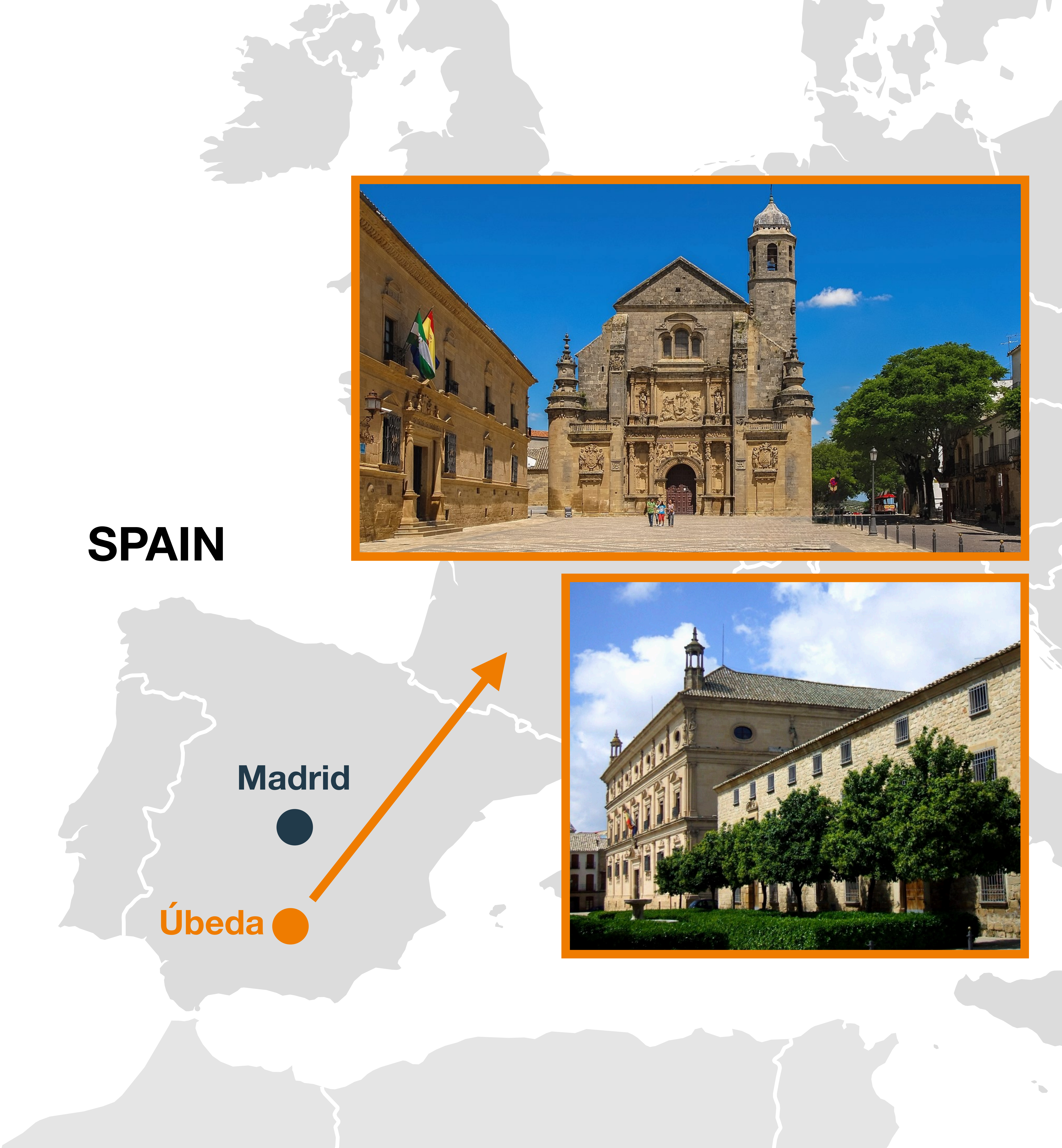

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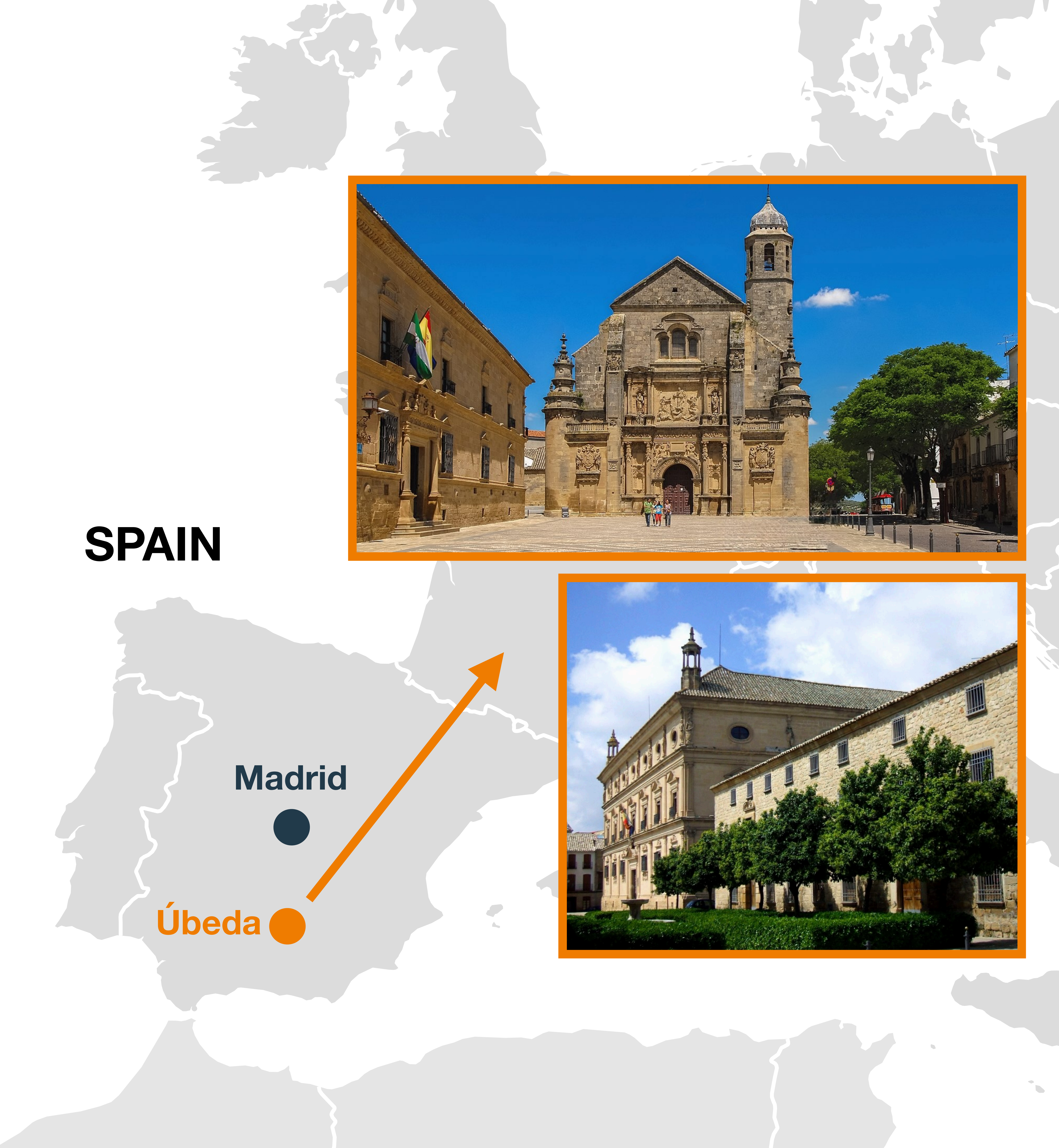
BEST THING TO DO IN SPAIN IS ~~LEAVING~~ EATING



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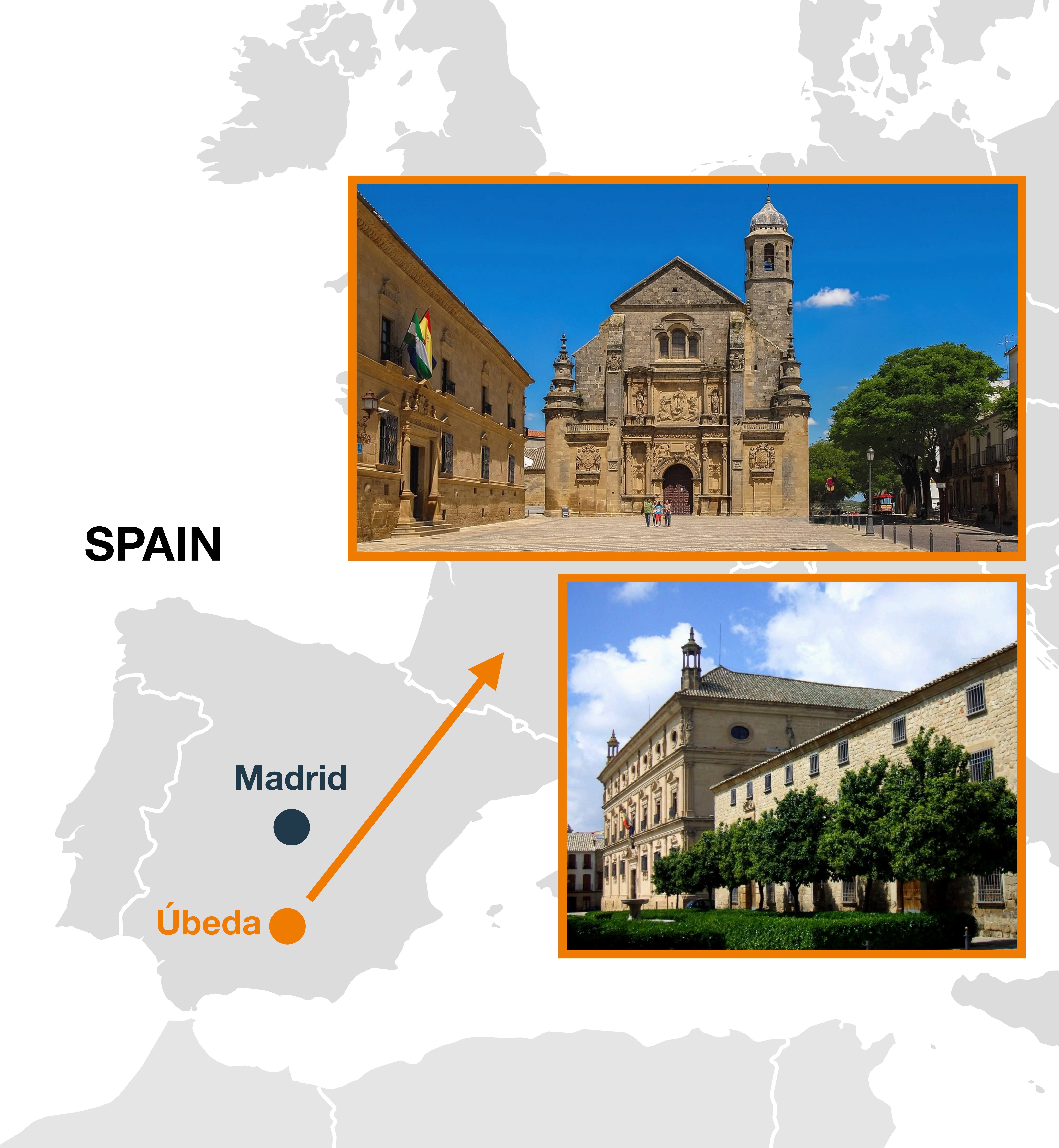
BEST THING TO DO IN SPAIN IS ~~LEAVING~~ EATING

Spanish omelette



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Ham "serrano"

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right way to do it? :(

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Olive oil

SPAIN

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random fact

the king of Spain is also

random fact

the king of Spain is also the king of Jerusalem 😐

???

random fact

the king of Spain is also the king of Jerusalem 😐

???

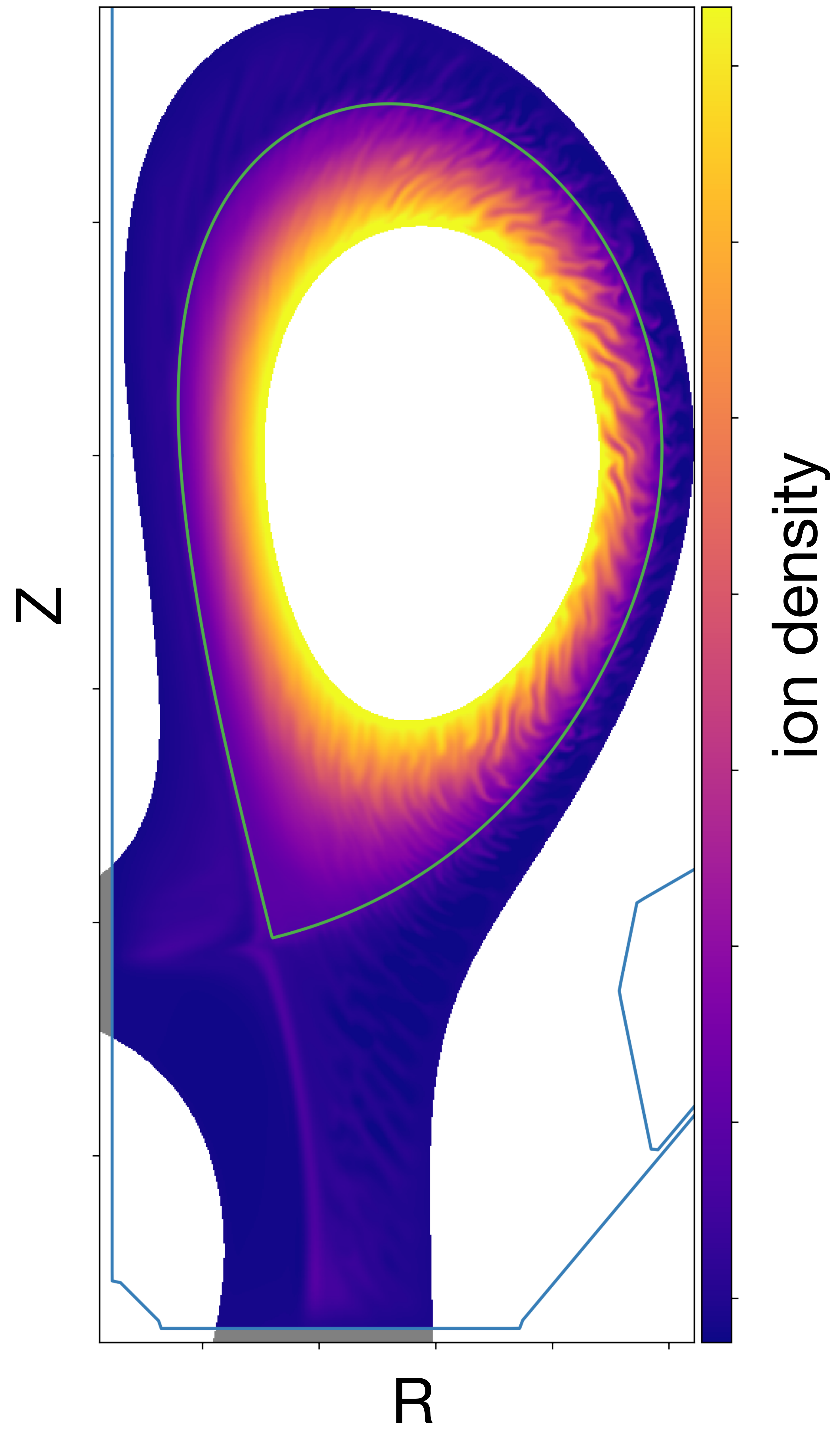
now blackouts too !!! 😃

PhD project

Motivation

MICRO TURBULENCE

- small structures lead to macroscopic losses in magnetically confined plasmas
- optimize energy confinement → losses minimization
- understand it in the plasma
  - **core** → short wavelength
  - **edge** → long wavelength



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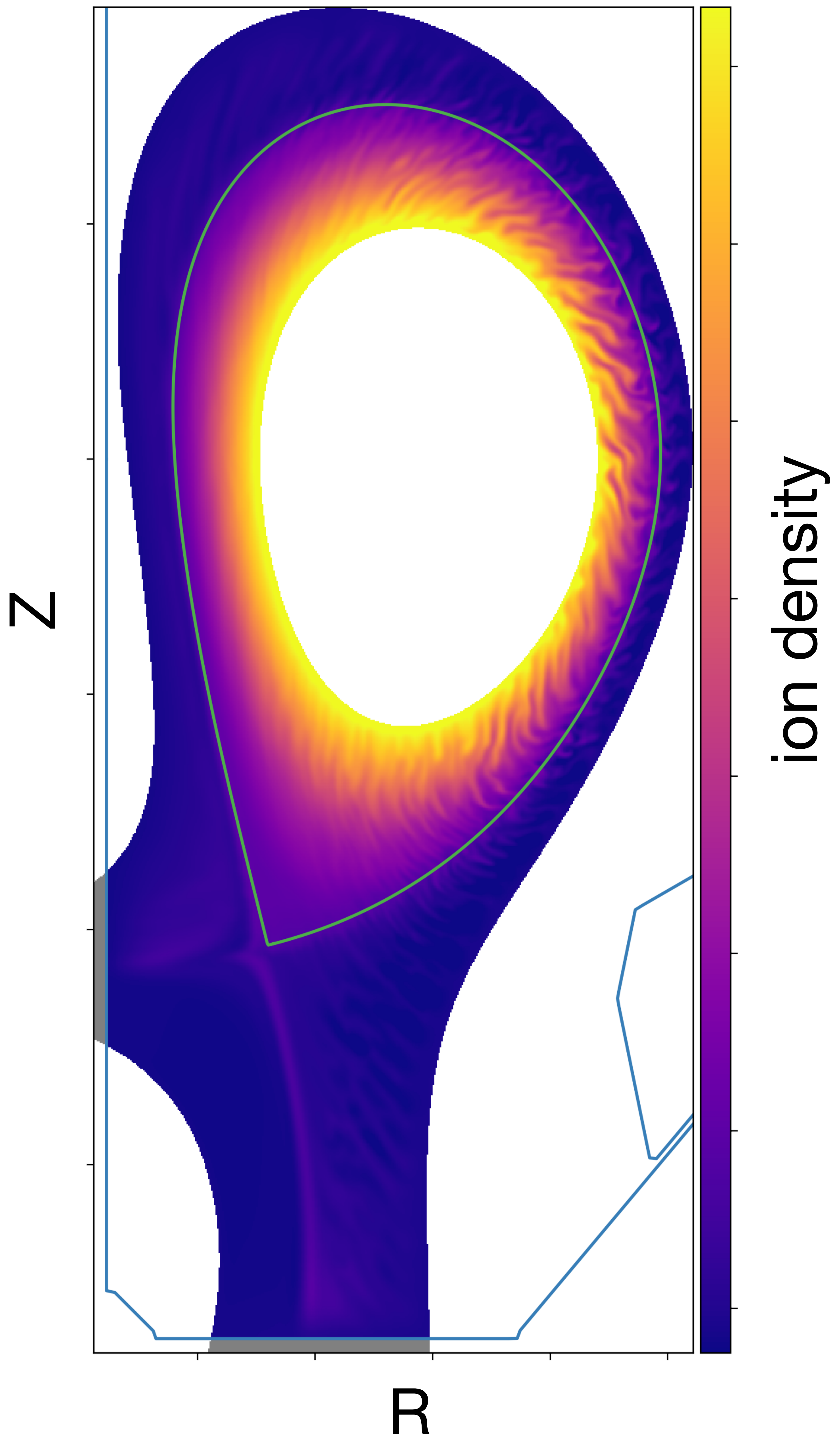
GENE-X

- Eulerian code that solves the gyrokinetic Vlasov eq. on a grid
- collisional, full-f, EM gyrokinetic turbulence model
- addresses the complexities of **edge turbulence** simulations



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MICRO TURBULENCE

- small structures lead to macroscopic losses in magnetically confined plasmas
- optimize energy confinement → losses minimization
- understand it in the plasma
  - **core** → short wavelength **?**
  - **edge** → long wavelength

GENE-X → **optimized for long wavelength turbulence!**

- Eulerian code that solves the gyrokinetic Vlasov eq. on a grid
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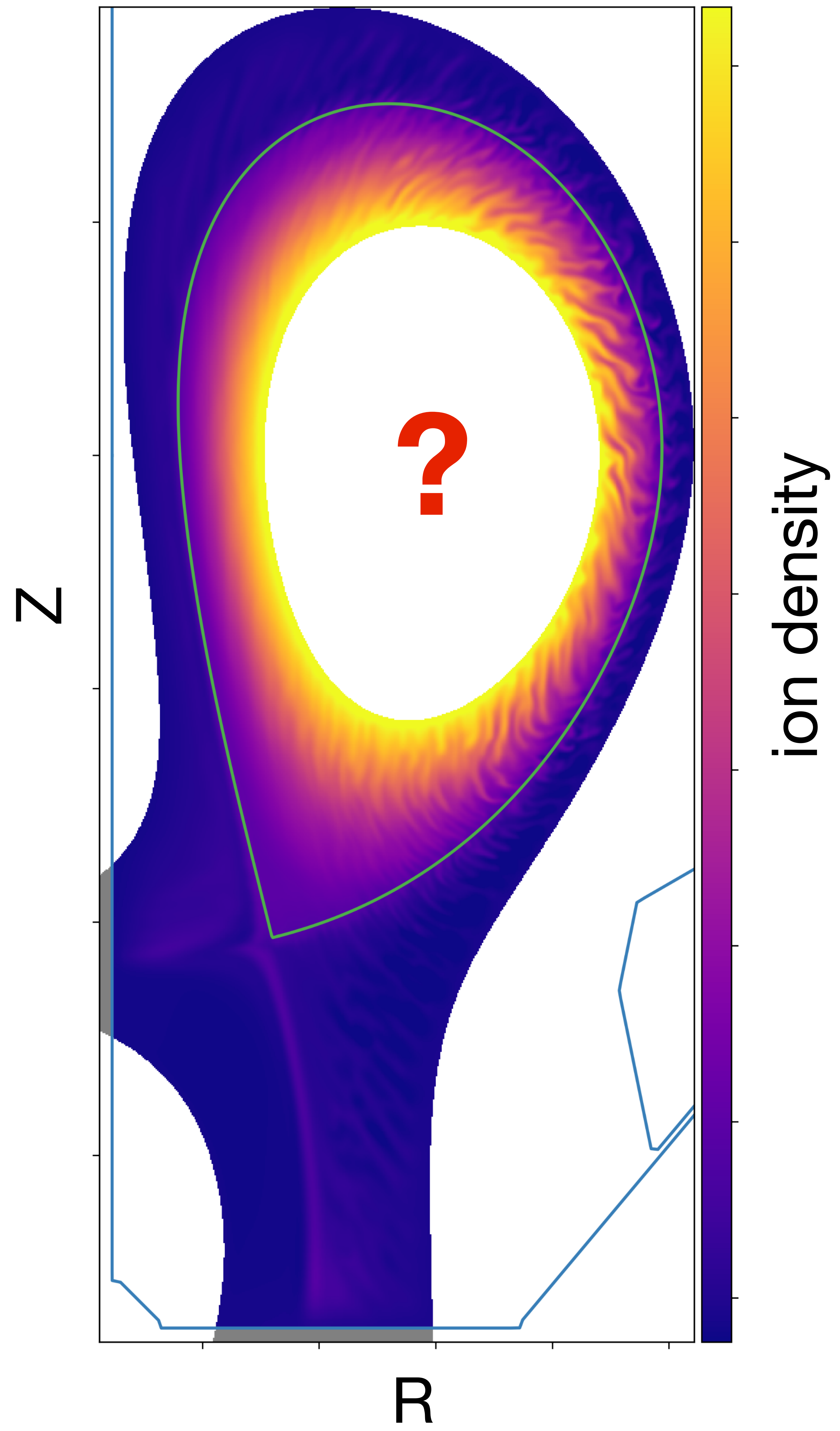
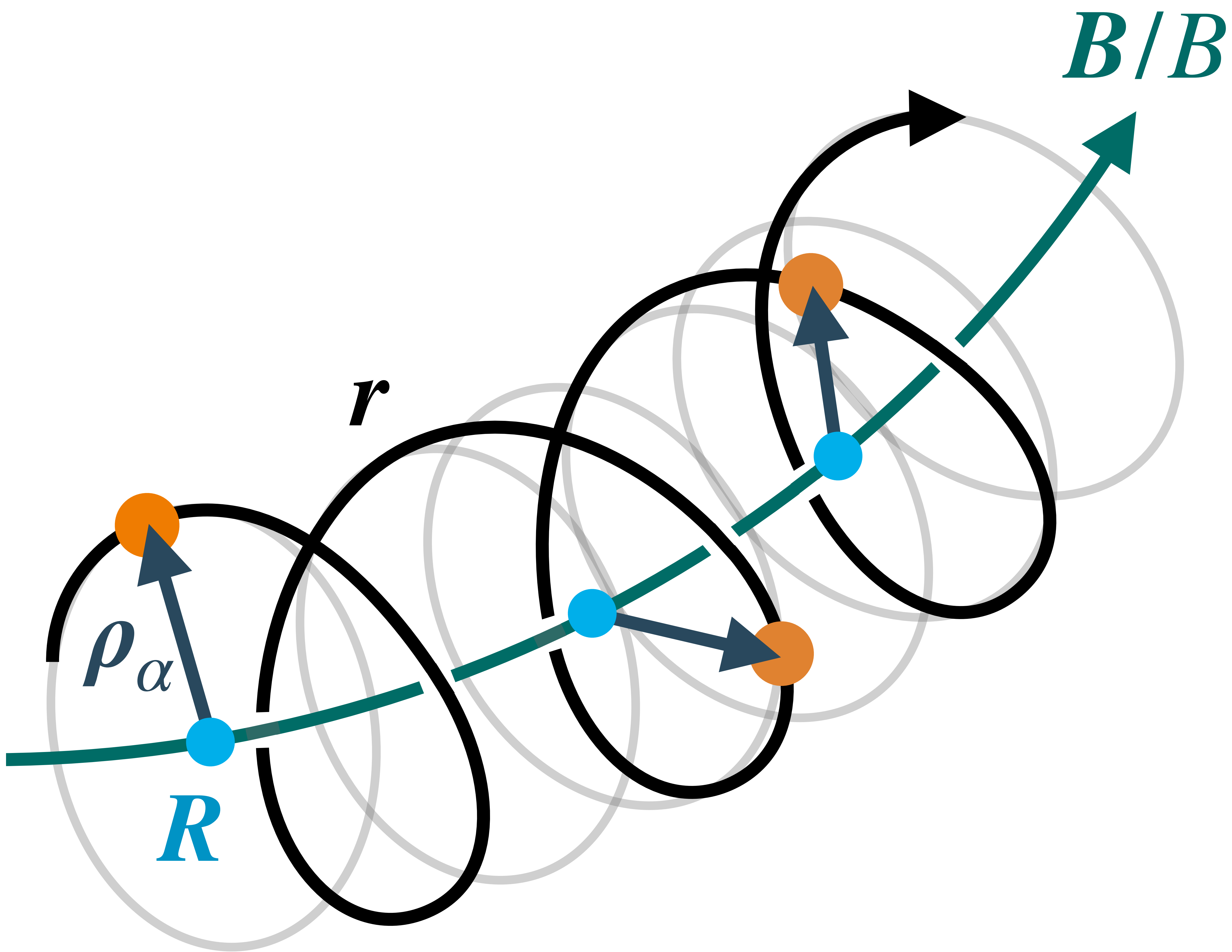


Finite $\underbrace{\text{Larmor radius}}_{\rho_\alpha}$ (FLR) effects

short wavelength turbulence



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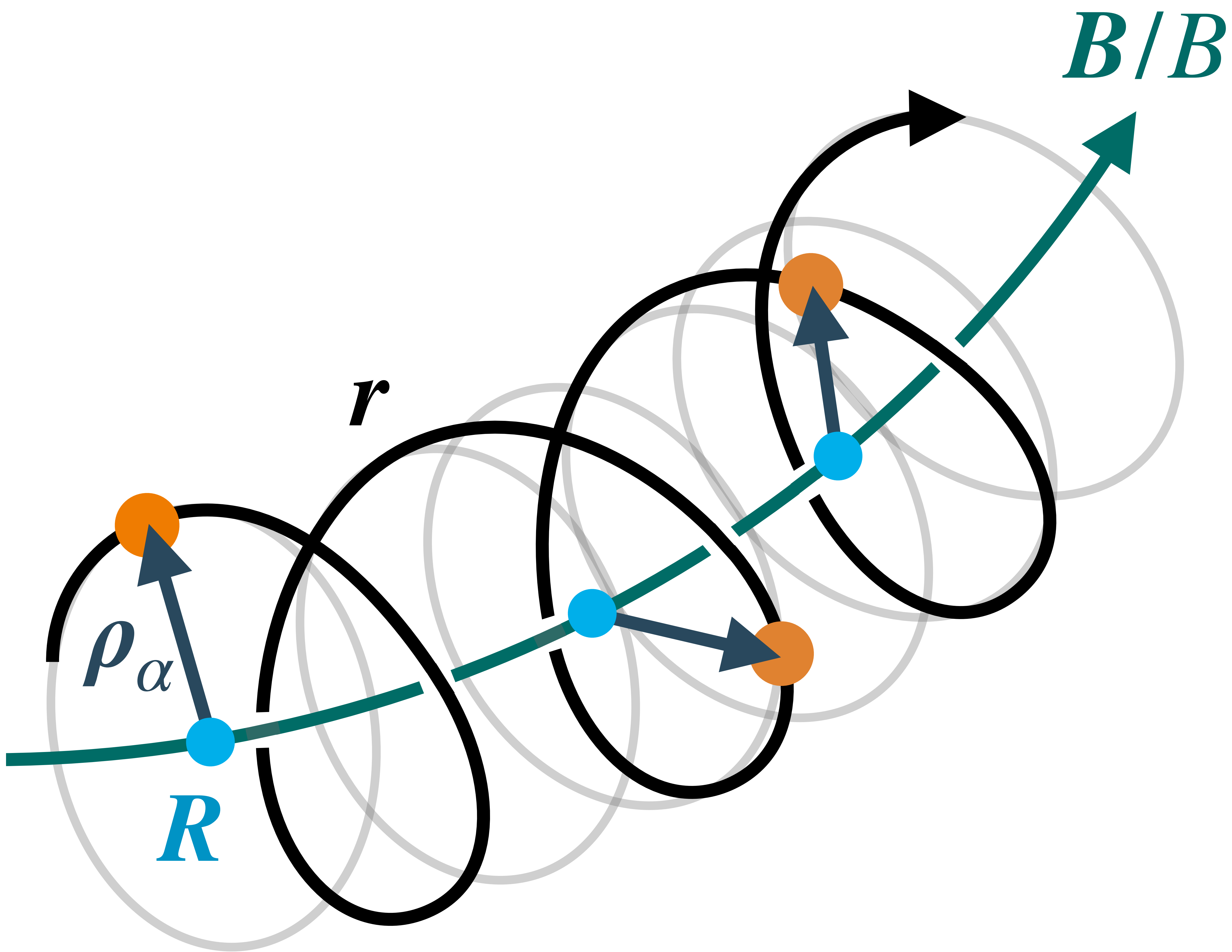

ρ_α **relevant in**
- plasma core
- core-edge transition
- near steep gradients

Finite $\underbrace{\text{Larmor radius}}_{\rho_\alpha}$ (FLR) effects

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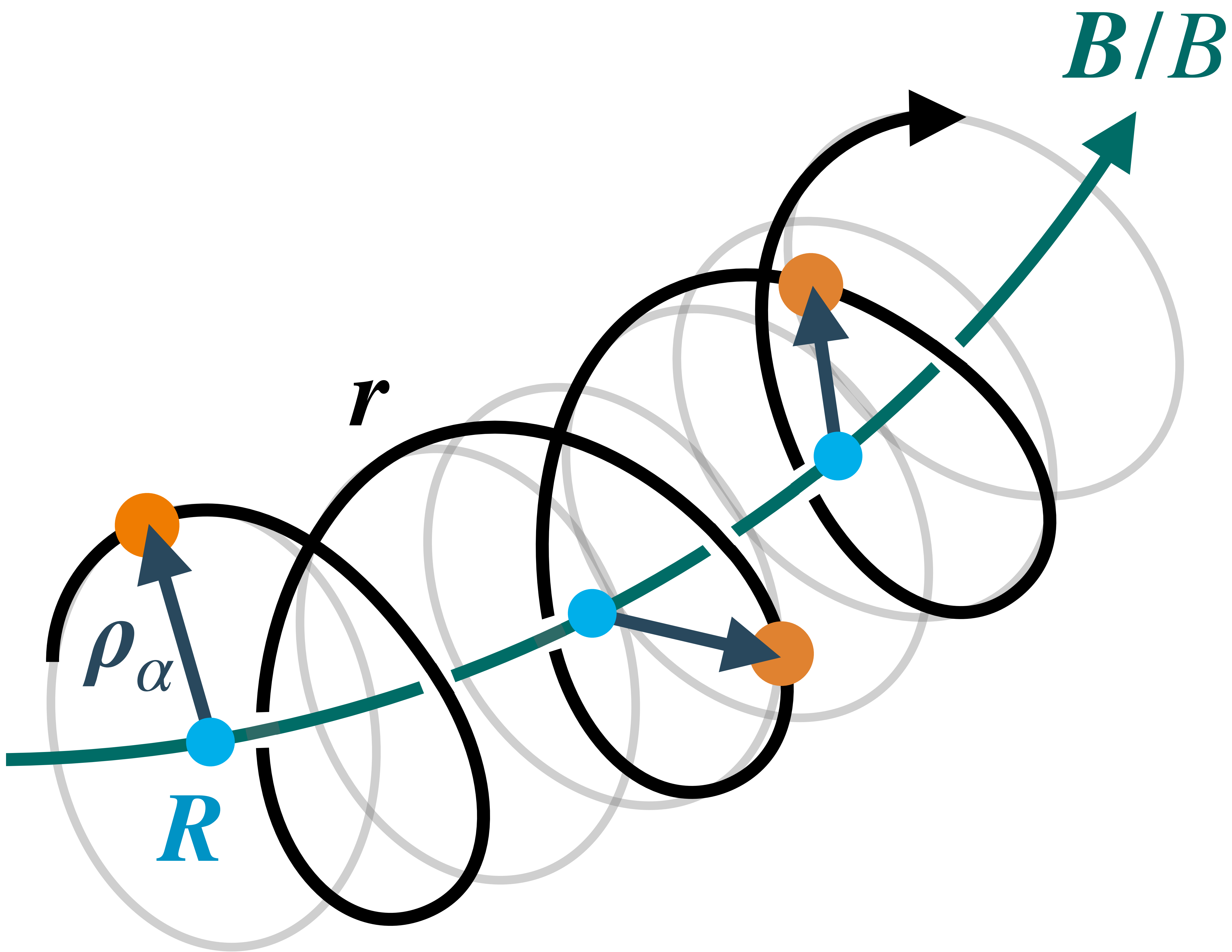


ρ_α **relevant in** →
- plasma core
- core-edge transition
- near steep gradients

FLR effects →
- to →
  - **simulations from the edge to the core**
  - **regions where gradients ~ ρ_α order**
  - ***etc.***
- through → **gyro-averages**

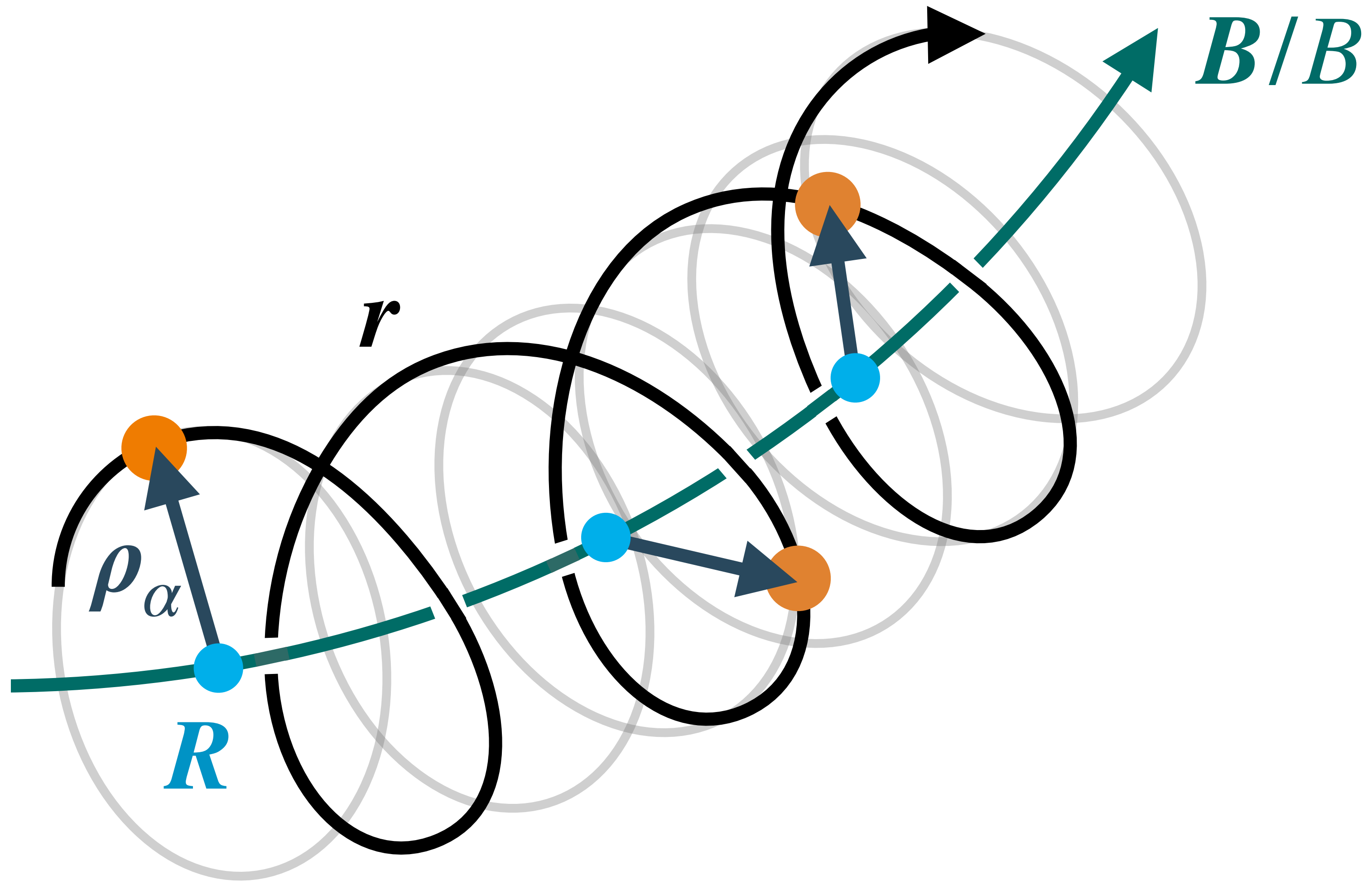
Gyro-averages and GENE-X equations

VLASOV EQ.

$$\frac{\partial f_\alpha}{\partial t} + \dot{\boldsymbol{R}} \cdot \nabla f_\alpha + \dot{v}_\parallel \frac{\partial f_\alpha}{\partial v_\parallel} = 0 \qquad \left(\boldsymbol{B} = B\,\boldsymbol{b}\right)$$

$$\dot{\boldsymbol{R}} \sim \boldsymbol{B}\, v_\parallel + \frac{m_\alpha c}{q_\alpha} v_\parallel^2 \nabla \times \boldsymbol{b} + v_\parallel \nabla \times \boldsymbol{A}_{1\parallel} + \frac{c}{q_\alpha} \boldsymbol{b} \times \mu \nabla B + c\, \boldsymbol{b} \times \nabla \phi_1$$

$$\dot{v}_\parallel \sim -\boldsymbol{Y} \left(\mu \nabla B + q_\alpha \nabla \phi_1 \right) - \frac{q_\alpha}{m_\alpha c} \frac{\partial A_{1\parallel}}{\partial t}$$



[1] D. Michels, PhD thesis (Technische Universität München, 2022).

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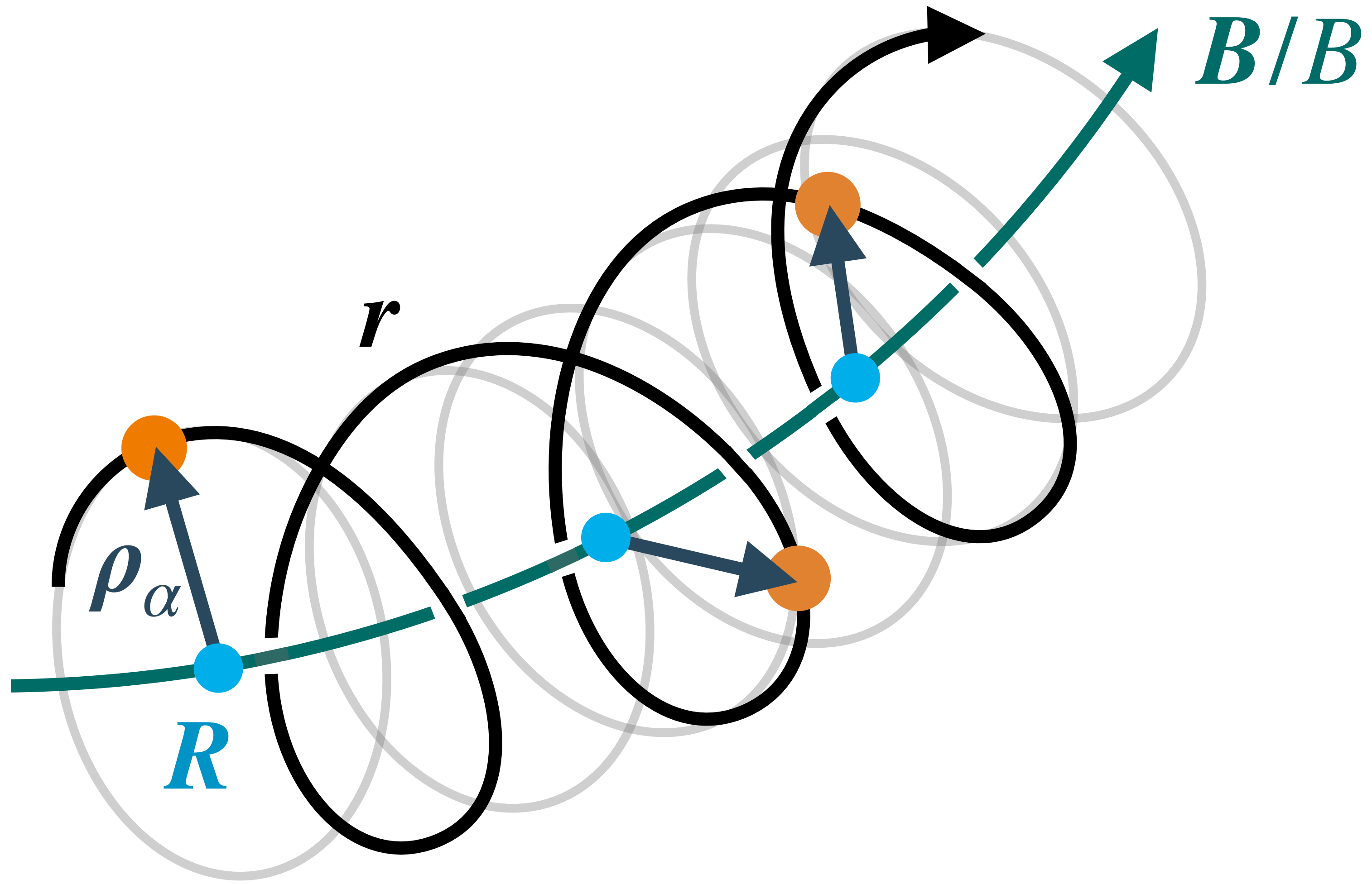

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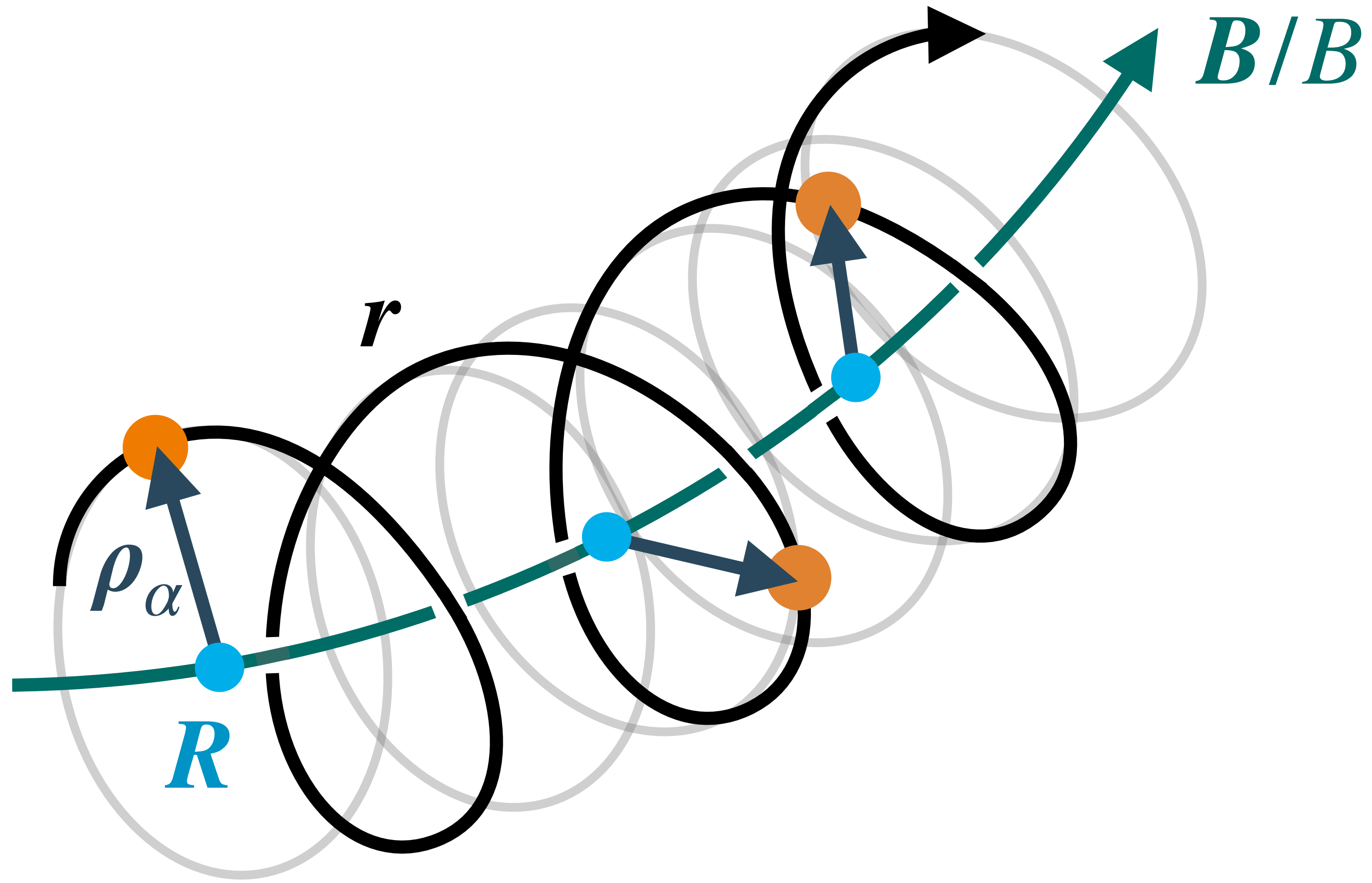

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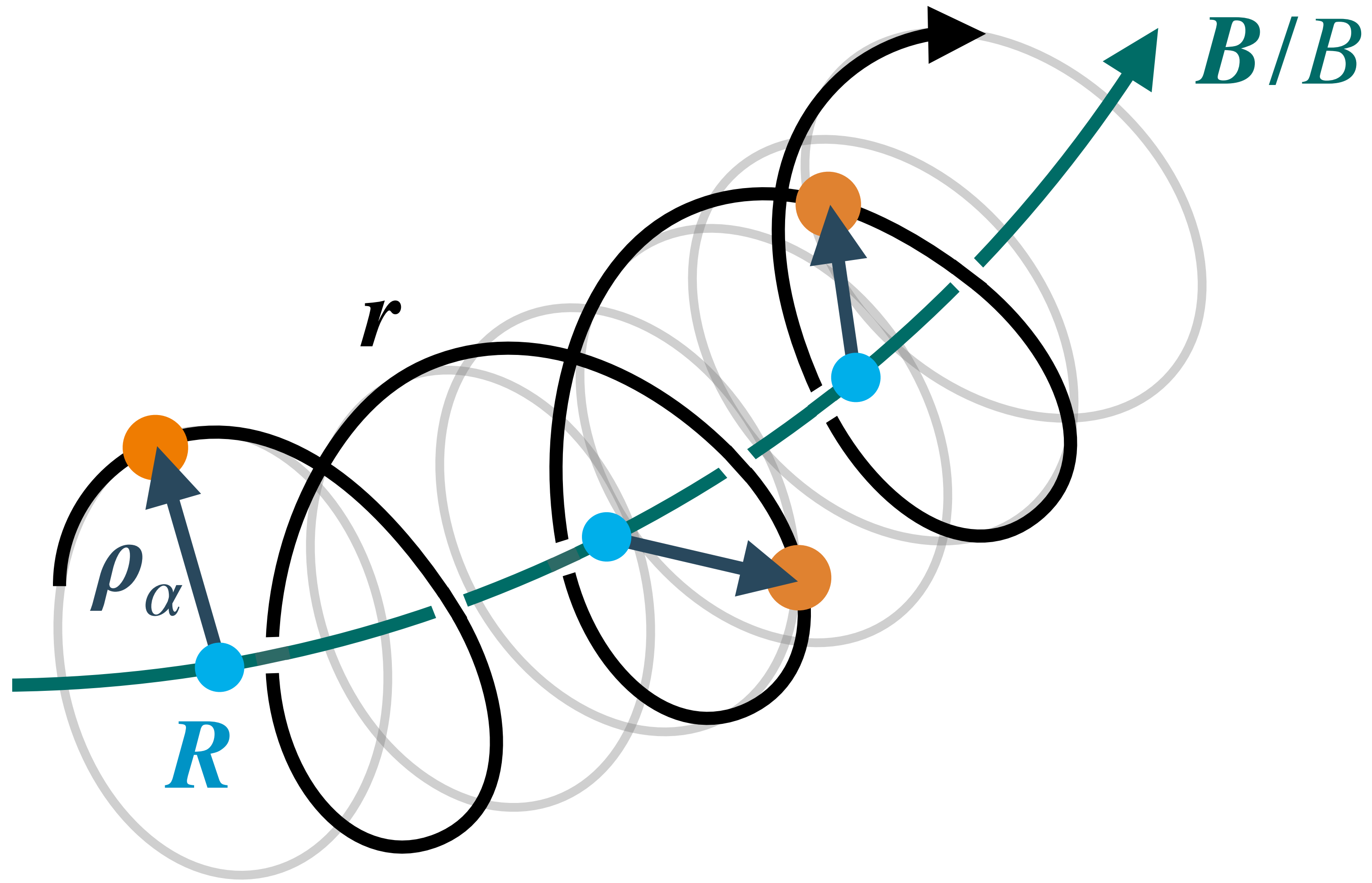

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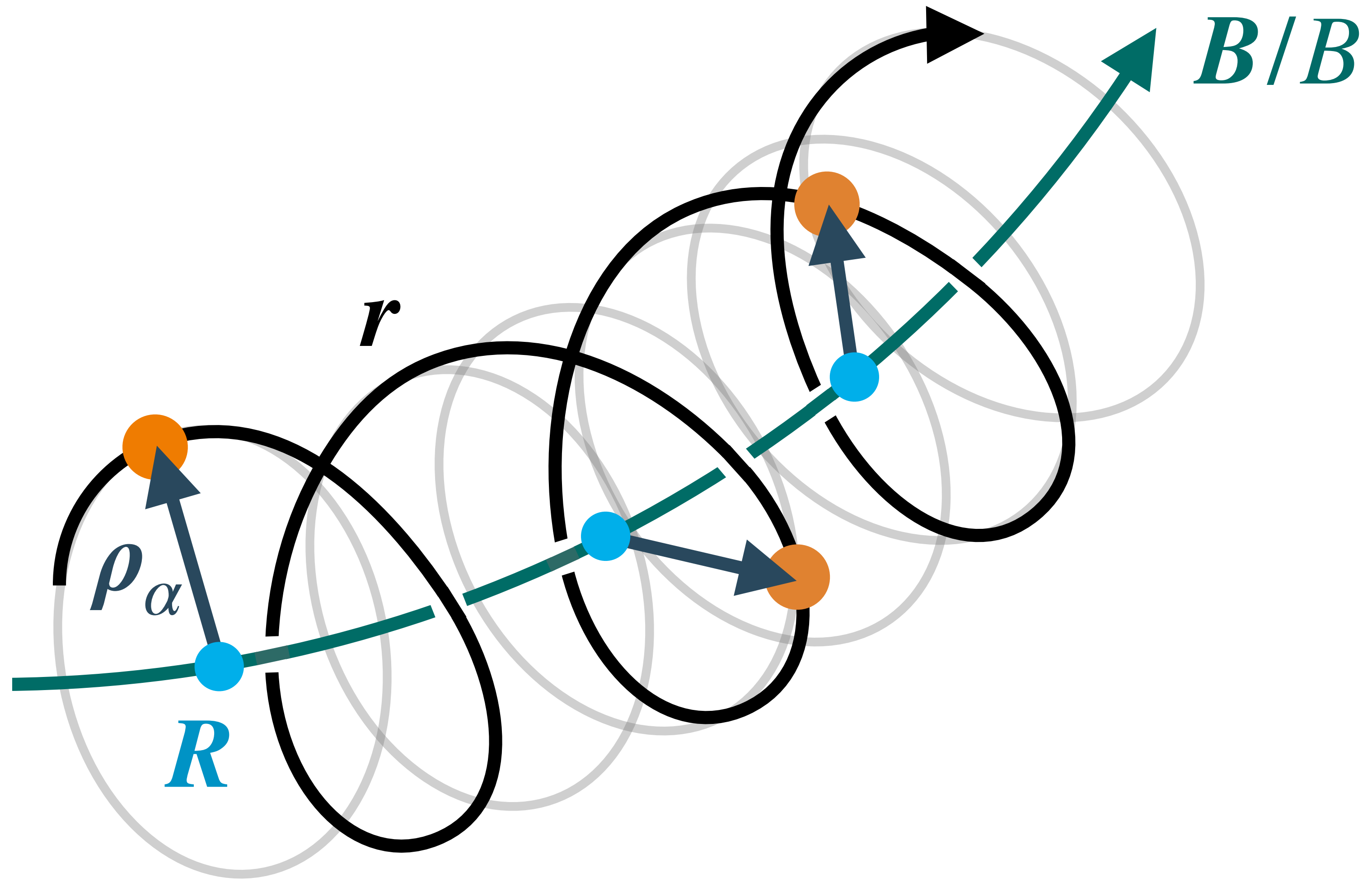

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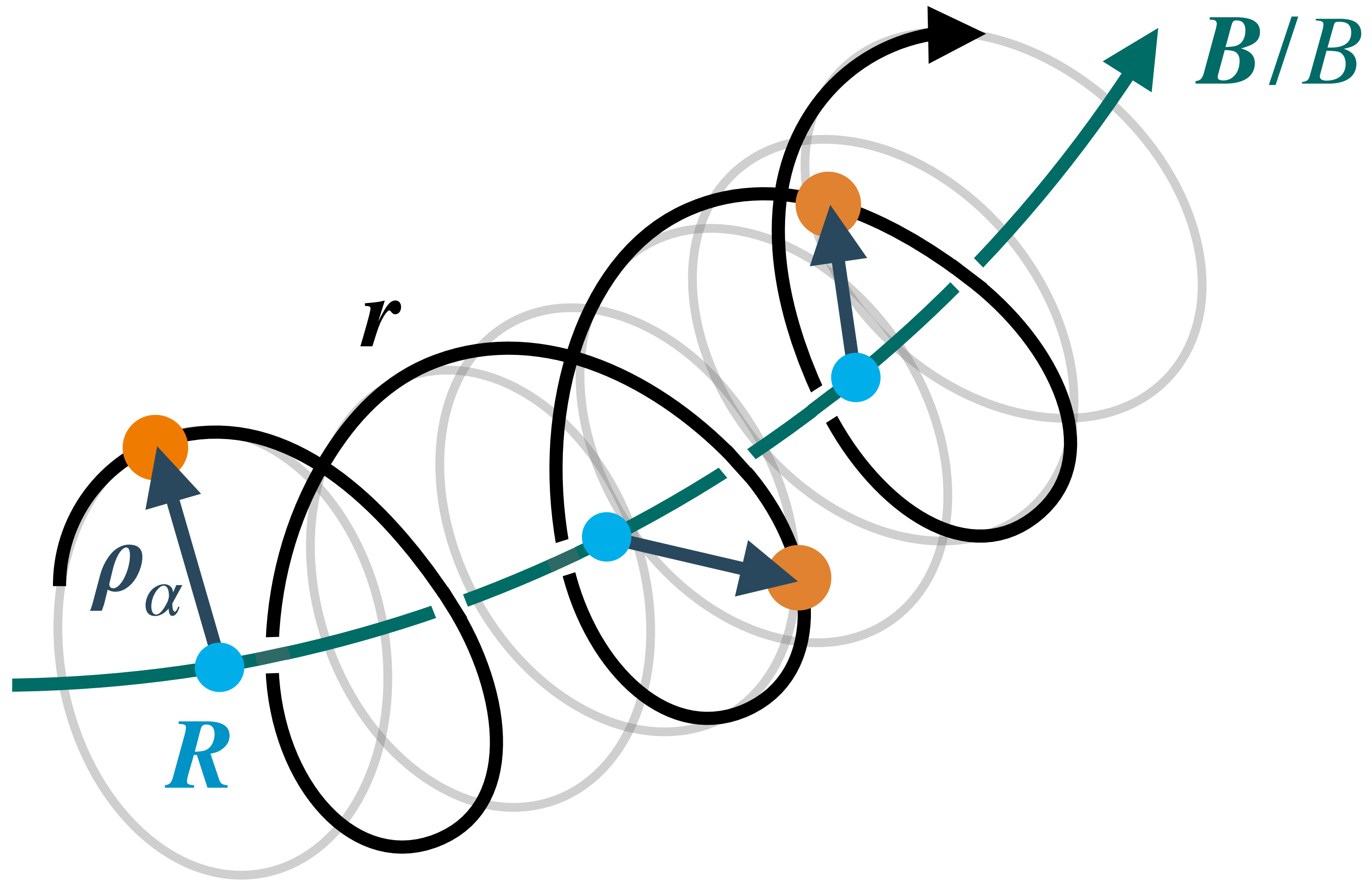

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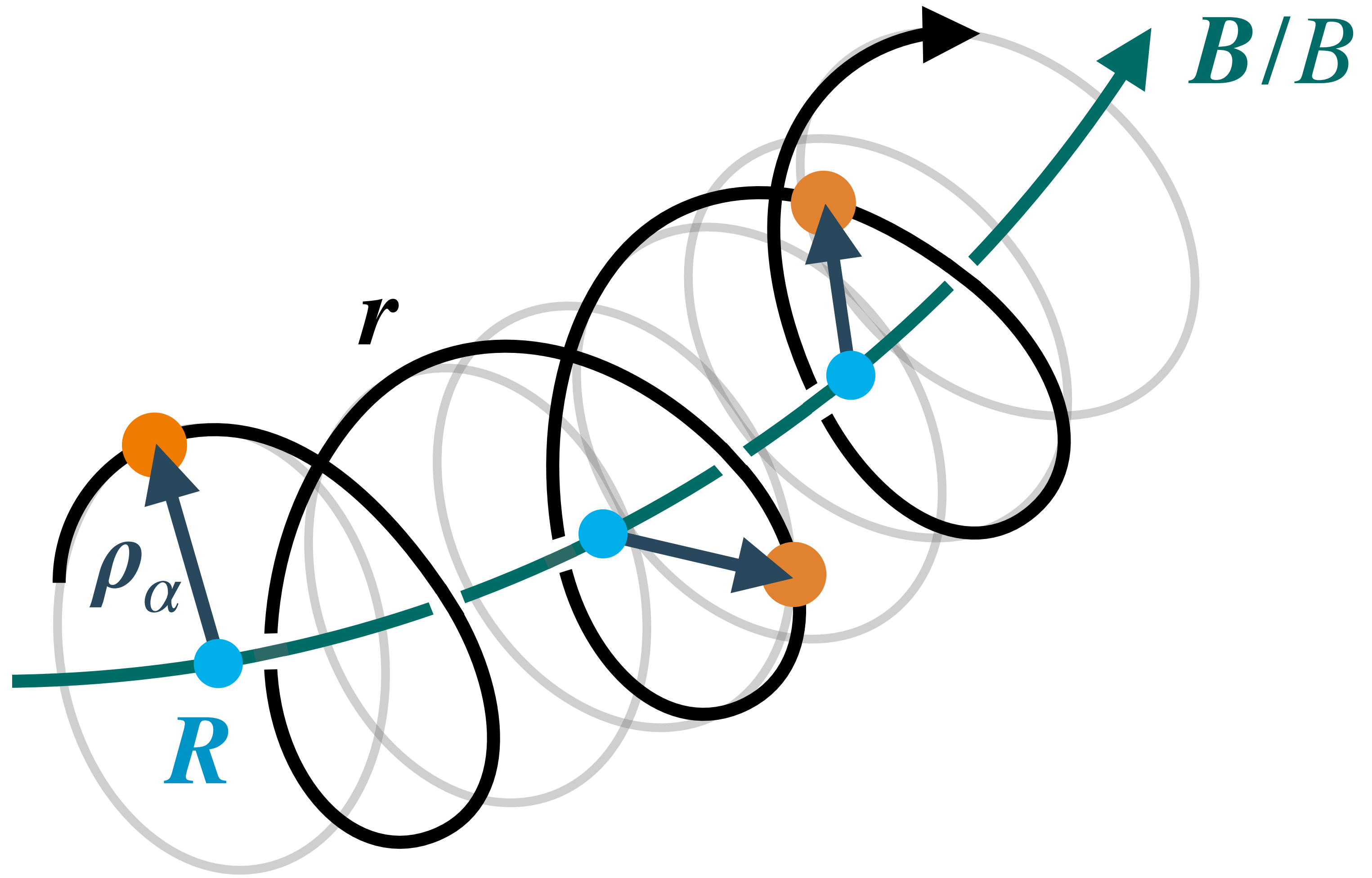

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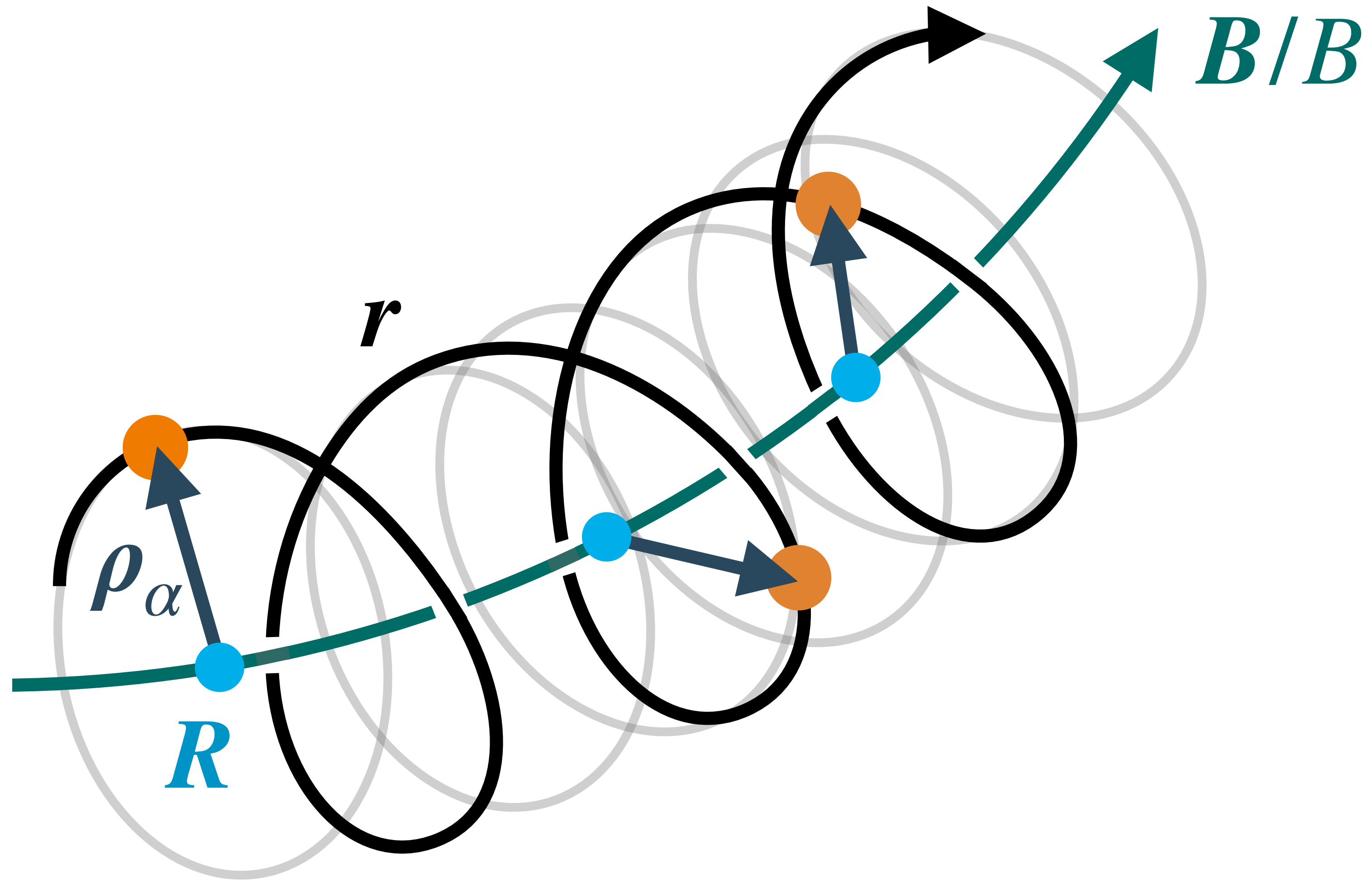

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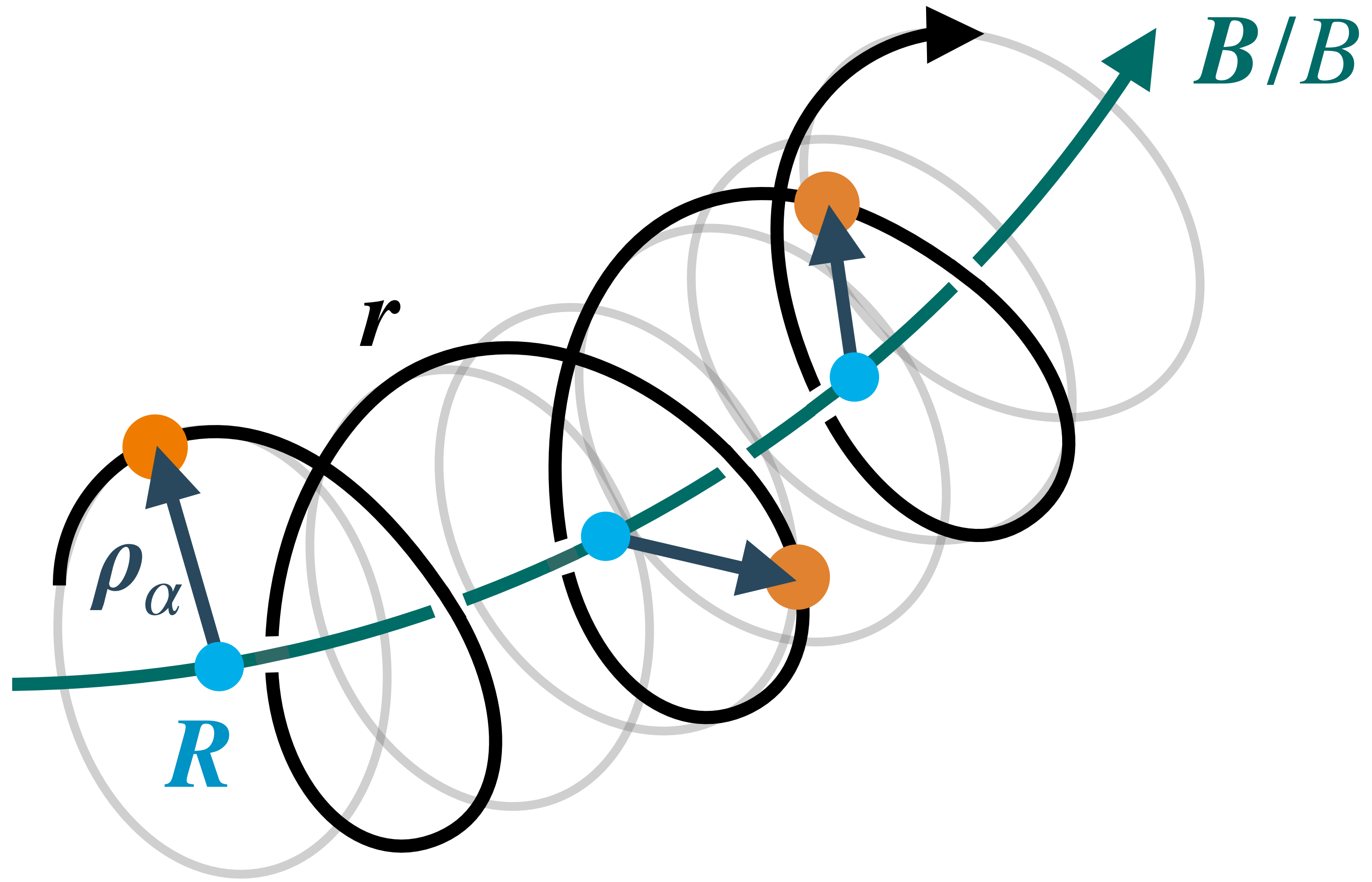

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gyro-average

$$\langle \phi(\boldsymbol{r})\rangle_{\boldsymbol{R}} = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi}\, e^{\boldsymbol{\rho}_\alpha\cdot\nabla}\, \phi(\boldsymbol{R})$$

curvature drift

⊥ drift ∇B

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B advection

⊥ EM flutter

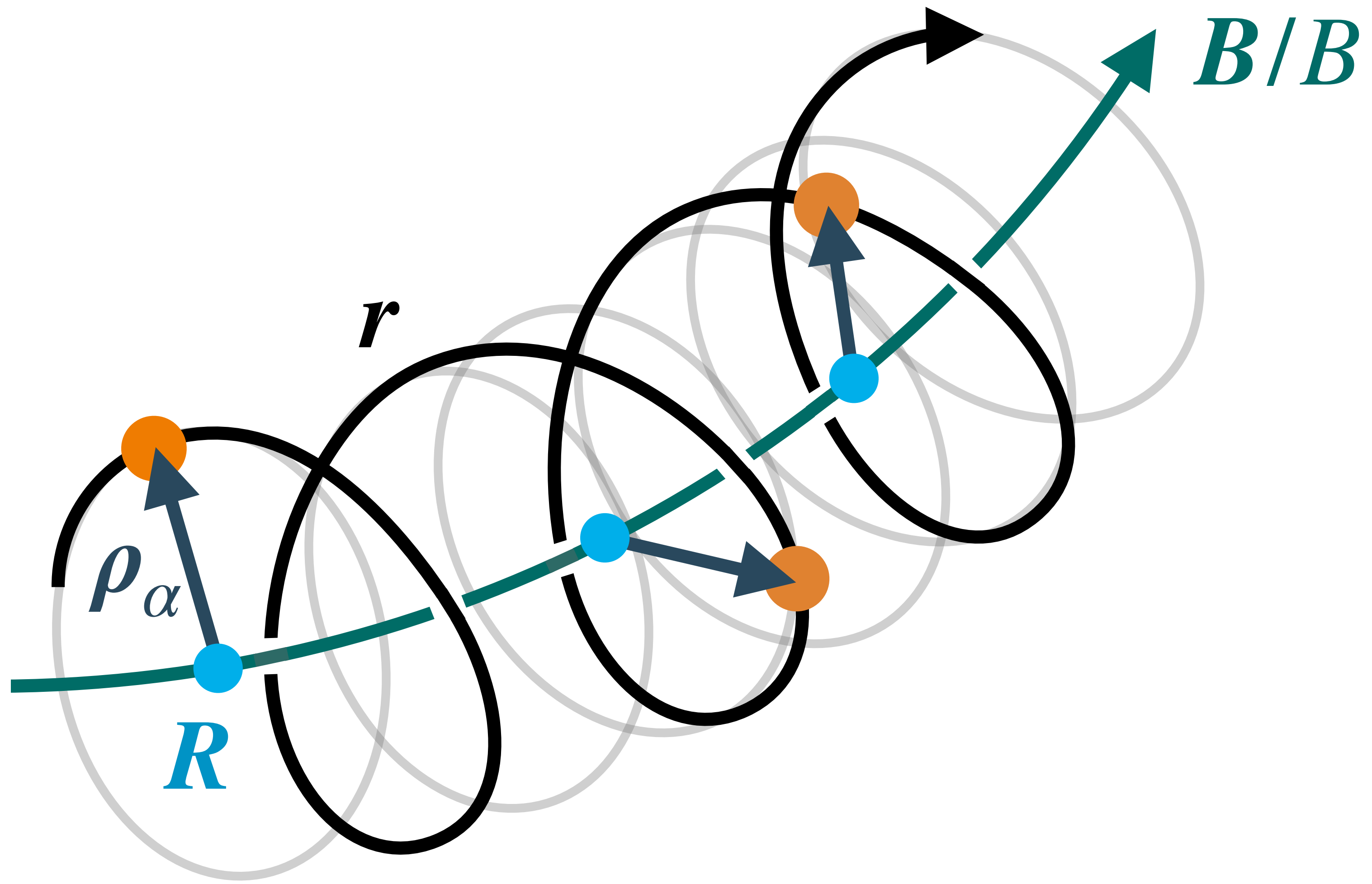
⊥ drift $\boldsymbol{E}\times\boldsymbol{B}$

$\boldsymbol{E}$ field acceler.

$$\dot{v}_\parallel \sim -\boldsymbol{Y}\left(\mu\,\nabla B + q_\alpha\,\nabla\ \ \phi_1\right) - \frac{q_\alpha}{m_\alpha c}\,\frac{\partial\ A_{1\parallel}}{\partial t}$$

mirror force

induction



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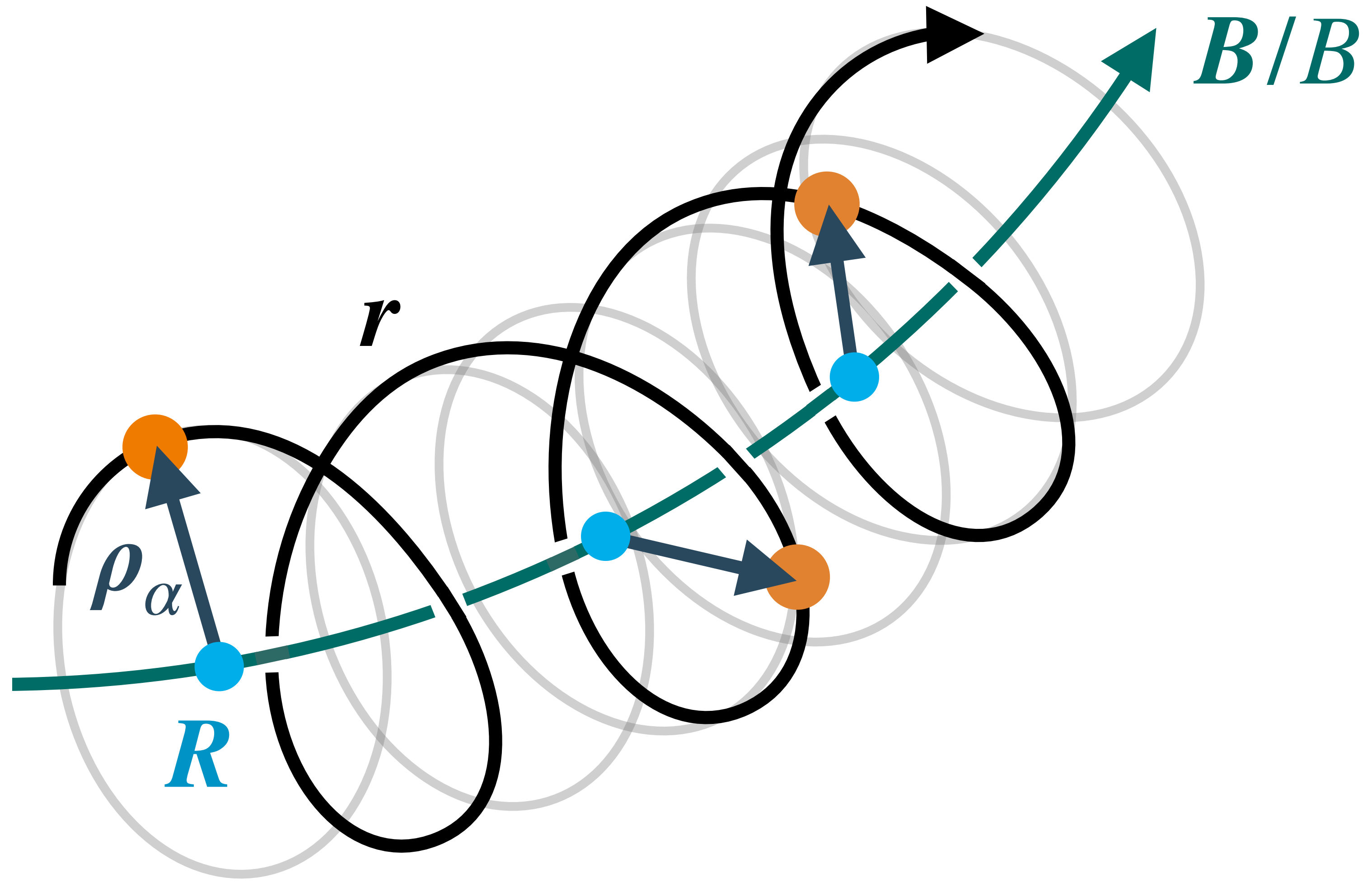

Padé approximant

gyro-average

$$\langle \phi(\boldsymbol{r}) \rangle_{\boldsymbol{R}} = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi} \, e^{\boldsymbol{\rho}_\alpha \cdot \nabla} \, \phi(\boldsymbol{R})$$

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$\downarrow$ $\mathscr{F}$ local gyrokinetics

$$\langle \phi(\boldsymbol{r}) \rangle_{\boldsymbol{R}} = \frac{1}{(2\pi)^3} \int \mathrm{d}\boldsymbol{k}\; J_0\left(\rho_\alpha k_\perp\right)\; e^{i\boldsymbol{k}\cdot\boldsymbol{R}}\; \hat{\phi}(\boldsymbol{k})$$

Padé approximant

gyro-average

$$\langle \phi(\boldsymbol{r}) \rangle_{\boldsymbol{R}} = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi} \, e^{\boldsymbol{\rho}_\alpha \cdot \nabla} \, \phi(\boldsymbol{R})$$

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↓

FLR operator in $\mathscr{F}$

Bessel func. of the 1st kind

Padé approximant

gyro-average

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FLR operator in $\mathscr{F}$

Bessel func. of the 1st kind

Taylor on $J_0 \xrightarrow{k_\perp \to \infty} \infty$

$\Downarrow$

another approx., *e.g.*, **Padé approximant**

$$J_0(\rho_\alpha k_\perp) \overset{P0/2}{\approx} \frac{1}{1 + \rho_\alpha^2 k_\perp^2 / 4}$$

Padé approximant

gyro-average

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$\Downarrow$

another approx., *e.g.*, **Padé approximant**

$$J_0(\rho_\alpha\, k_\perp) \overset{P0/2}{\approx} \frac{1}{1+\rho_\alpha^2\, k_\perp^2/4}$$

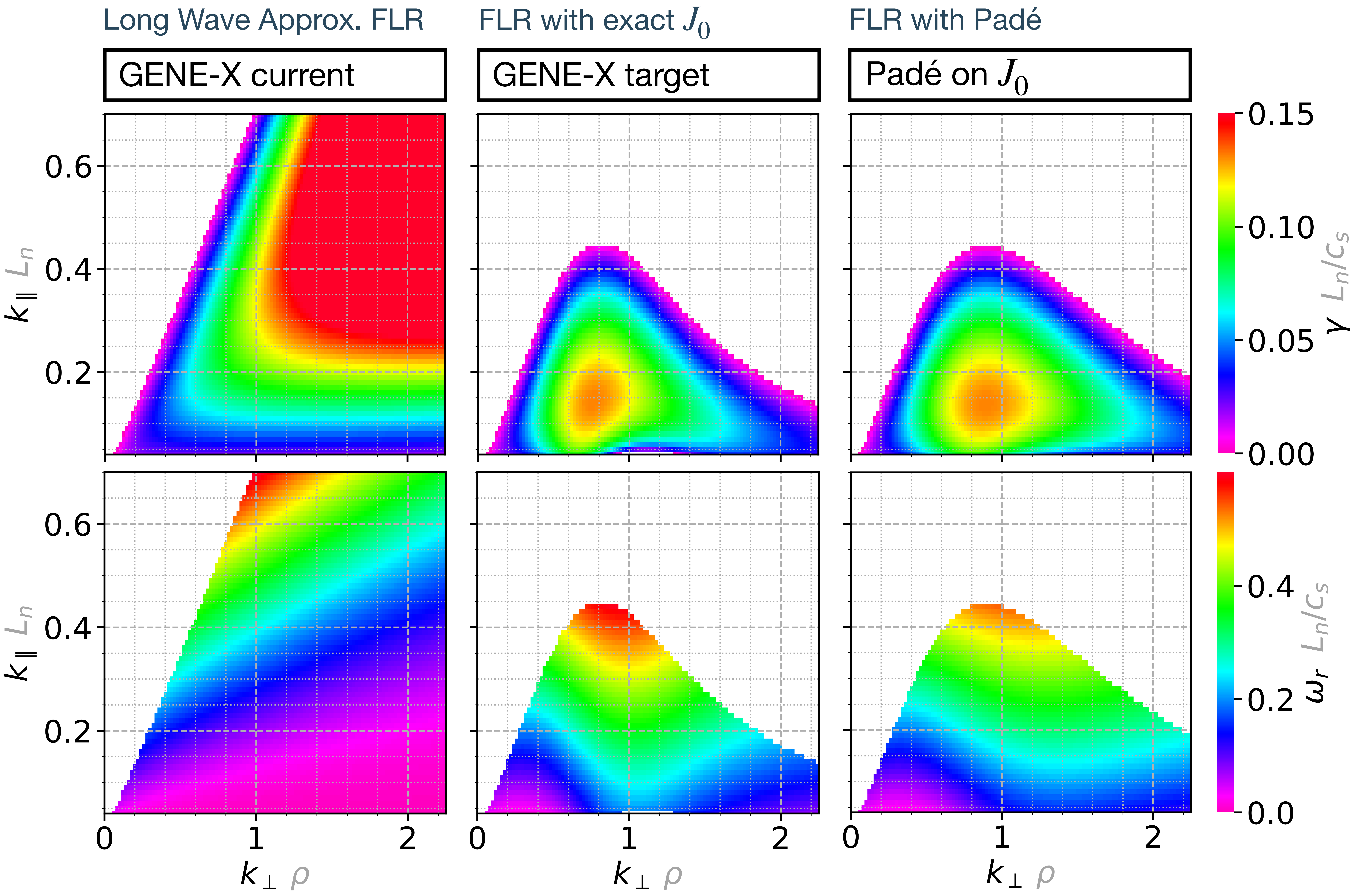
$\downarrow$ $\mathscr{F}^{-1}$ global gyrokinetics

$$\boxed{\left(1-\frac{1}{4}\,\rho_\alpha^2\,\nabla_\perp^2\right)\langle\phi(\boldsymbol{r})\rangle_{\boldsymbol{R}} = \phi(\boldsymbol{R})}$$

differential equation for a gyro-average

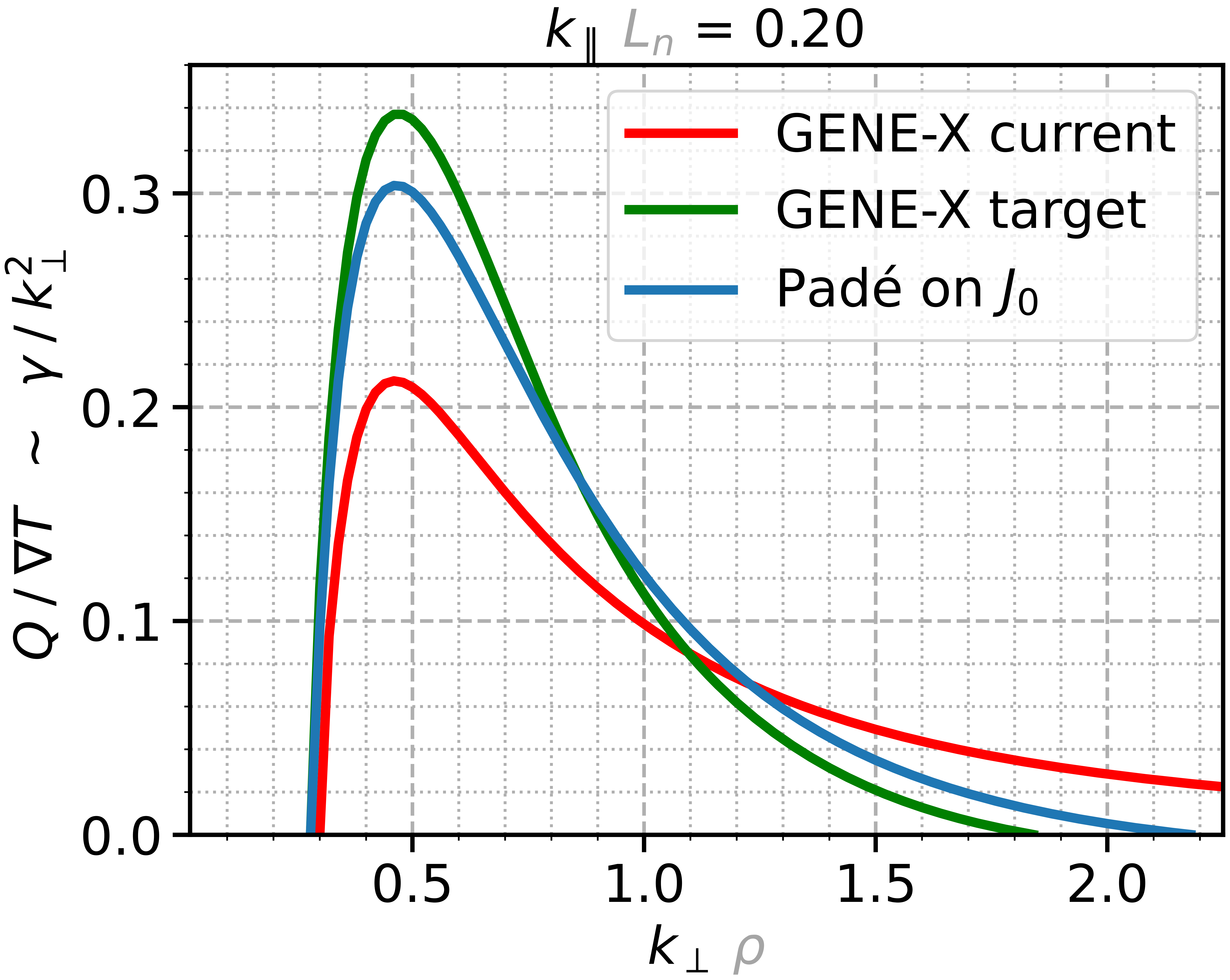
Impact of FLR models

EXAMPLE in the **electrostatic slab ion temperature gradient (ITG)** dispersion relation in Fourier space **with diffusion** in ∥ and ⊥ dirs.



Impact of FLR models — ITG disp. rel.

GENE-X current → Long Wave App. FLR
GENE-X target → FLR with exact J_0
Padé on J_0 → FLR with Padé



analysis of quasi-linear flux

[2] A. Mariani *et al.*, Physics of Plasmas **25**, 012313 (2018).

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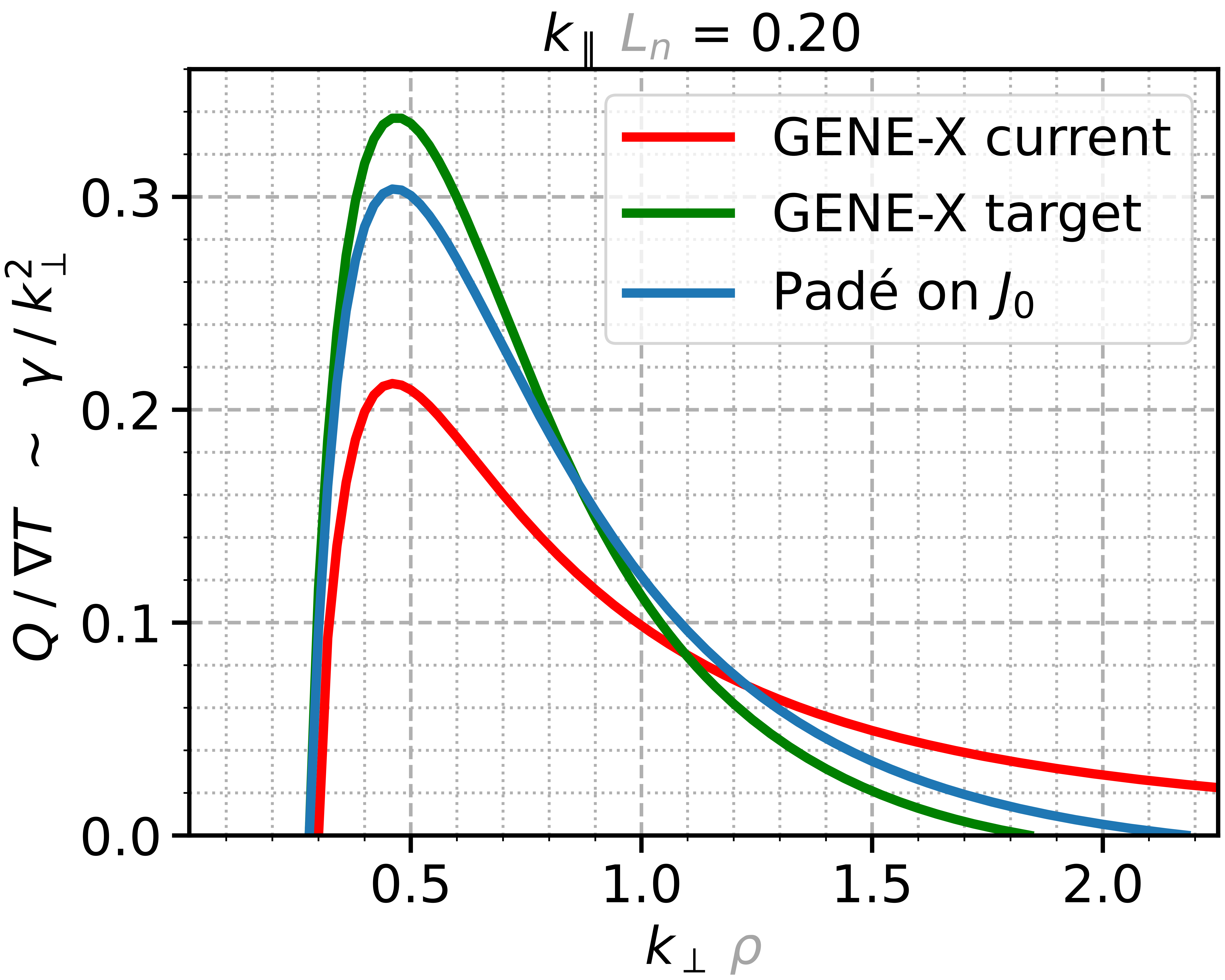

analysis of quasi-linear flux

→ major Physics change from GENE-X **current** to **target**

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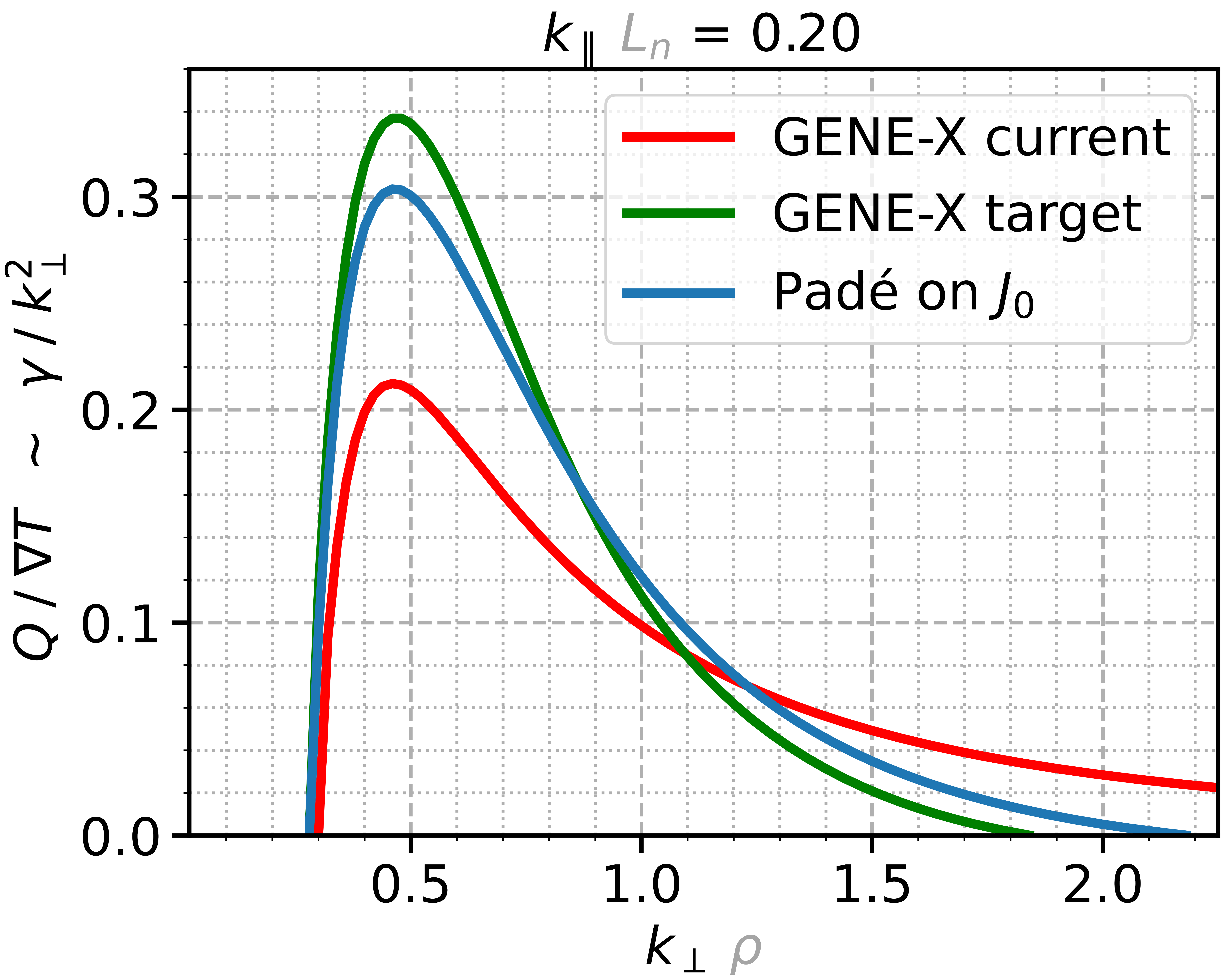

analysis of quasi-linear flux

- major Physics change from GENE-X **current** to **target**
- **Padé on J_0** not far-fetched from **target** and provides very convenient algorithm

$$\boxed{\left(1-\frac{1}{4}\,\rho_\alpha^2\nabla_\perp^2\right)\ \langle\phi(\boldsymbol{r})\rangle_{\boldsymbol{R}}=\phi(\boldsymbol{R})}$$

differential eq. for gyro-average

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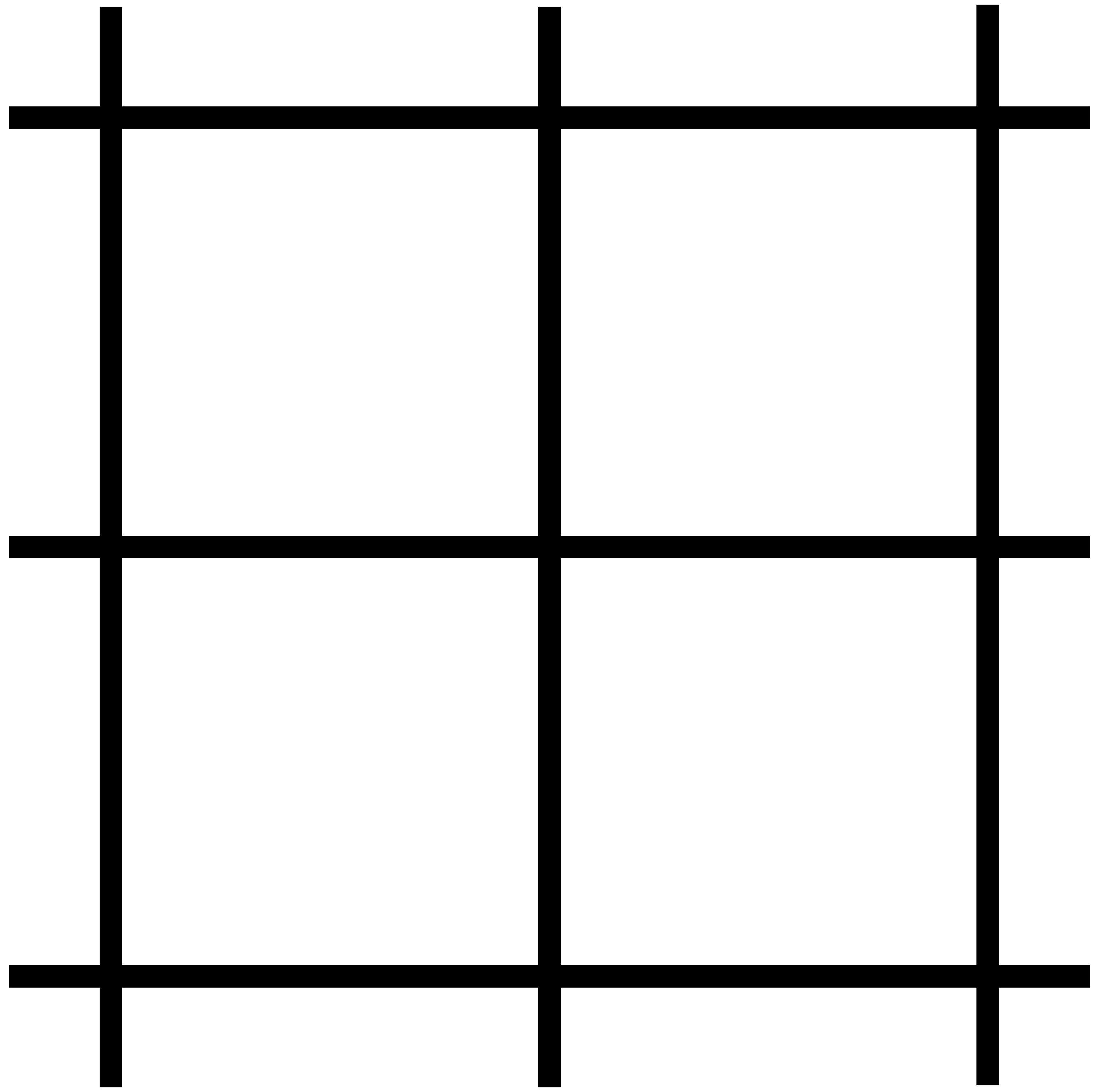
Gyro-matrix approach

gyro-average

$$\langle \phi(\boldsymbol{r}) \rangle_{\boldsymbol{R}} = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi} \, e^{\rho_\alpha \cdot \nabla} \, \phi(\boldsymbol{R})$$



[3] T. Görler et al., Journal of Computational Phys. **230**, 7053–7071 (2011).

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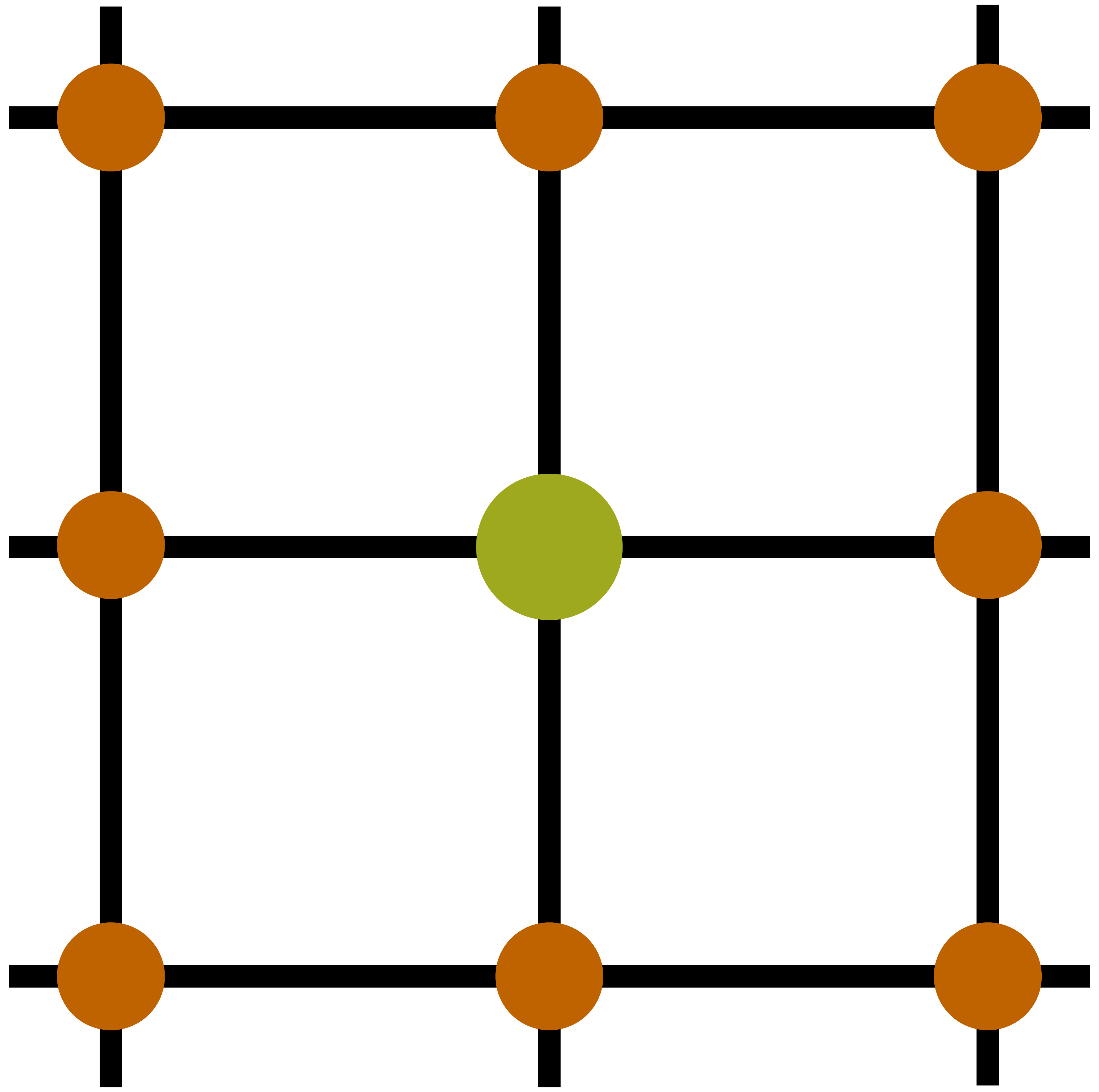
⟶ **grid** of **values**



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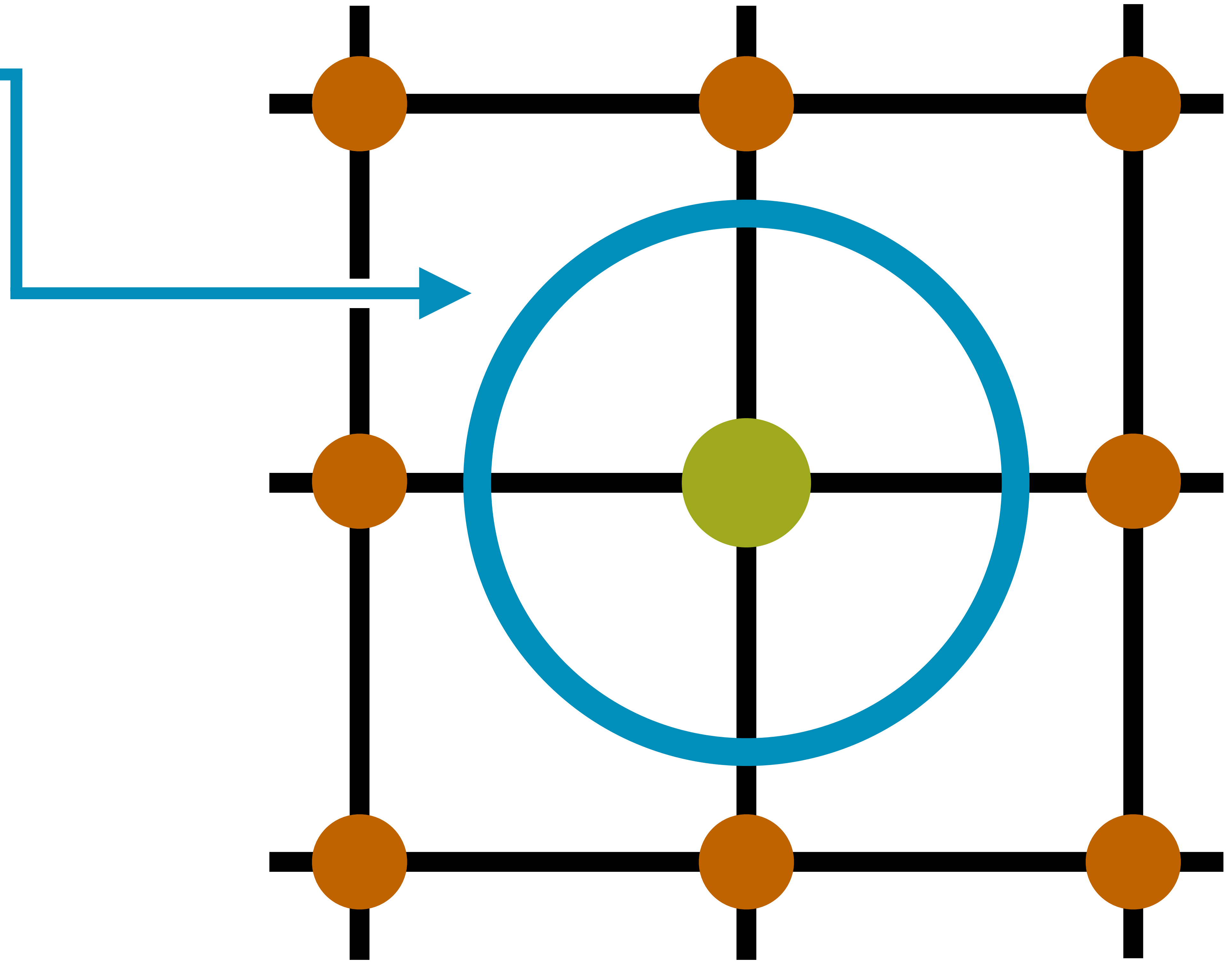

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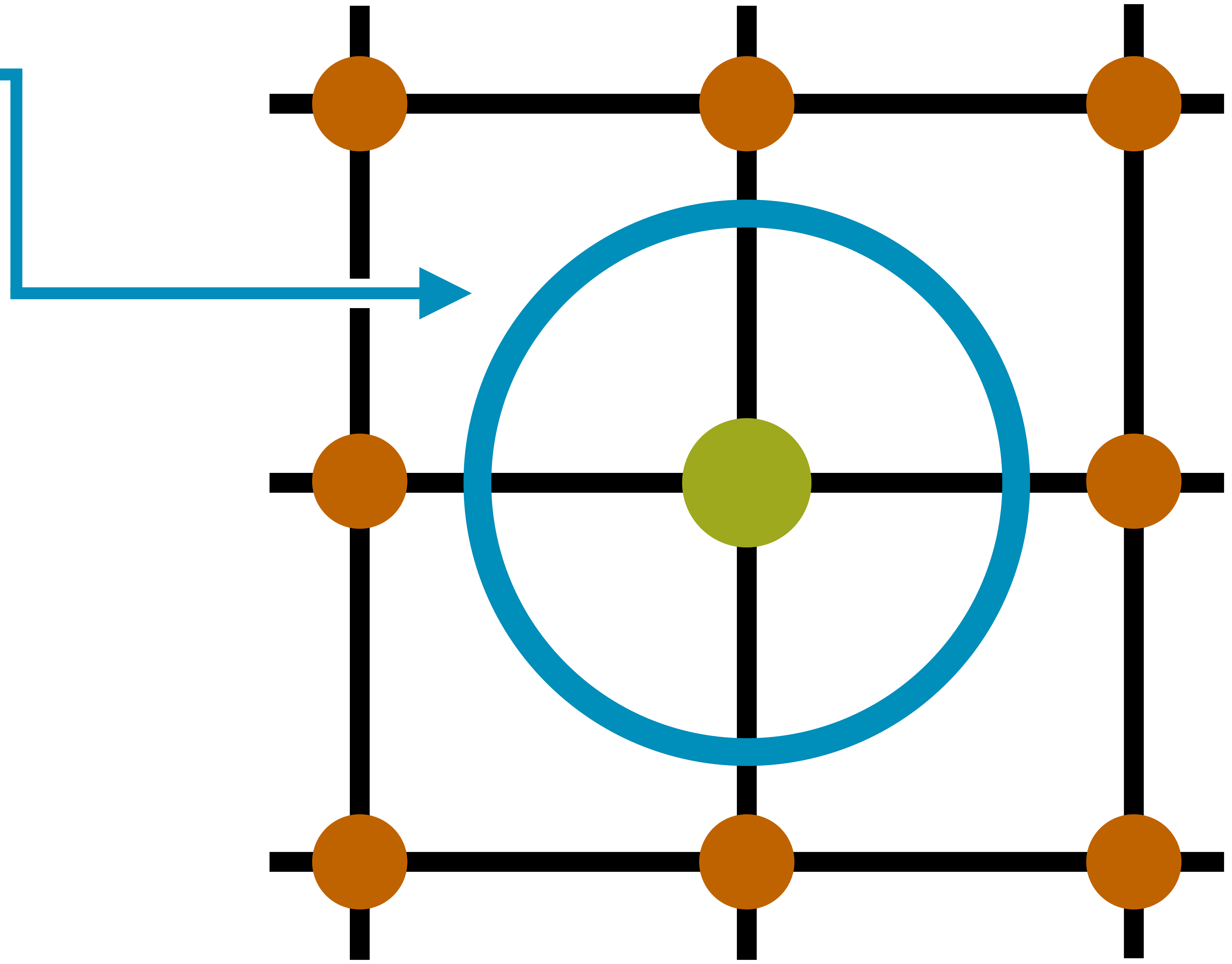
→ matrix combining interpolation and gyro-average

→ **gyro-matrix** $\boxed{\langle \boldsymbol{\phi} \rangle_{\boldsymbol{R}} = G(\mu)\, \boldsymbol{\phi}}$

calculates gyro-averages for all orders up to the discretization error



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Gyro-matrix approach

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$$\langle \phi(\boldsymbol{r})\rangle_{\boldsymbol{R}} = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi}\, e^{\rho_\alpha\cdot\nabla}\, \phi(\boldsymbol{R})$$

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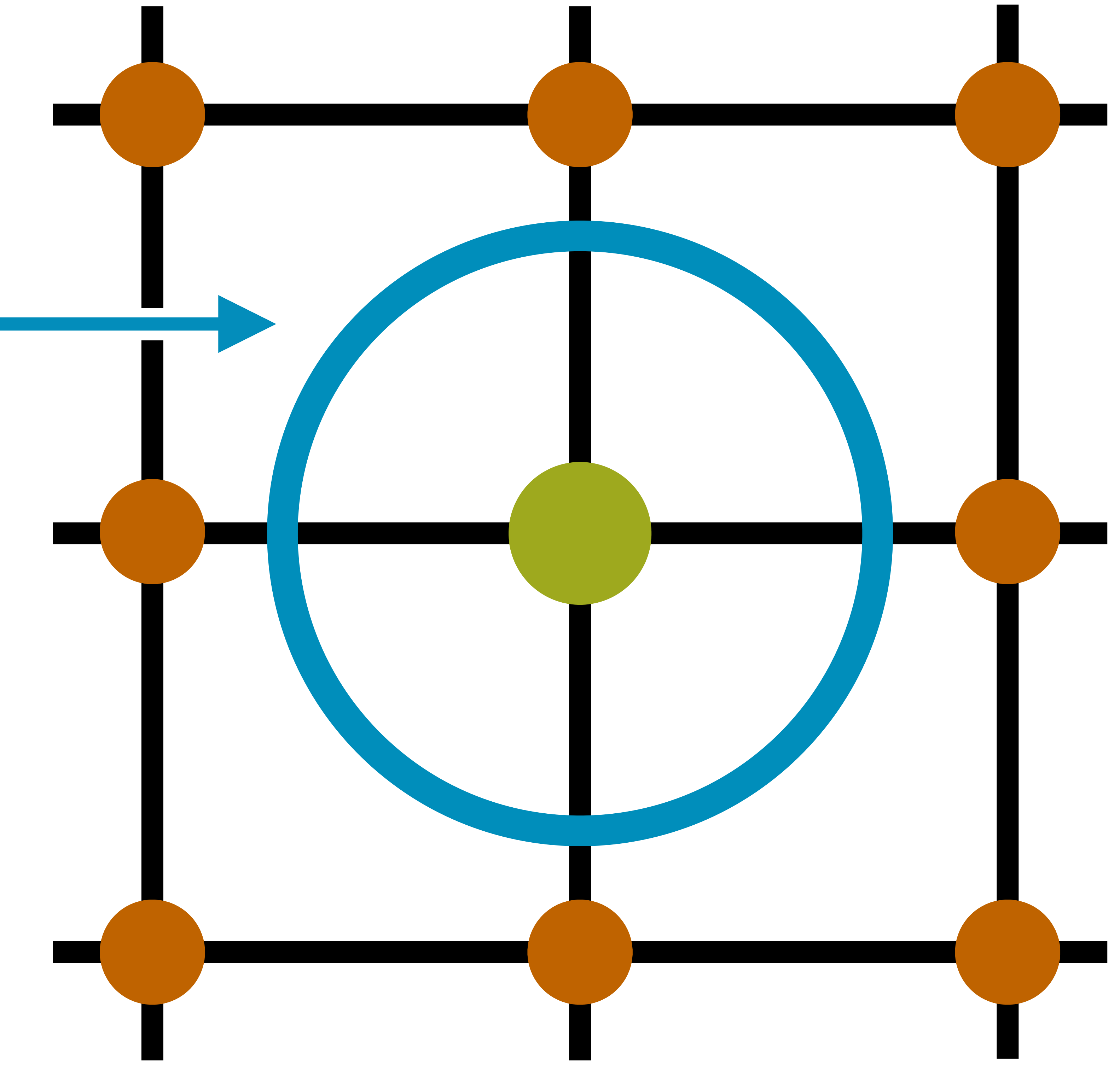

CAVEATS: intricate algorithm, expensive in resources…

[3] T. Görler et al., Journal of Computational Phys. **230**, 7053–7071 (2011).

Study of finite Larmor radius (FLR) physics in tokamaks and stellarators with the gyrokinetic turbulence code GENE-X

SUMMARY

→ Re-derivation of GENE-X eqs. accounting for FLR effects

  → Padé approx. of J_0

→ Example of FLR models in the ES slab ITG disp. relation

  → Padé on J_0 is Physically close
  and provides a convenient algorithm for FLR effects

→ Gyro-matrix approach

THANK YOU

ADDENDUM

Gyro-averages and GENE-X equations

gyro-average

$$\langle \phi(\boldsymbol{r}) \rangle_{\boldsymbol{R}} = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi}\, e^{\boldsymbol{\rho}_\alpha \cdot \nabla}\, \phi(\boldsymbol{R})$$

$$\boxed{\mathscr{L} = \frac{q_\alpha}{c}\boldsymbol{A}\cdot\dot{\boldsymbol{R}} + \frac{q_\alpha}{c}\left\langle A_{1\parallel}\right\rangle_{\boldsymbol{R}} \boldsymbol{b}\cdot\dot{\boldsymbol{R}} + m_\alpha v_\parallel \boldsymbol{b}\cdot\dot{\boldsymbol{R}} + \frac{\mu B}{\Omega_\alpha}\dot{\theta} - q_\alpha \left\langle \phi_1 \right\rangle_{\boldsymbol{R}} - \frac{1}{2} m_\alpha v_\parallel^2 - \mu B + \frac{m_\alpha c^2}{2 B^2}\left|\nabla_\perp \phi_1\right|^2}$$

VLASOV EQ.

$$\frac{\partial f_\alpha}{\partial t} + \dot{\boldsymbol{R}}\cdot\nabla f_\alpha + \dot{v}_\parallel \frac{\partial f_\alpha}{\partial v_\parallel} = 0$$

$$\left(\boldsymbol{B}^* \sim \boldsymbol{B} + \frac{m_\alpha c}{q_\alpha} v_\parallel \nabla\times\boldsymbol{b} + \left\langle \nabla\times\boldsymbol{A}_1 \right\rangle\right)$$

using the GENE-X Poisson bracket **[1]** for our coordinates

$$\dot{\boldsymbol{R}} = \frac{\boldsymbol{B}^*}{B_\parallel^*} v_\parallel + \frac{c}{q_\alpha B_\parallel^*}\boldsymbol{b}\times\left(\mu\nabla B + q_\alpha \nabla\left\langle \phi_1\right\rangle_{\boldsymbol{R}}\right)$$

$$\dot{v}_\parallel = -\frac{\boldsymbol{B}^*}{m_\alpha B_\parallel^*}\left(\mu\nabla B + q_\alpha\nabla\left\langle\phi_1\right\rangle_{\boldsymbol{R}}\right) - \frac{q_\alpha}{m_\alpha c}\frac{\partial\left\langle A_{1\parallel}\right\rangle}{\partial t}$$

[1] D. Michels, PhD thesis (Technische Universität München, 2022).

Gyro-averages and GENE-X equations

gyro-average

$$\langle \phi(\boldsymbol{r})\rangle_{\boldsymbol{R}} = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi}\, e^{\boldsymbol{\rho}_\alpha\cdot\nabla}\,\phi(\boldsymbol{R})$$

adjoint operator

$$\langle \phi(\boldsymbol{R})\rangle^{\dagger}_{\boldsymbol{r}} = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi} \int \mathrm{d}\boldsymbol{R}\,\delta\left(\boldsymbol{R} + \boldsymbol{\rho}_\alpha - \boldsymbol{r}\right)\,\phi(\boldsymbol{R})$$

$$\boxed{\mathscr{L} = \frac{q_\alpha}{c}\boldsymbol{A}\cdot\dot{\boldsymbol{R}} + \frac{q_\alpha}{c}\left\langle A_{1\parallel}\right\rangle_{\boldsymbol{R}}\boldsymbol{b}\cdot\dot{\boldsymbol{R}} + m_\alpha v_\parallel \boldsymbol{b}\cdot\dot{\boldsymbol{R}} + \frac{\mu B}{\Omega_\alpha}\dot{\theta} - q_\alpha\left\langle\phi_1\right\rangle_{\boldsymbol{R}} - \frac{1}{2}m_\alpha v_\parallel^2 - \mu B + \frac{m_\alpha c^2}{2B^2}\left|\nabla_\perp\phi_1\right|^2}$$

least-action principle for

$\phi \longrightarrow$ **QUASI-NEUTRALITY EQ.**

$$\sum_\alpha q_\alpha \left\langle n_\alpha\right\rangle^{\dagger}_{\boldsymbol{r}} = -\nabla\cdot\left(\sum_\alpha n_\alpha \frac{m_\alpha c^2}{B^2}\nabla_\perp\phi_1\right)$$

$A \longrightarrow$ **AMPÈRE'S LAW**

$$\sum_\alpha \frac{q_\alpha}{c}\left\langle\int \mathrm{d}W f_\alpha v_\parallel\right\rangle^{\dagger}_{\boldsymbol{r}} = -\frac{1}{4\pi}\left|\nabla_\perp^2 A_{1\parallel}\right|$$

Gyro-averages and GENE-X equations

gyro-average

$$\langle \phi(\boldsymbol{r}) \rangle_{\boldsymbol{R}} = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi}\, e^{\boldsymbol{\rho}_\alpha \cdot \nabla}\, \phi(\boldsymbol{R})$$

adjoint operator

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ENERGY CONSERVATION

$$\sum_\alpha \int \mathrm{d}V \int \mathrm{d}W\, q_\alpha\, f_\alpha \left\langle \frac{\partial \phi_1}{\partial t} - \frac{v_\parallel}{c} \frac{\partial A_{1\parallel}}{\partial t} \right\rangle_{\boldsymbol{R}} = \sum_\alpha \int \mathrm{d}V\, q_\alpha \left(\left\langle \int \mathrm{d}W f_\alpha \right\rangle^{\dagger}_{\boldsymbol{r}} \frac{\partial \phi_1}{\partial t} - \left\langle \int \mathrm{d}W f_\alpha \frac{v_\parallel}{c} \right\rangle^{\dagger}_{\boldsymbol{r}} \frac{\partial A_{1\parallel}}{\partial t} \right)$$

Gyro-averages and GENE-X equations

gyro-average

$$\langle\phi(\boldsymbol{r})\rangle_{\boldsymbol{R}} = \int_0^{2\pi}\frac{\mathrm{d}\theta}{2\pi}\, e^{\boldsymbol{\rho}_\alpha\cdot\nabla}\,\phi(\boldsymbol{R})$$

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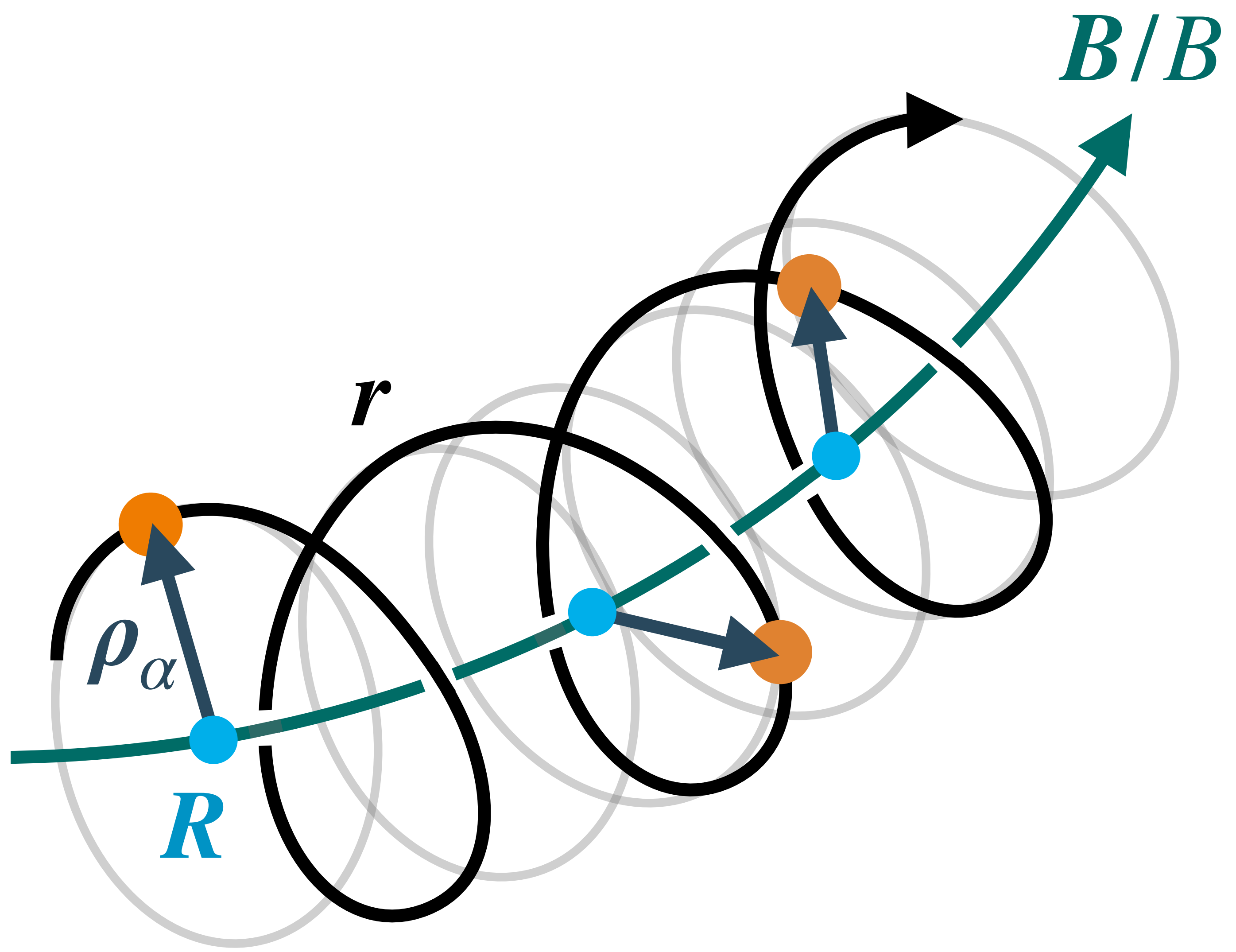
$$\langle\phi(\boldsymbol{R})\rangle^\dagger_{\boldsymbol{r}} = \int_0^{2\pi}\frac{\mathrm{d}\theta}{2\pi}\int \mathrm{d}\boldsymbol{R}\;\delta\left(\boldsymbol{R}+\boldsymbol{\rho}_\alpha-\boldsymbol{r}\right)\,\phi(\boldsymbol{R})$$

[1]

$$\mathscr{L}\sim \;\phi_1\;,\;A_{1\|}$$

ENERGY CONSERVATION

$$f_1\left(\langle\ldots\rangle_{\boldsymbol{R}}\right) = f_2\left(\langle\ldots\rangle^\dagger_{\boldsymbol{r}}\right)$$



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$$\frac{\partial f_\alpha}{\partial t} + \dot{\boldsymbol{R}}\cdot\nabla f_\alpha + \dot{v}_\| \frac{\partial f_\alpha}{\partial v_\|} = 0$$

- $\dot{\boldsymbol{R}} \sim \left\langle\phi_1\right\rangle_{\boldsymbol{R}}$
- $\dot{v}_\| \sim \left\langle\phi_1\right\rangle_{\boldsymbol{R}}, \left\langle A_{1\|}\right\rangle$

least-action principle for

- ϕ → **QUASI-NEUTRALITY** $\sim \left\langle n_\alpha\right\rangle^\dagger_{\boldsymbol{r}}$
- A → **AMPÈRE'S LAW** $\sim \left\langle \int \mathrm{d}W f_\alpha v_\| \right\rangle^\dagger_{\boldsymbol{r}}$

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FLR models

FLR… where?

orders of the Hamiltonian $\mathscr{H}$

$\mathscr{H}_0 , \mathscr{H}_1$ → full-FLR

$\mathscr{H}_2$ → Long Wavelength Approx. (LWA)

FLR IN ⟶	$\mathscr{H}_0 , \mathscr{H}_1$	$\mathscr{H}_2$
GENE-X current	LWA	LWA
GENE-X target	FULL	LWA
FULL FLR	FULL	FULL

FLR models

tested in the **electrostatic slab ion temperature gradient (ITG)** dispersion relation in Fourier space **with diffusion** in || and ⊥ dirs.

$$\sum_\alpha \frac{1}{T_\alpha}\Gamma_0 + \sum_\alpha \frac{1}{T_\alpha}\Gamma_0 \xi_{\text{eff}} Z - \sum_\alpha \frac{1}{\sqrt{2\,T_\alpha}}\frac{k_\perp}{k_\parallel}\left(\Gamma_0 Z + \eta\left(\Gamma_0 \xi_{\text{eff}}\left(1+\xi_{\text{eff}} Z\right) - \frac{1}{2}\Gamma_0 Z + a\left(\Gamma_1 - \Gamma_0\right) Z\right)\right)$$

+ polarization term $= 0$

- if full-FLR in $\mathscr{H}_2$ $\Rightarrow$ $\equiv \sum_\alpha \tau_\alpha^{-1}\left(1-\Gamma_0\right)$
- if LWA in $\mathscr{H}_2$ $\Rightarrow$ $\equiv \sum_\alpha k_\perp^2$

merging Vlasov eq. + quasi-neutrality eq.
$\Rightarrow$ J_0 respective operators combine to $\Gamma_0 \sim J_0^2$

Physics in Γ_0 !

Padé on J_0 from where we get Γ_0
or Padé on Γ_0 directly

FLR models

tested in the **electrostatic slab ion temperature gradient (ITG)** dispersion relation in Fourier space **with diffusion** in || and ⊥ dirs.

FLR models

tested in the **electrostatic slab ion temperature gradient (ITG)** dispersion relation in Fourier space **with diffusion** in ∥ and ⊥ dirs.

LWA in $\mathscr{H}_0, \mathscr{H}_1$

full-FLR in $\mathscr{H}_0, \mathscr{H}_1$

Padé

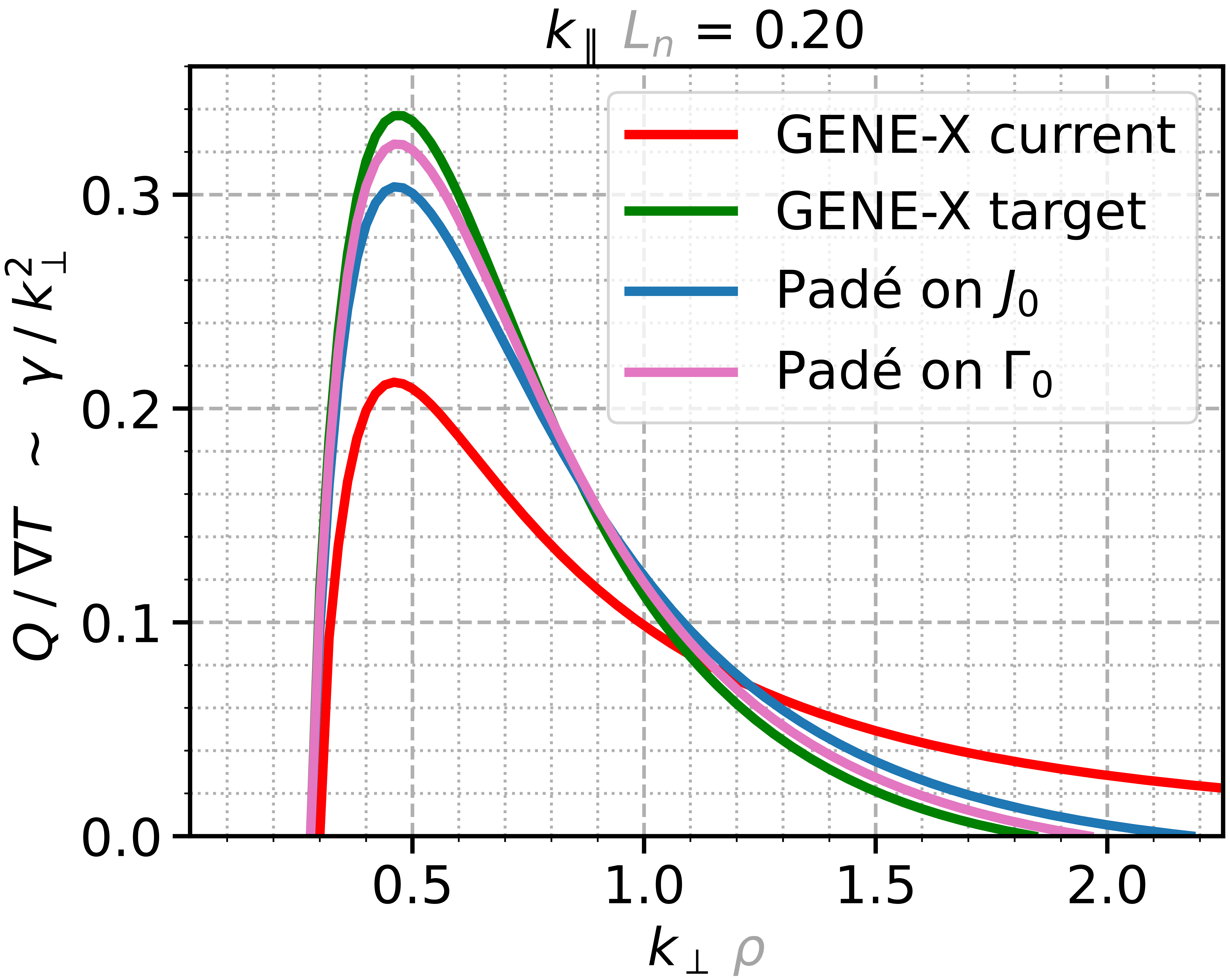
GENE-X current

GENE-X target

Padé on J_0

Padé on Γ_0

FLR models — ITG dispersion relation



analysis of quasi-linear flux **[1]**

- major Physics change from GENE-X **current** to **target**
- **Padé on** Γ_0 more Physically sensible
- **Padé on** J_0 not far-fetched and provides very convenient algorithm

$$\boxed{\left(1-\frac{1}{4}\,\rho_\alpha^2\nabla_\perp^2\right)\,\langle\phi(\boldsymbol{r})\rangle_\theta=\phi(\boldsymbol{R})}$$

differential eq. for gyro-average

[1] A. Mariani *et al.*, Physics of Plasmas **25**, 012313 (2018).

MAX-PLANCK-INSTITUT
FÜR PLASMAPHYSIK

Study of finite Larmor radius (FLR) physics in tokamaks and stellarators with the gyrokinetic turbulence code GENE-X

José Capitán, Philipp Ulbl, Baptiste J. Frei, Frank Jenko

GENE-X

HEPP Introductory Talk

EUROfusion

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