MAX-PLANCK-INSTITUT
FÜR PLASMAPHYSIK

Study of finite Larmor radius (FLR) physics in tokamaks and stellarators with the gyrokinetic turbulence code GENE-X

José Capitán, Philipp Ulbl, Baptiste J. Frei, Frank Jenko

HEPP Introductory Talk

EUROfusion

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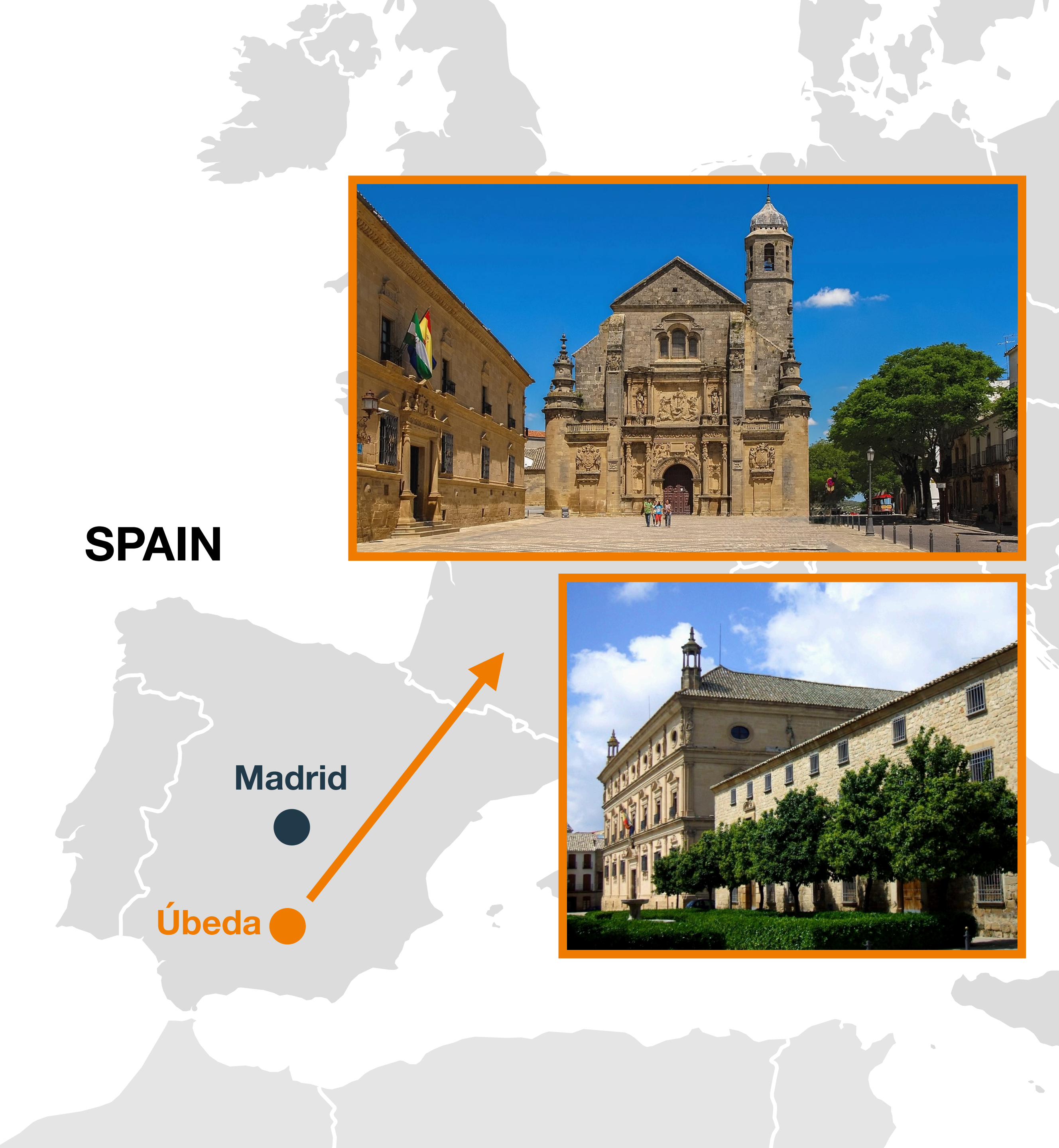
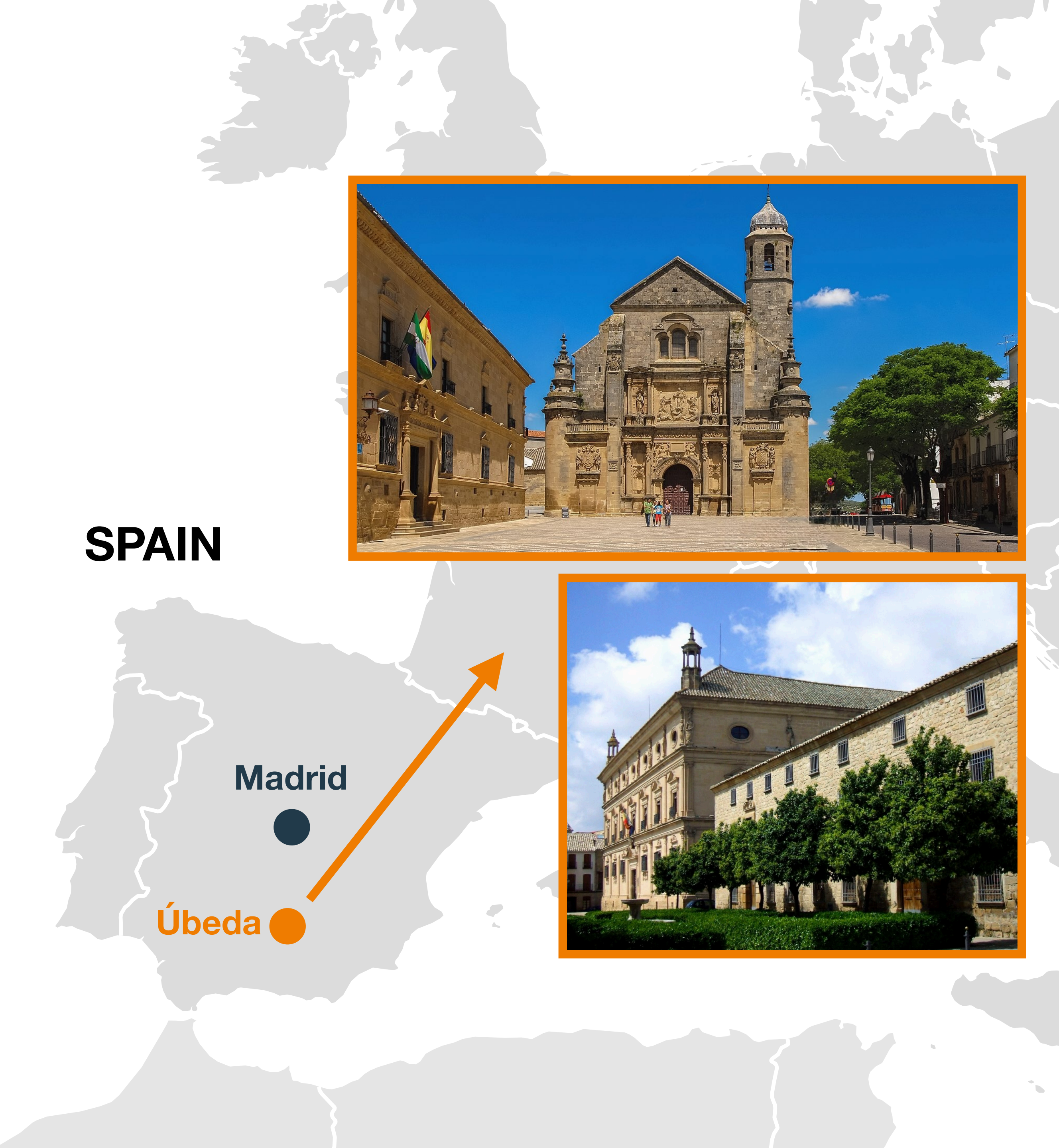
Bachelor → Physics

Master → thesis in **CIEMAT**
doing simulations with `stella` in
W7-X high mirror and **CIEMAT-QI**
configs.


SPAIN
Madrid
Úbeda

BEST THING TO DO IN SPAIN IS ~~LEAVING~~ EATING



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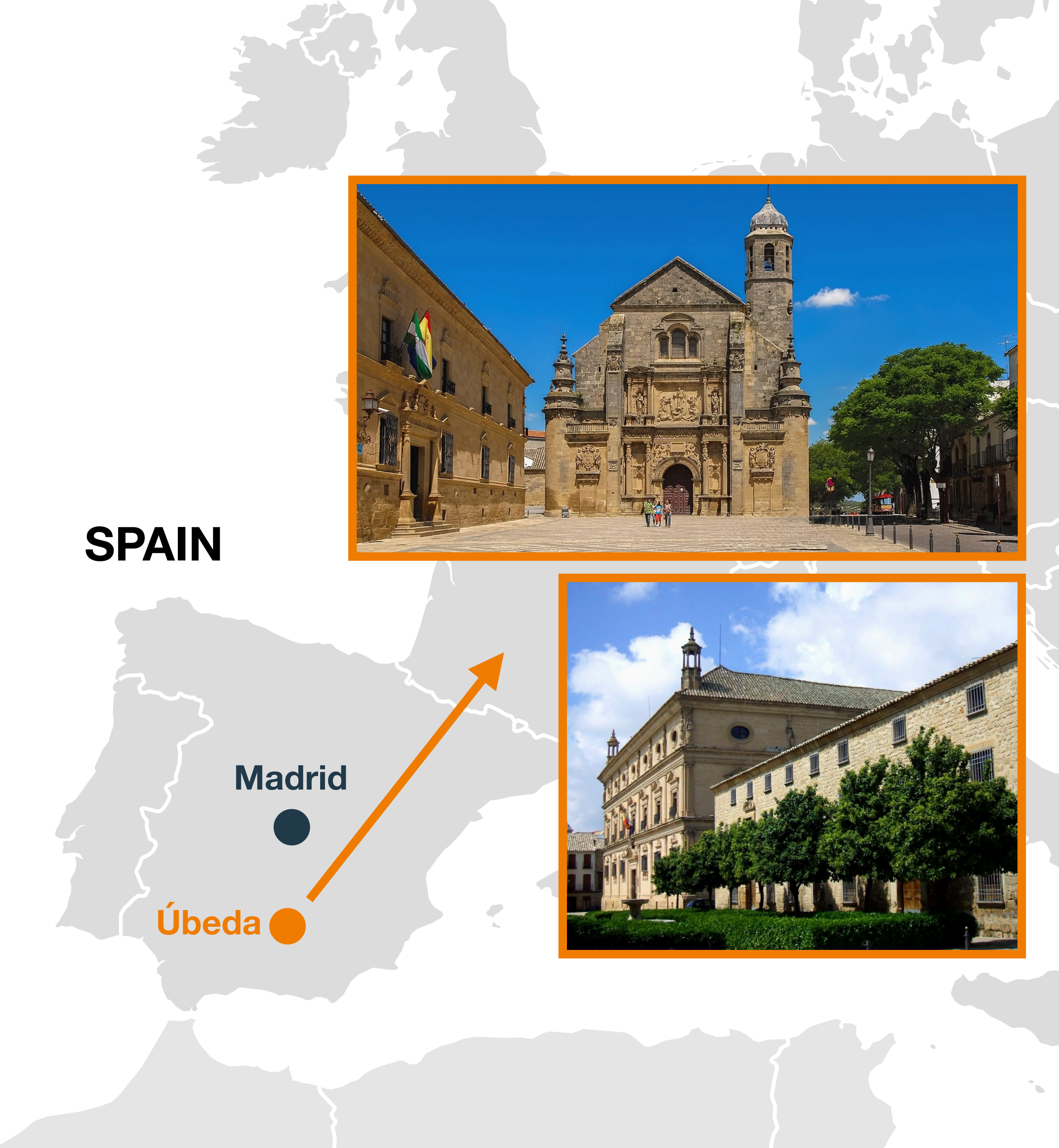
BEST THING TO DO IN SPAIN IS ~~LEAVING~~ EATING

Spanish omelette



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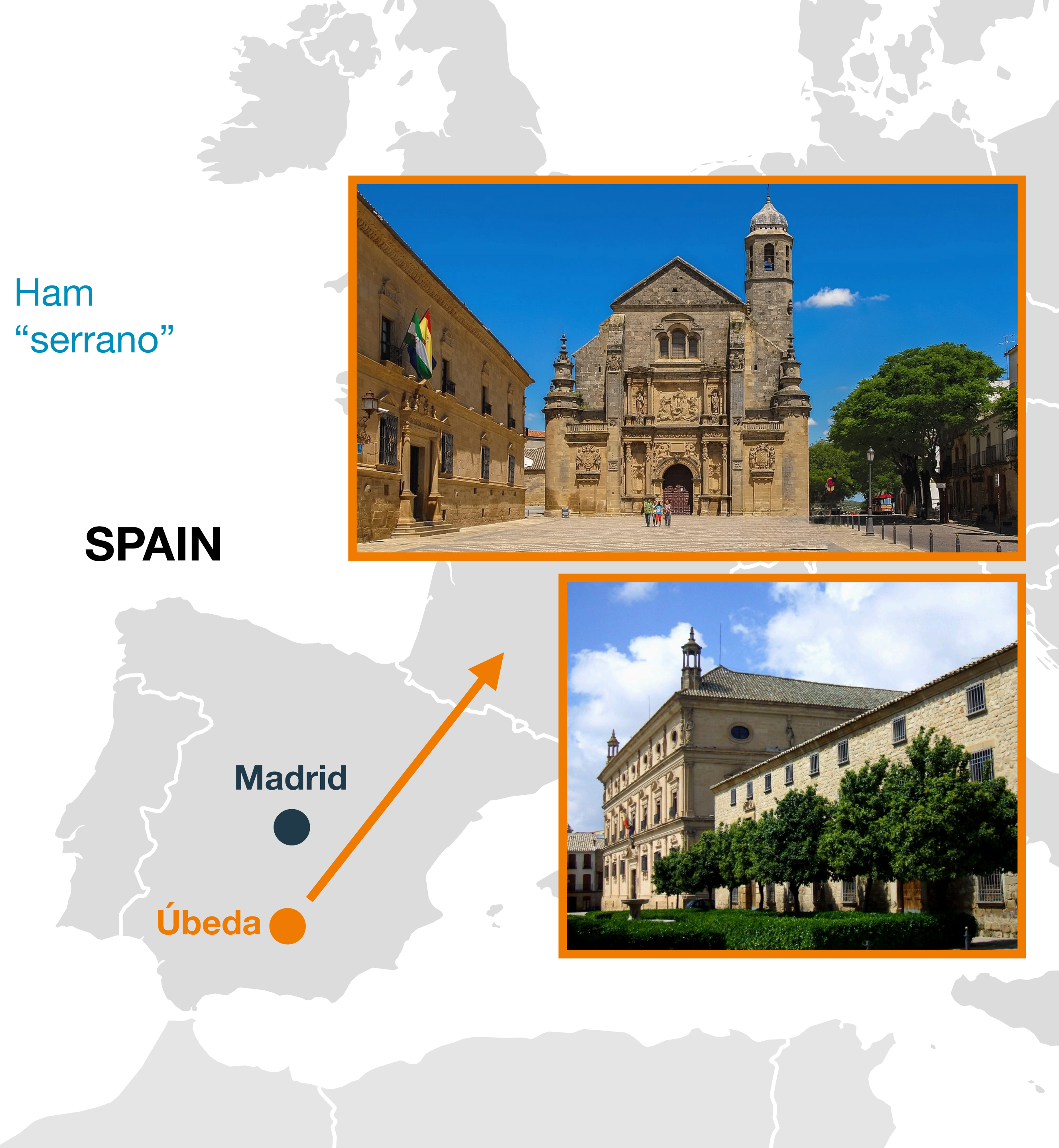
BEST THING TO DO IN SPAIN IS ~~LEAVING~~ EATING

Spanish omelette

Ham “serrano”



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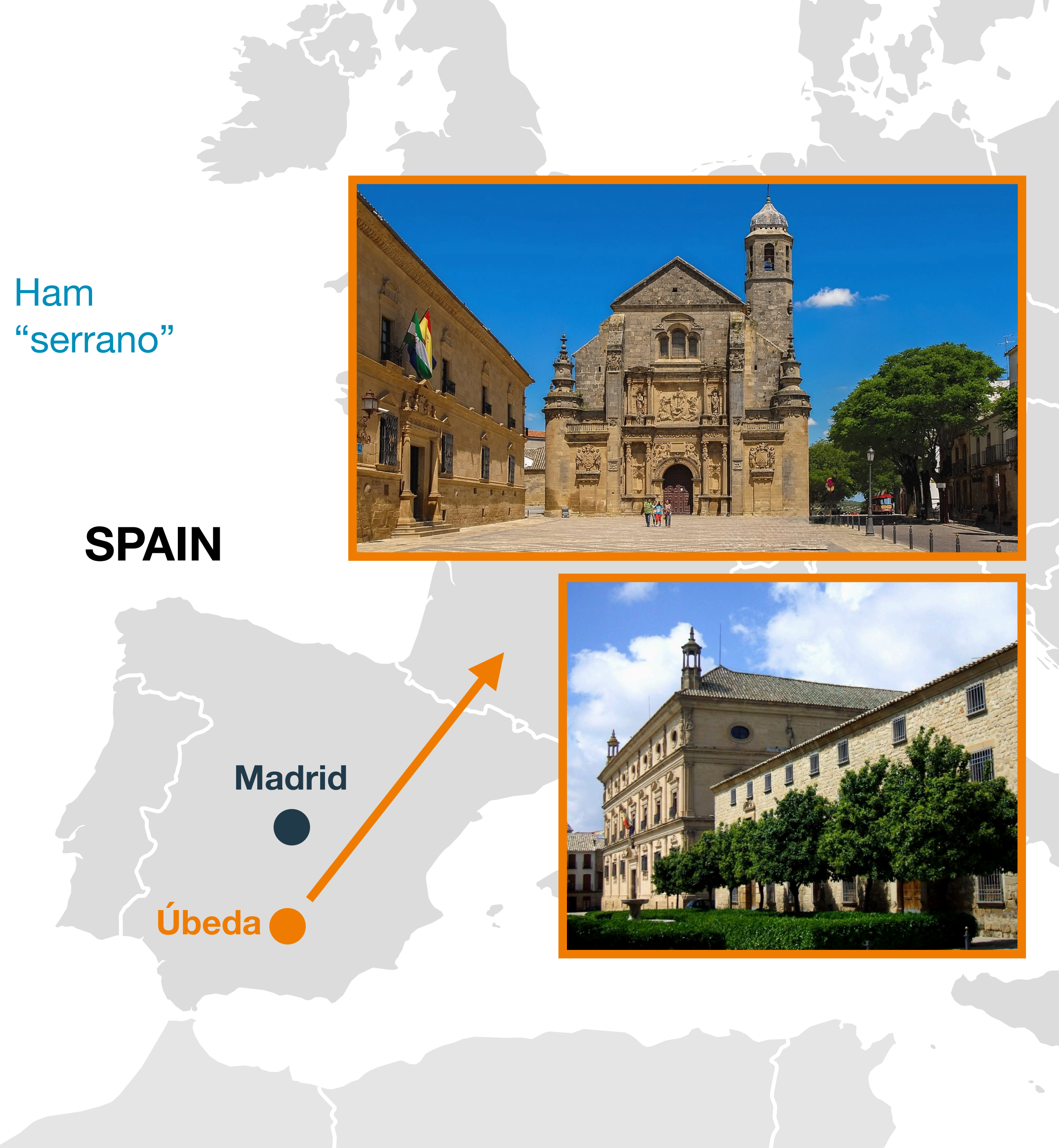
Paella



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Olive oil

random fact

the king of Spain is also

random fact

the king of Spain is also the king of Jerusalem 😐

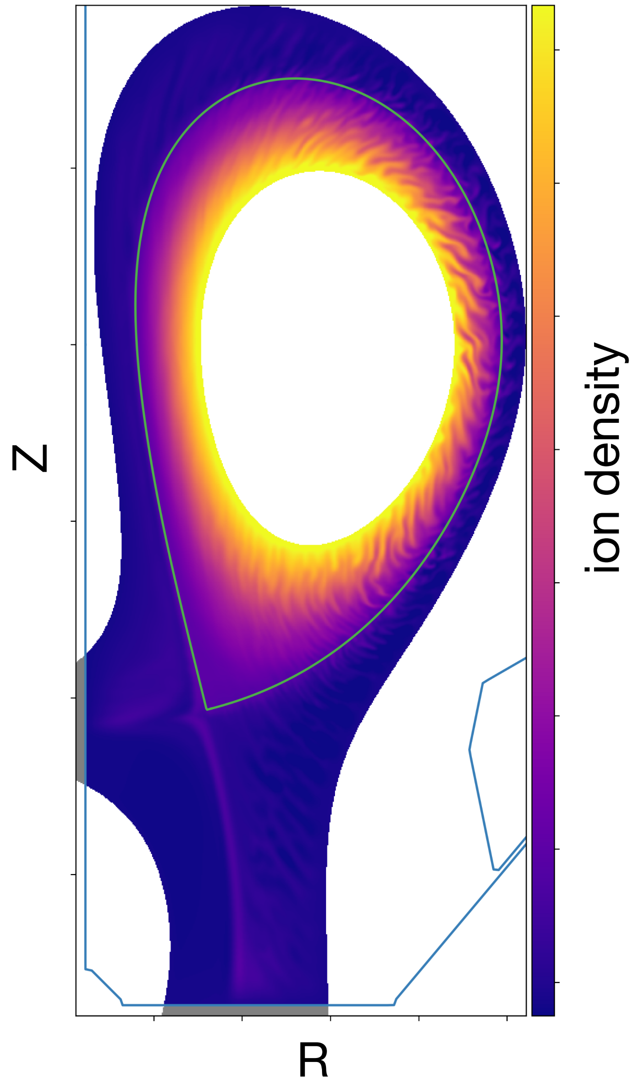
???

PhD project

Motivation

MICRO TURBULENCE

- small structures lead to macroscopic losses in magnetically confined plasmas
- optimize energy confinement → losses minimization
- understand it in the plasma
  - **core** → short wavelength
  - **edge** → long wavelength



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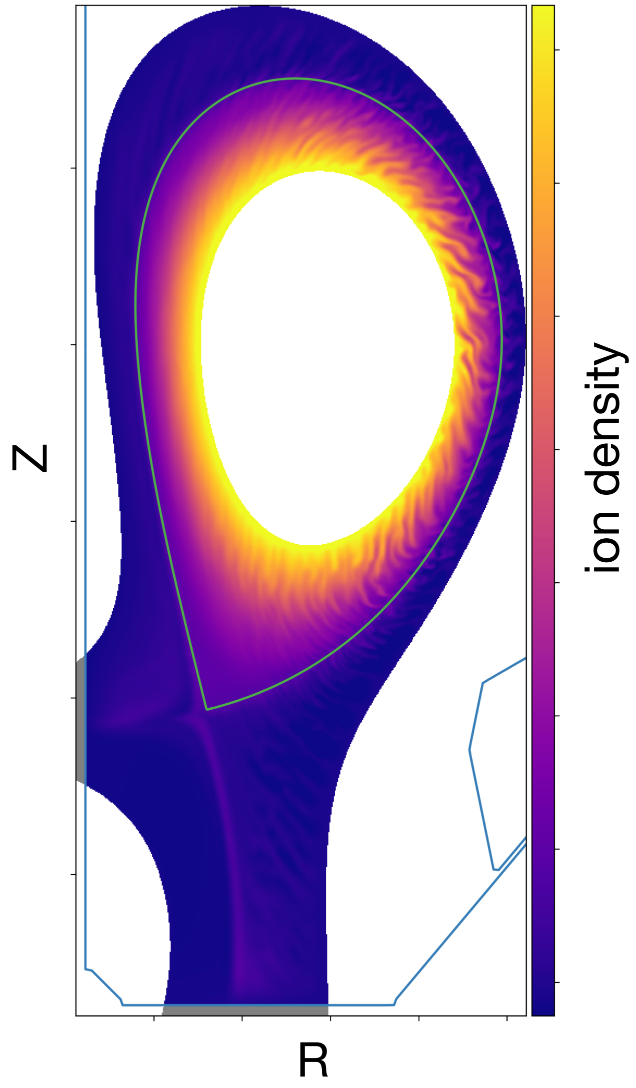
GENE-X

- Eulerian code that solves the gyrokinetic Vlasov eq. on a grid
- collisional, full-f, EM gyrokinetic turbulence model
- addresses the complexities of **edge turbulence** simulations



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MICRO TURBULENCE

- small structures lead to macroscopic losses in magnetically confined plasmas
- optimize energy confinement → losses minimization
- understand it in the plasma
  - **core** → short wavelength **?**
  - **edge** → long wavelength

GENE-X → **optimized for long wavelength turbulence!**

- Eulerian code that solves the gyrokinetic Vlasov eq. on a grid
- collisional, full-f, EM gyrokinetic turbulence model
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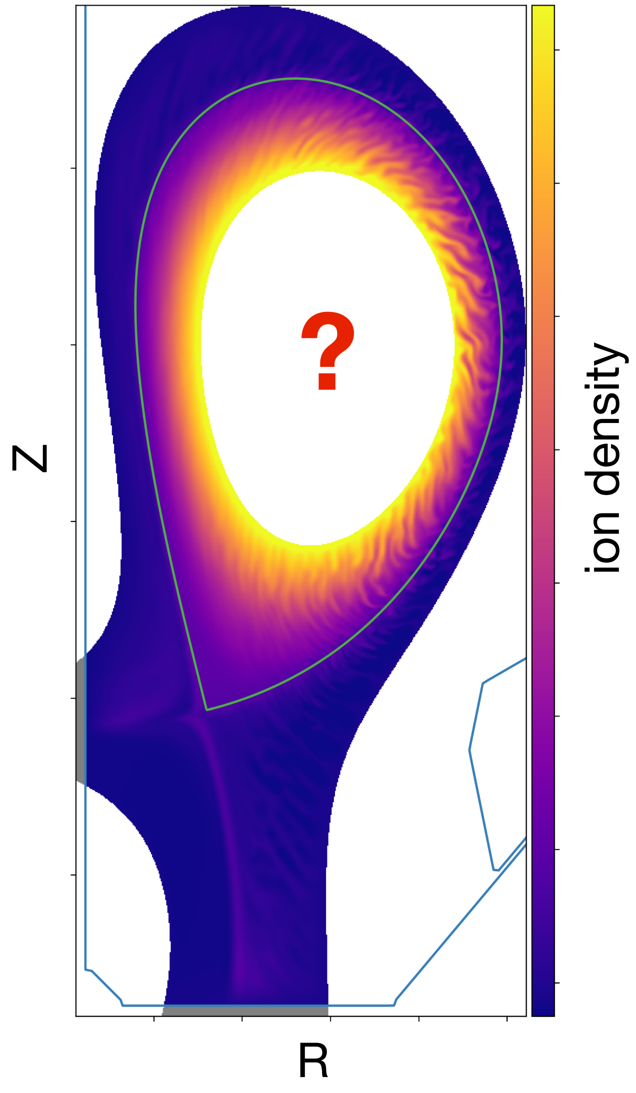
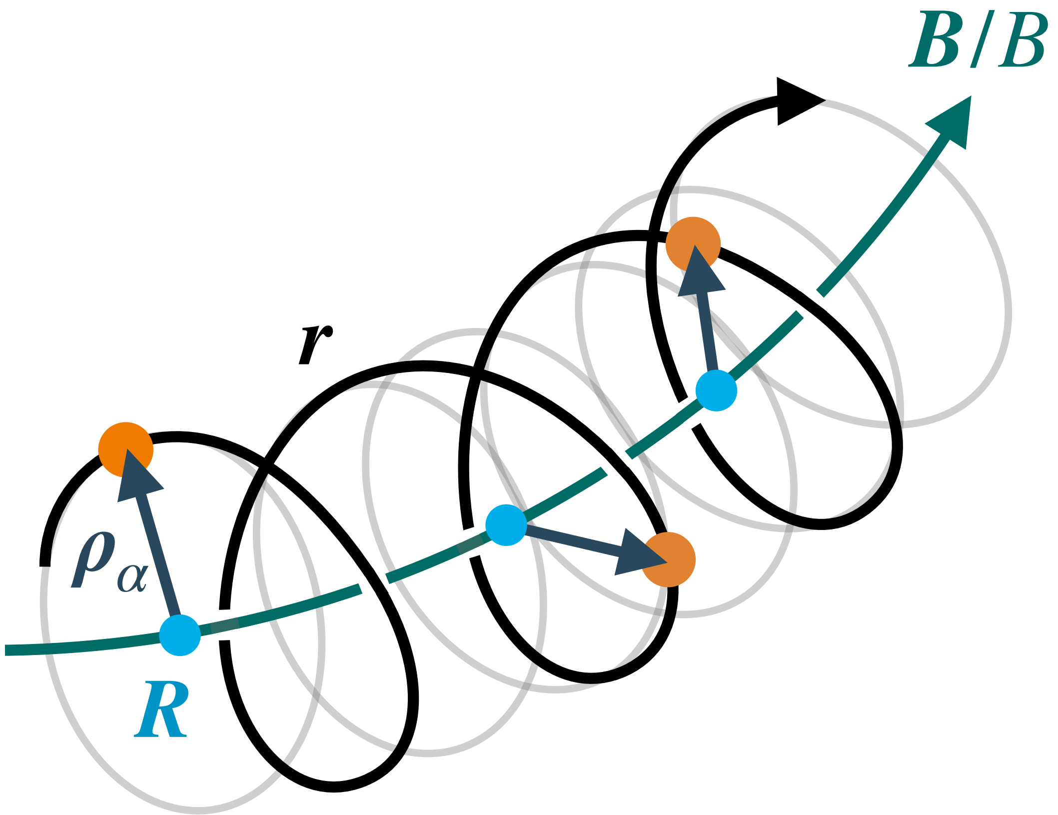


Finite Larmor radius (FLR) effects

ρ_α

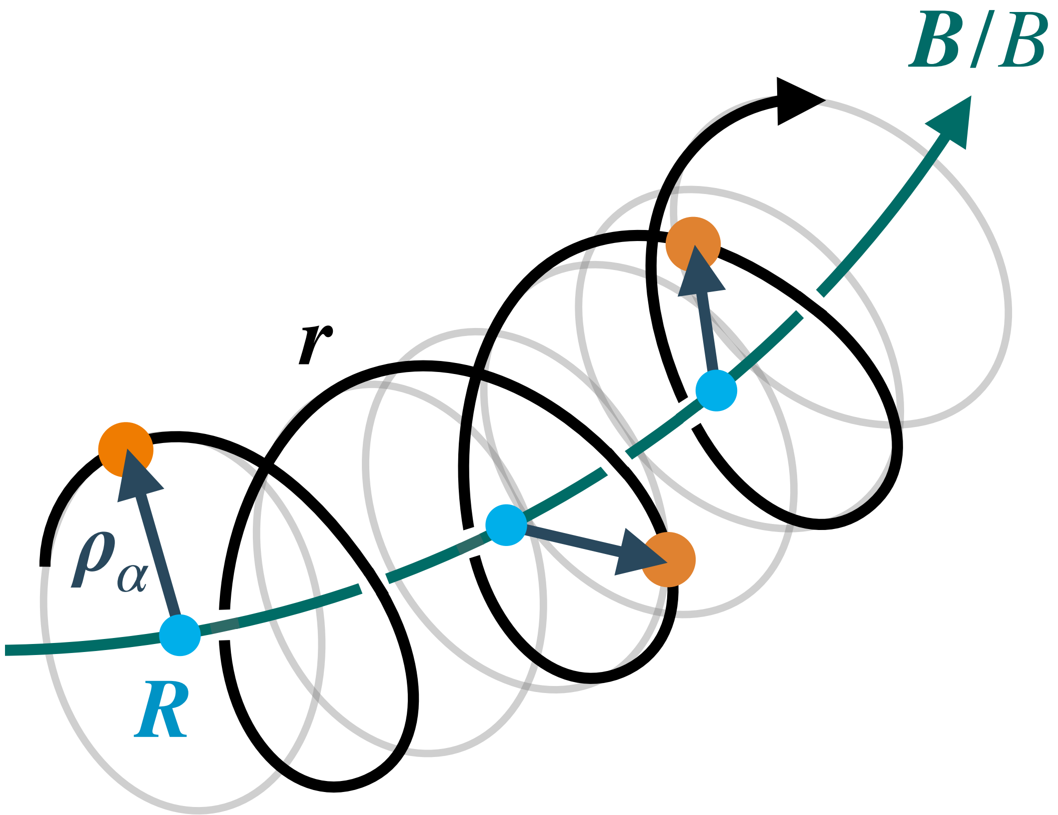
short wavelength turbulence



Finite Larmor radius (FLR) effects

ρ_α (Larmor radius)

short wavelength turbulence



ρ_α **relevant in**
- plasma core
- core-edge transition
- near steep gradients

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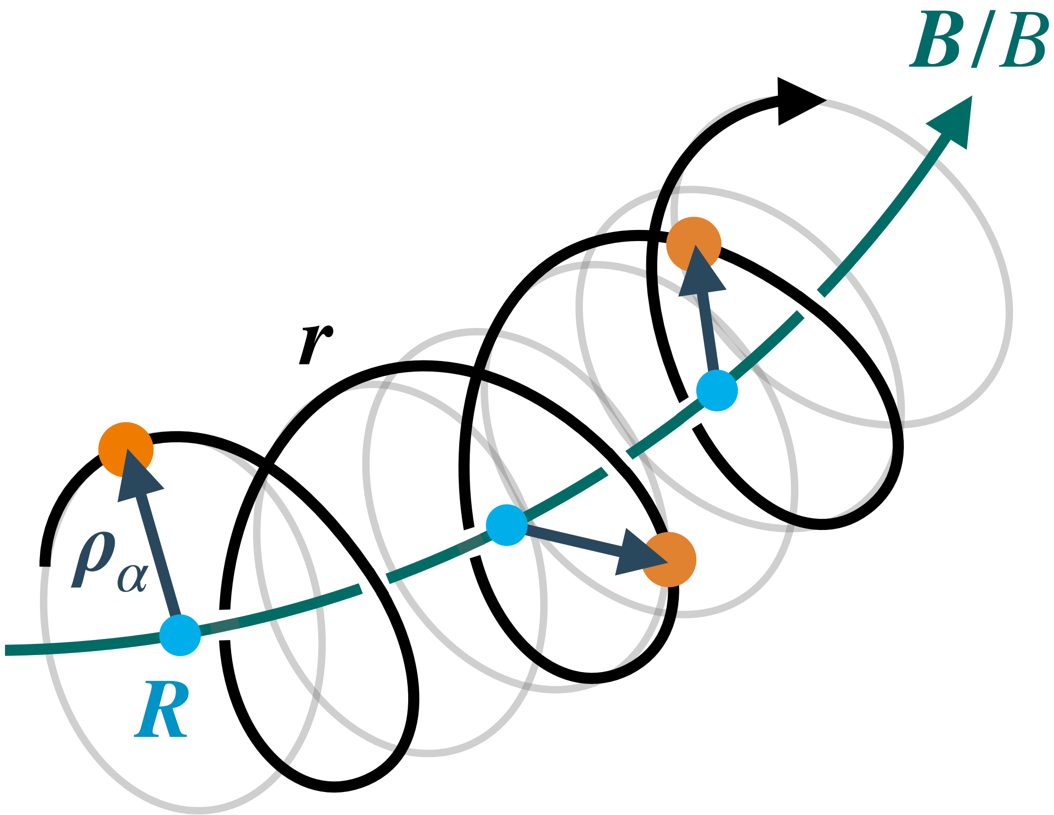

ρ_α **relevant in**
- plasma core
- core-edge transition
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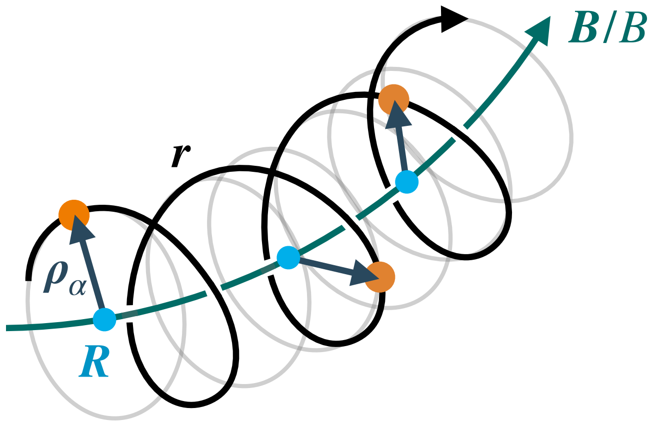
FLR effects
- to
  - **simulations from the edge to the core**
  - **regions where gradients ~ ρ_α order**
  - ***etc.***
- through
  - **gyro-averages**

Gyro-averages and GENE-X equations

VLASOV EQ. $$\frac{\partial f_\alpha}{\partial t} + \dot{\boldsymbol{R}} \cdot \nabla f_\alpha + \dot{v}_\parallel \frac{\partial f_\alpha}{\partial v_\parallel} = 0$$

$$\dot{\boldsymbol{R}} \sim \boldsymbol{B}\, v_\parallel + \frac{m_\alpha c}{q_\alpha} v_\parallel^2 \nabla \times \boldsymbol{b} + v_\parallel \quad \nabla \times \boldsymbol{A}_{1\parallel} \qquad + \frac{c}{q_\alpha} \boldsymbol{b} \times \mu \nabla B + c\, \boldsymbol{b} \times \nabla \quad \phi_1$$

$$\dot{v}_\parallel \sim -\boldsymbol{Y} \left(\mu \nabla B + q_\alpha \nabla \quad \phi_1 \qquad \right) - \frac{q_\alpha}{m_\alpha c} \frac{\partial \quad A_{1\parallel}}{\partial t}$$



[1] D. Michels, PhD thesis (Technische Universität München, 2022).

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B advection

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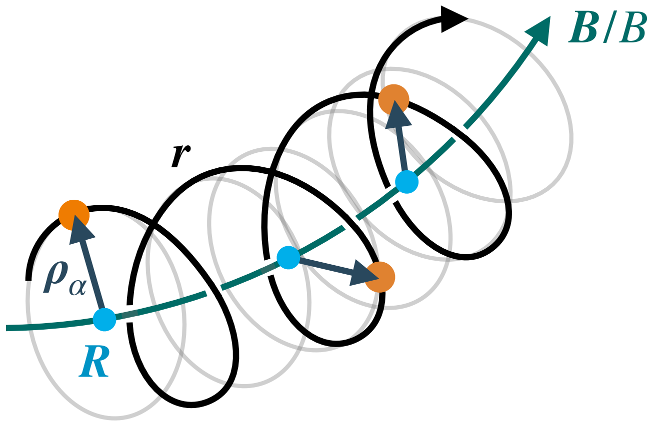

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curvature drift

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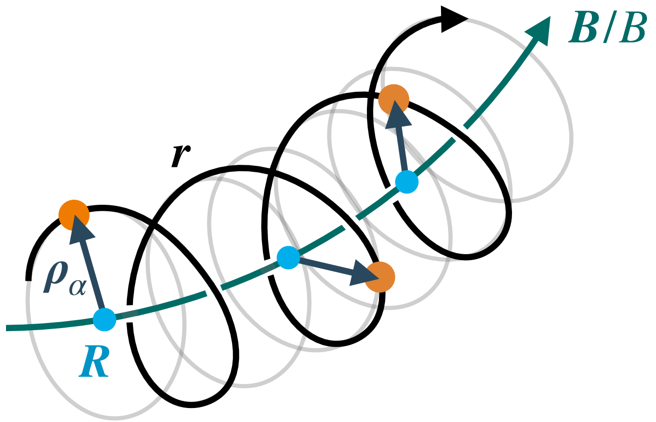

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$\perp$ EM flutter

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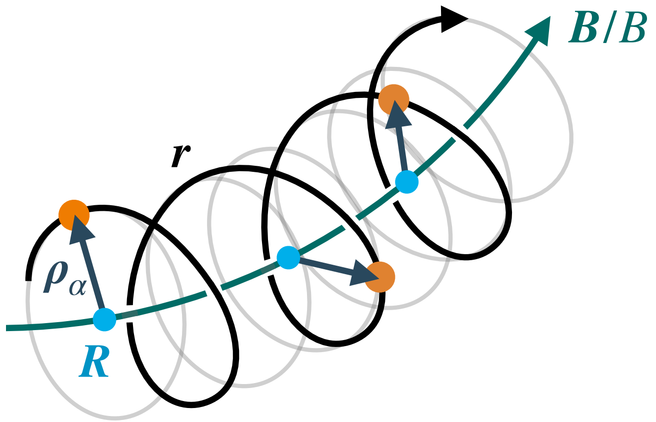

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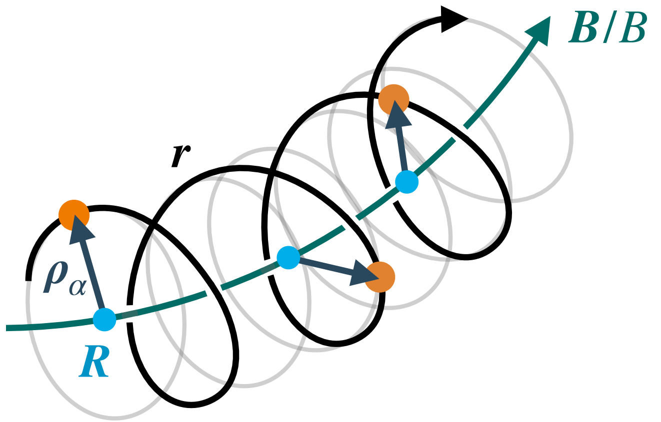

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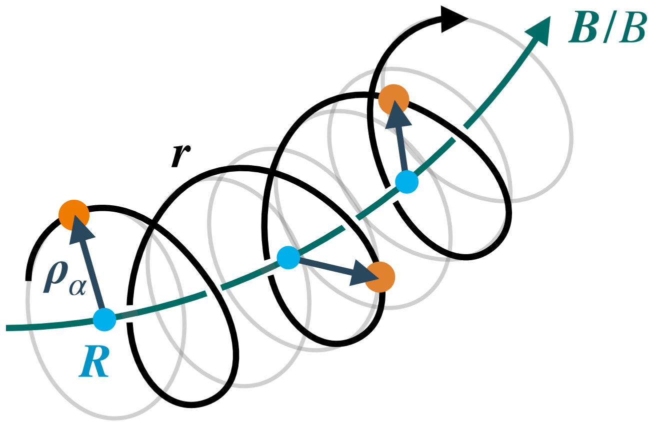

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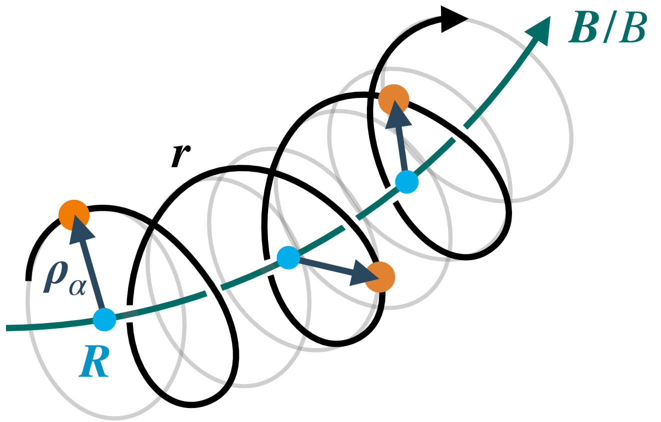
mirror force



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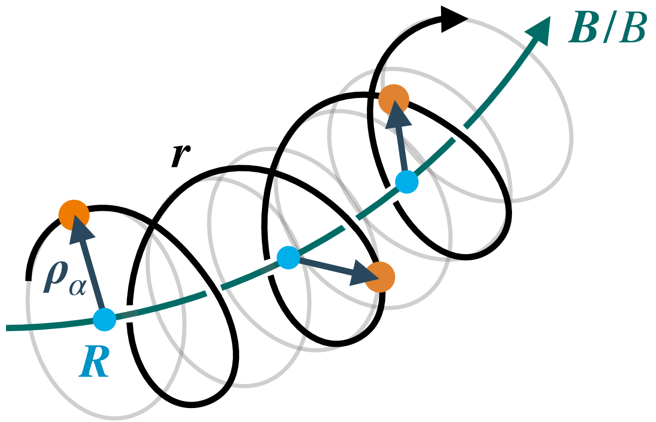
mirror force | $\boldsymbol{E}$ field acceler.



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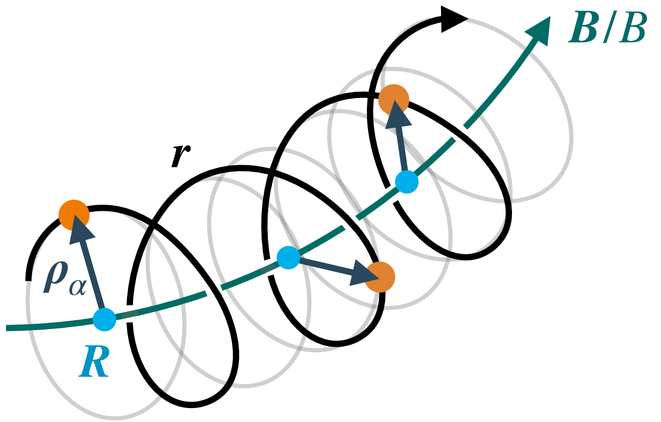

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Gyro-averages and GENE-X equations

gyro-average

$$\langle \phi(\boldsymbol{r}) \rangle_{\boldsymbol{R}} = \int_0^{2\pi} \frac{d\theta}{2\pi}\, e^{\boldsymbol{\rho}_\alpha \cdot \nabla}\, \phi(\boldsymbol{R})$$

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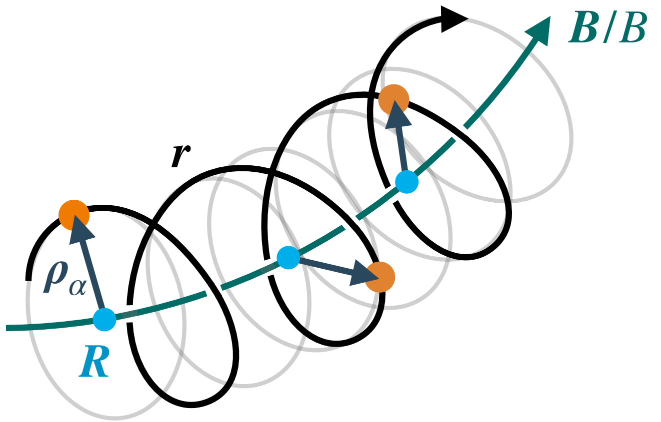

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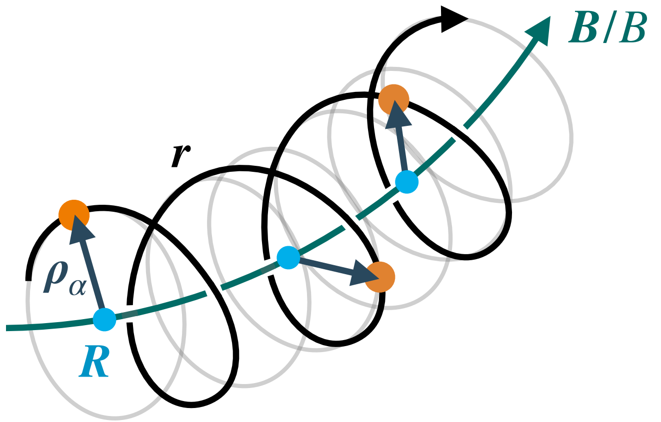
mirror force; $\boldsymbol{E}$ field acceler.; induction



Padé approximant

gyro-average

$$\langle \phi(\boldsymbol{r}) \rangle_{\boldsymbol{R}} = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi}\, e^{\boldsymbol{\rho}_\alpha \cdot \nabla}\, \phi(\boldsymbol{R})$$

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$\downarrow \mathscr{F}$

$$\langle \phi(\boldsymbol{r}) \rangle_{\boldsymbol{R}} = \frac{1}{(2\pi)^3} \int \mathrm{d}\boldsymbol{k}\; J_0\left(\rho_\alpha k_\perp\right)\; e^{i\boldsymbol{k}\cdot\boldsymbol{R}}\, \hat{\phi}(\boldsymbol{k})$$

Padé approximant

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FLR operator in $\mathscr{F}$

Bessel func. of the 1st kind

Padé approximant

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FLR operator in $\mathscr{F}$

Bessel func. of the 1st kind

Taylor on $J_0 \xrightarrow{k_\perp \to \infty} \infty$

$\Downarrow$

another approx., *e.g.*, **Padé approximant**

$$J_0(\rho_\alpha k_\perp) \overset{P0/2}{\approx} \frac{1}{1 + \rho_\alpha^2 k_\perp^2 / 4}$$

Padé approximant

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$$J_0(\rho_\alpha k_\perp) \overset{P0/2}{\approx} \frac{1}{1 + \rho_\alpha^2 k_\perp^2 / 4}$$

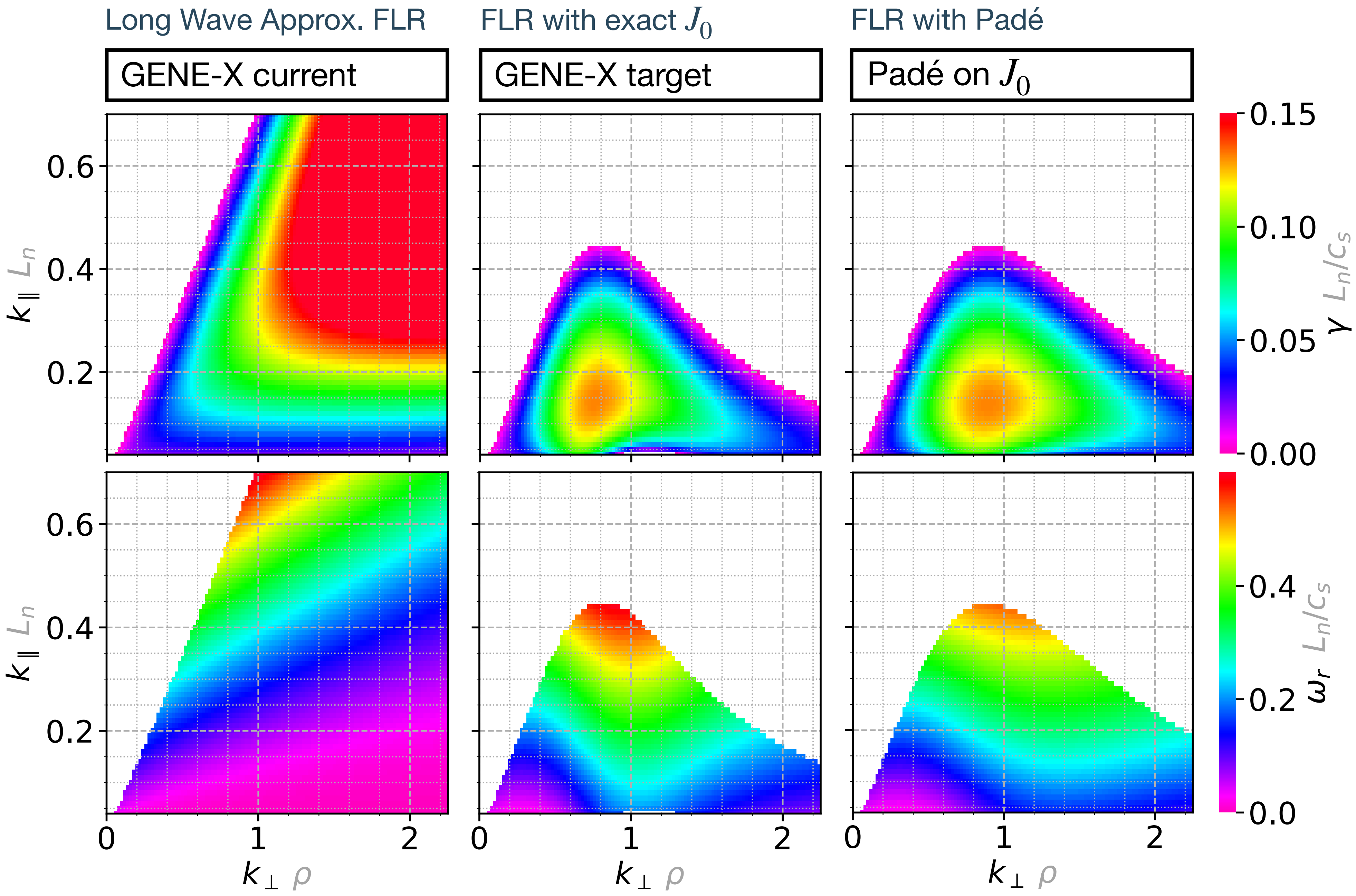
$\downarrow \mathscr{F}^{-1}$

$$\boxed{\left(1 - \frac{1}{4}\,\rho_\alpha^2 \nabla_\perp^2\right) \langle \phi(\boldsymbol{r}) \rangle_{\boldsymbol{R}} = \phi(\boldsymbol{R})}$$

differential equation for a gyro-average

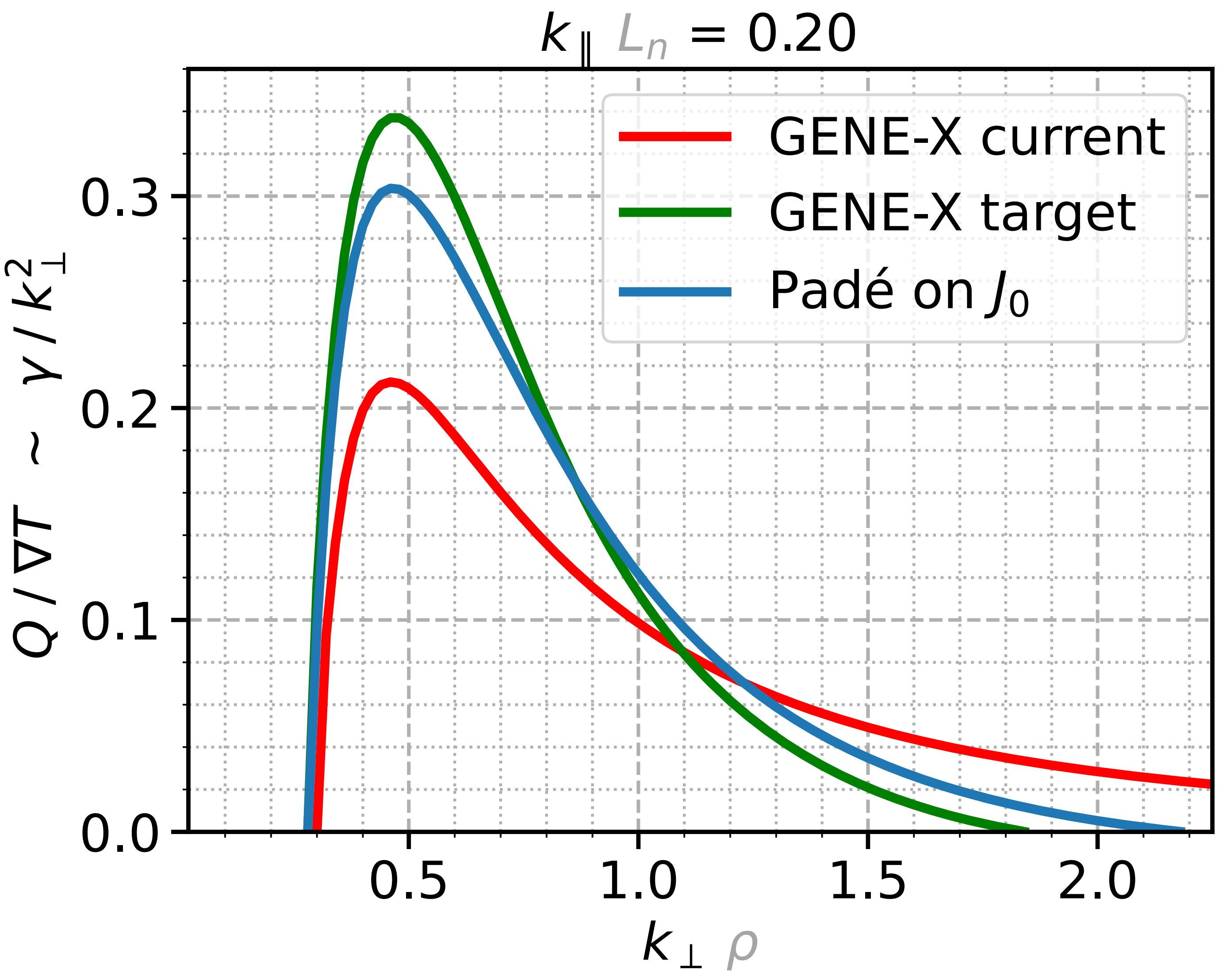
Impact of FLR models

EXAMPLE in the **electrostatic slab ion temperature gradient (ITG)** dispersion relation in Fourier space **with diffusion** in || and ⊥ dirs.



Impact of FLR models — ITG disp. rel.

GENE-X current → Long Wave App. FLR
GENE-X target → FLR with exact J_0
Padé on J_0 → FLR with Padé



analysis of quasi-linear flux

[2] A. Mariani *et al.*, Physics of Plasmas **25**, 012313 (2018).

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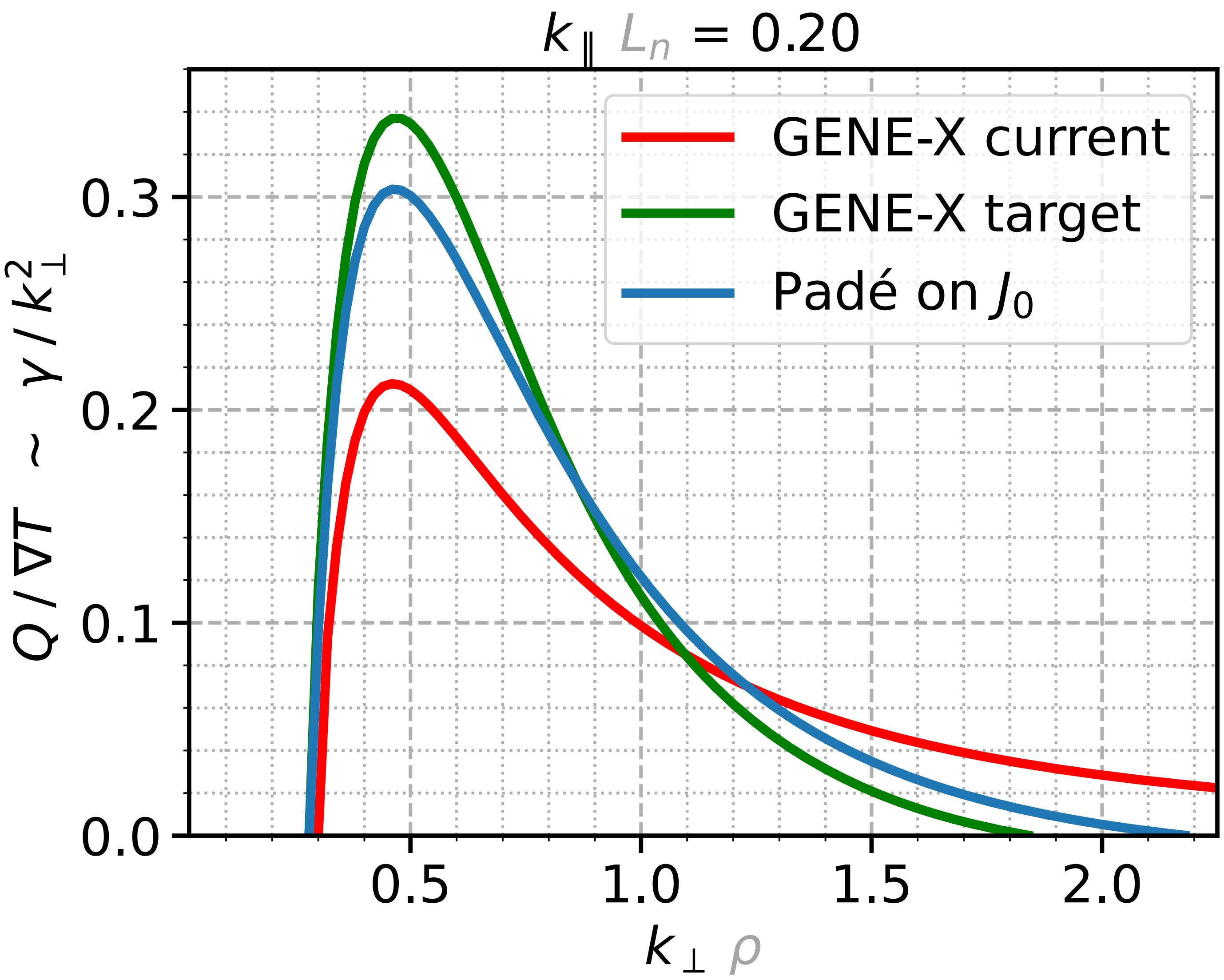


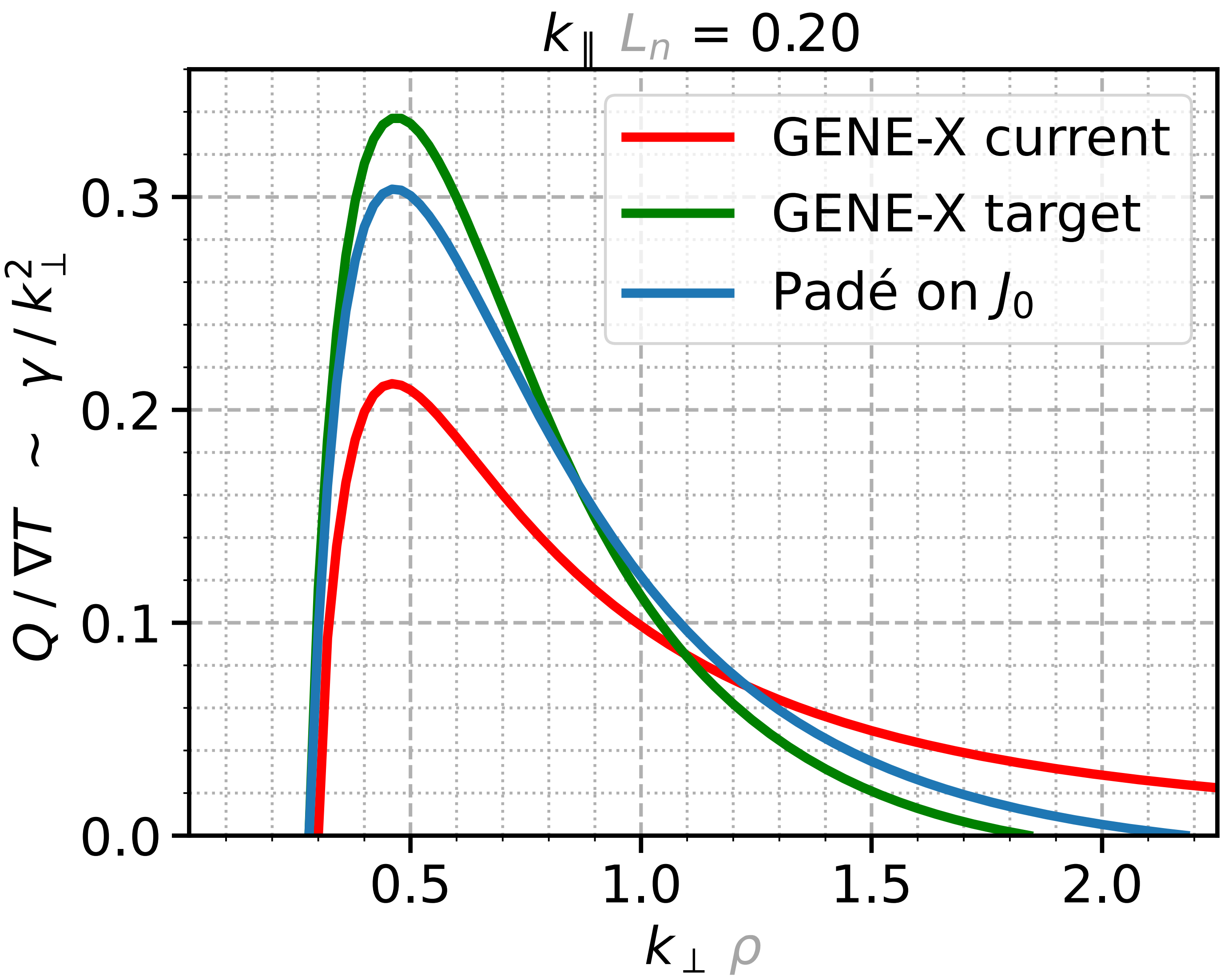
analysis of quasi-linear flux

↳ major Physics change from GENE-X **current** to **target**

[2] A. Mariani *et al.*, Physics of Plasmas **25**, 012313 (2018).

Impact of FLR models — ITG disp. rel.

GENE-X current → Long Wave App. FLR
GENE-X target → FLR with exact J_0
Padé on J_0 → FLR with Padé



analysis of quasi-linear flux

- major Physics change from GENE-X **current** to **target**
- **Padé on J_0** not far-fetched from **target** and provides very convenient algorithm

$$\left(1 - \frac{1}{4}\,\rho_\alpha^2 \nabla_\perp^2\right) \langle \phi(\boldsymbol{r}) \rangle_{\boldsymbol{R}} = \phi(\boldsymbol{R})$$

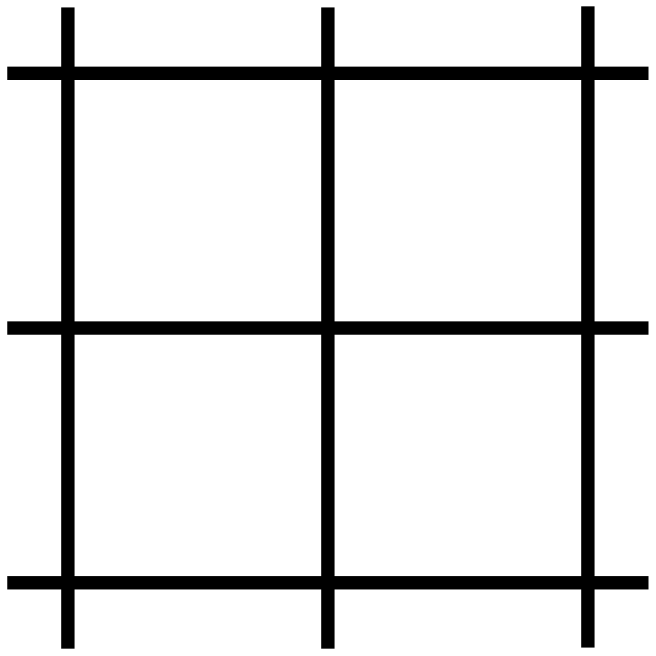
differential eq. for gyro-average

[2] A. Mariani *et al.*, Physics of Plasmas **25**, 012313 (2018).

Gyro-matrix approach

gyro-average

$$\langle \phi(\boldsymbol{r}) \rangle_{\boldsymbol{R}} = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi}\, e^{\rho_\alpha \cdot \nabla}\, \phi(\boldsymbol{R})$$



[3] T. Görler et al., Journal of Computational Phys. **230**, 7053–7071 (2011).

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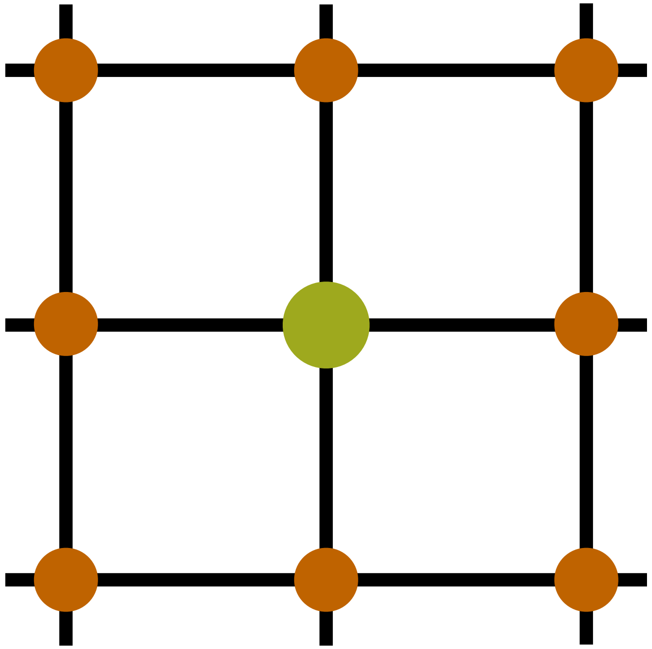
⟶ **grid** of **values**



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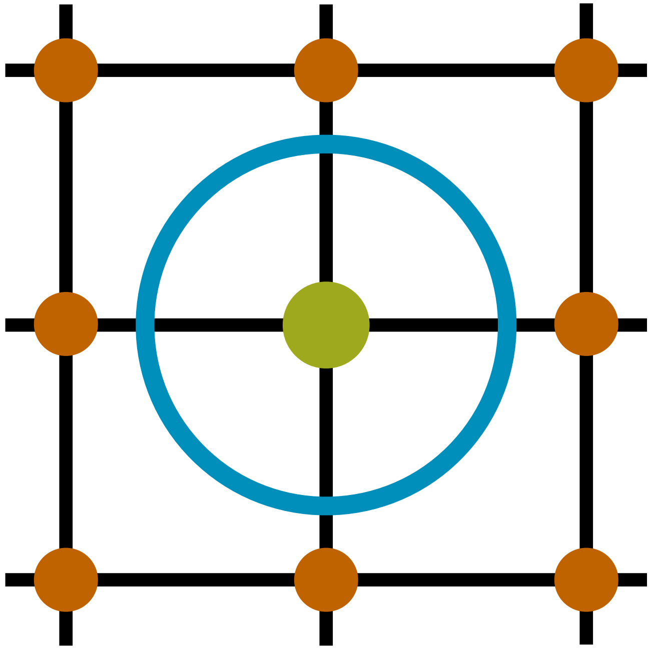
→ **grid** of **values**

→ **interpolated quantities** around **each point**



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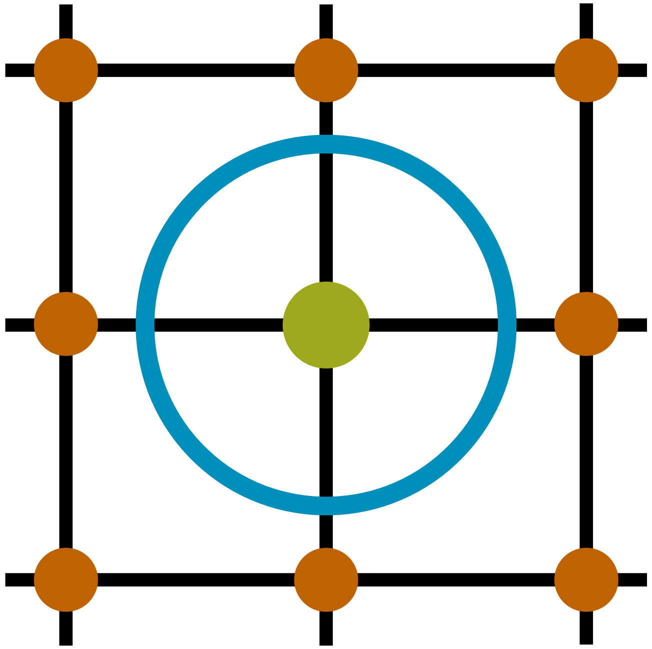
→ matrix combining interpolation and gyro-average

→ **gyro-matrix** $$\langle \boldsymbol{\phi} \rangle_{\boldsymbol{R}} = G(\mu)\, \boldsymbol{\phi}$$

calculates gyro-averages for all orders up to the discretization error



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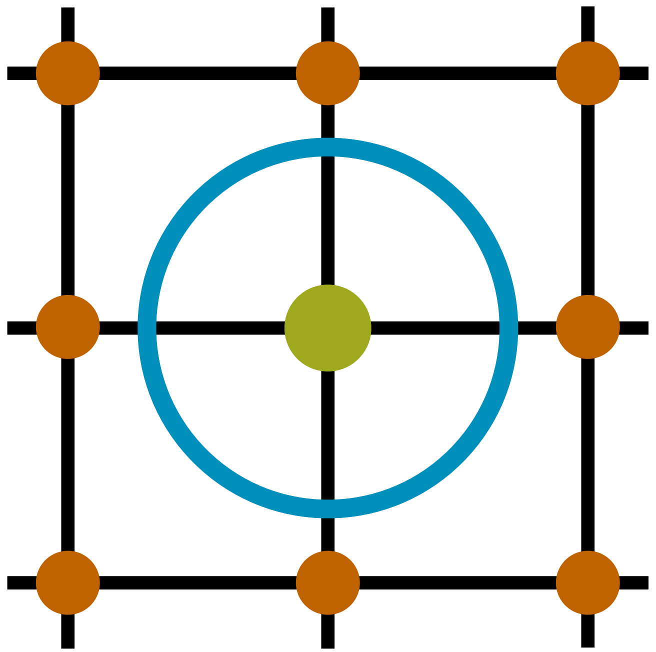

CAVEATS: intricate algorithm, expensive in resources…

[3] T. Görler et al., Journal of Computational Phys. **230**, 7053–7071 (2011).

Study of finite Larmor radius (FLR) physics in tokamaks and stellarators with the gyrokinetic turbulence code GENE-X

GENE-X

SUMMARY

→ Re-derivation of GENE-X eqs. accounting for FLR effects

→ Padé approx. of J_0

→ Example of FLR models in the ES slab ITG disp. relation

→ Padé on J_0 is Physically close and provides a convenient algorithm for FLR effects

→ Gyro-matrix approach

THANK YOU

ADDENDUM

Gyro-averages and GENE-X equations

gyro-average

$$\langle \phi(\boldsymbol{r}) \rangle_{\boldsymbol{R}} = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi}\, e^{\boldsymbol{\rho}_\alpha \cdot \nabla}\, \phi(\boldsymbol{R})$$

$$\mathscr{L} = \frac{q_\alpha}{c} \boldsymbol{A} \cdot \dot{\boldsymbol{R}} + \frac{q_\alpha}{c} \left\langle A_{1\parallel} \right\rangle_{\boldsymbol{R}} \boldsymbol{b} \cdot \dot{\boldsymbol{R}} + m_\alpha v_\parallel \boldsymbol{b} \cdot \dot{\boldsymbol{R}} + \frac{\mu B}{\Omega_\alpha} \dot{\theta} - q_\alpha \left\langle \phi_1 \right\rangle_{\boldsymbol{R}} - \frac{1}{2} m_\alpha v_\parallel^2 - \mu B + \frac{m_\alpha c^2}{2 B^2} \left| \nabla_\perp \phi_1 \right|^2$$

VLASOV EQ. $$\frac{\partial f_\alpha}{\partial t} + \dot{\boldsymbol{R}} \cdot \nabla f_\alpha + \dot{v}_\parallel \frac{\partial f_\alpha}{\partial v_\parallel} = 0$$

$$\left(\boldsymbol{B}^* \sim \boldsymbol{B} + \frac{m_\alpha c}{q_\alpha} v_\parallel \nabla \times \boldsymbol{b} + \left\langle \nabla \times \boldsymbol{A}_1 \right\rangle \right)$$

using the GENE-X Poisson bracket [1] for our coordinates

$$\dot{\boldsymbol{R}} = \frac{\boldsymbol{B}^*}{B_\parallel^*} v_\parallel + \frac{c}{q_\alpha B_\parallel^*} \boldsymbol{b} \times \left(\mu \nabla B + q_\alpha \nabla \left\langle \phi_1 \right\rangle_{\boldsymbol{R}} \right)$$

$$\dot{v}_\parallel = - \frac{\boldsymbol{B}^*}{m_\alpha B_\parallel^*} \left(\mu \nabla B + q_\alpha \nabla \left\langle \phi_1 \right\rangle_{\boldsymbol{R}} \right) - \frac{q_\alpha}{m_\alpha c} \frac{\partial \left\langle A_{1\parallel} \right\rangle}{\partial t}$$

[1] D. Michels, PhD thesis (Technische Universität München, 2022).

Gyro-averages and GENE-X equations

gyro-average

$$\langle \phi(\boldsymbol{r}) \rangle_{\boldsymbol{R}} = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi}\, e^{\boldsymbol{\rho}_\alpha \cdot \nabla}\, \phi(\boldsymbol{R})$$

adjoint operator

$$\langle \phi(\boldsymbol{R}) \rangle_{\boldsymbol{r}}^{\dagger} = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi} \int \mathrm{d}\boldsymbol{R}\, \delta\left(\boldsymbol{R} + \boldsymbol{\rho}_\alpha - \boldsymbol{r}\right)\, \phi(\boldsymbol{R})$$

$$\mathscr{L} = \frac{q_\alpha}{c} \boldsymbol{A} \cdot \dot{\boldsymbol{R}} + \frac{q_\alpha}{c} \left\langle A_{1\|} \right\rangle_{\boldsymbol{R}} \boldsymbol{b} \cdot \dot{\boldsymbol{R}} + m_\alpha v_\| \boldsymbol{b} \cdot \dot{\boldsymbol{R}} + \frac{\mu B}{\Omega_\alpha} \dot{\theta} - q_\alpha \left\langle \phi_1 \right\rangle_{\boldsymbol{R}} - \frac{1}{2} m_\alpha v_\|^2 - \mu B + \frac{m_\alpha c^2}{2 B^2} \left| \nabla_\perp \phi_1 \right|^2$$

least-action principle for

- ϕ → **QUASI-NEUTRALITY EQ.**

$$\sum_\alpha q_\alpha \left\langle n_\alpha \right\rangle_{\boldsymbol{r}}^{\dagger} = -\nabla \cdot \left(\sum_\alpha n_\alpha \frac{m_\alpha c^2}{B^2} \nabla_\perp \phi_1 \right)$$

- A → **AMPÈRE'S LAW**

$$\sum_\alpha \frac{q_\alpha}{c} \left\langle \int \mathrm{d}W f_\alpha v_\| \right\rangle_{\boldsymbol{r}}^{\dagger} = -\frac{1}{4\pi} \left| \nabla_\perp^2 A_{1\|} \right|$$

Gyro-averages and GENE-X equations

gyro-average

$$\langle \phi(\boldsymbol{r}) \rangle_{\boldsymbol{R}} = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi} \, e^{\boldsymbol{\rho}_\alpha \cdot \nabla} \, \phi(\boldsymbol{R})$$

adjoint operator

$$\langle \phi(\boldsymbol{R}) \rangle_{\boldsymbol{r}}^{\dagger} = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi} \int \mathrm{d}\boldsymbol{R} \, \delta\left(\boldsymbol{R} + \boldsymbol{\rho}_\alpha - \boldsymbol{r}\right) \, \phi(\boldsymbol{R})$$

ENERGY CONSERVATION

$$\sum_\alpha \int \mathrm{d}V \int \mathrm{d}W \, q_\alpha \, f_\alpha \left\langle \frac{\partial \phi_1}{\partial t} - \frac{v_\parallel}{c} \frac{\partial A_{1\parallel}}{\partial t} \right\rangle_{\boldsymbol{R}} = \sum_\alpha \int \mathrm{d}V \, q_\alpha \left(\left\langle \int \mathrm{d}W f_\alpha \right\rangle_{\boldsymbol{r}}^{\dagger} \frac{\partial \phi_1}{\partial t} - \left\langle \int \mathrm{d}W f_\alpha \frac{v_\parallel}{c} \right\rangle_{\boldsymbol{r}}^{\dagger} \frac{\partial A_{1\parallel}}{\partial t} \right)$$

Gyro-averages and GENE-X equations

gyro-average
$$\langle \phi(\boldsymbol{r}) \rangle_{\boldsymbol{R}} = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi}\, e^{\boldsymbol{\rho}_\alpha \cdot \nabla}\, \phi(\boldsymbol{R})$$

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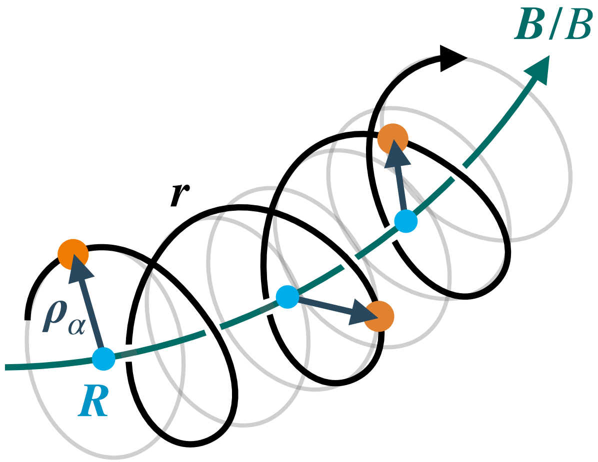
[1]

$$\mathscr{L} \sim \phi_1 \,,\; A_{1\|}$$

ENERGY CONSERVATION $f_1\left(\langle \dots \rangle_{\boldsymbol{R}}\right) = f_2\left(\langle \dots \rangle_{\boldsymbol{r}}^{\dagger}\right)$



VLASOV EQ. $$\frac{\partial f_\alpha}{\partial t} + \dot{\boldsymbol{R}} \cdot \nabla f_\alpha + \dot{v}_\| \frac{\partial f_\alpha}{\partial v_\|} = 0$$

- $\dot{\boldsymbol{R}} \sim \langle \phi_1 \rangle_{\boldsymbol{R}}$
- $\dot{v}_\| \sim \langle \phi_1 \rangle_{\boldsymbol{R}} \,, \langle A_{1\|} \rangle$

least-action principle for

- ϕ → **QUASI-NEUTRALITY** $\sim \langle n_\alpha \rangle_{\boldsymbol{r}}^{\dagger}$
- A → **AMPÈRE'S LAW** $\sim \left\langle \int \mathrm{d}W f_\alpha v_\| \right\rangle_{\boldsymbol{r}}^{\dagger}$

[1] D. Michels, PhD thesis (Technische Universität München, 2022).

FLR models

FLR… where?

orders of the Hamiltonian $\mathscr{H}$

- $\mathscr{H}_0, \mathscr{H}_1$ → full-FLR
- $\mathscr{H}_2$ → Long Wavelength Approx. (LWA)

FLR IN ⟶	$\mathscr{H}_0, \mathscr{H}_1$	$\mathscr{H}_2$
GENE-X current	LWA	LWA
GENE-X target	FULL	LWA
FULL FLR	FULL	FULL

FLR models

tested in the **electrostatic slab ion temperature gradient (ITG)** dispersion relation in Fourier space **with diffusion** in || and ⊥ dirs.

$$\sum_\alpha \frac{1}{T_\alpha}\Gamma_0 + \sum_\alpha \frac{1}{T_\alpha}\Gamma_0 \xi_{\text{eff}} Z - \sum_\alpha \frac{1}{\sqrt{2T_\alpha}} \frac{k_\perp}{k_\parallel}\left(\Gamma_0 Z + \eta\left(\Gamma_0 \xi_{\text{eff}}\left(1+\xi_{\text{eff}} Z\right) - \frac{1}{2}\Gamma_0 Z + a\left(\Gamma_1 - \Gamma_0\right) Z\right)\right)$$

+ polarization term $= 0$

- if full-FLR in $\mathscr{H}_2$ $\Rightarrow$ $\equiv \sum_\alpha \tau_\alpha^{-1}\left(1-\Gamma_0\right)$
- if LWA in $\mathscr{H}_2$ $\Rightarrow$ $\equiv \sum_\alpha k_\perp^2$

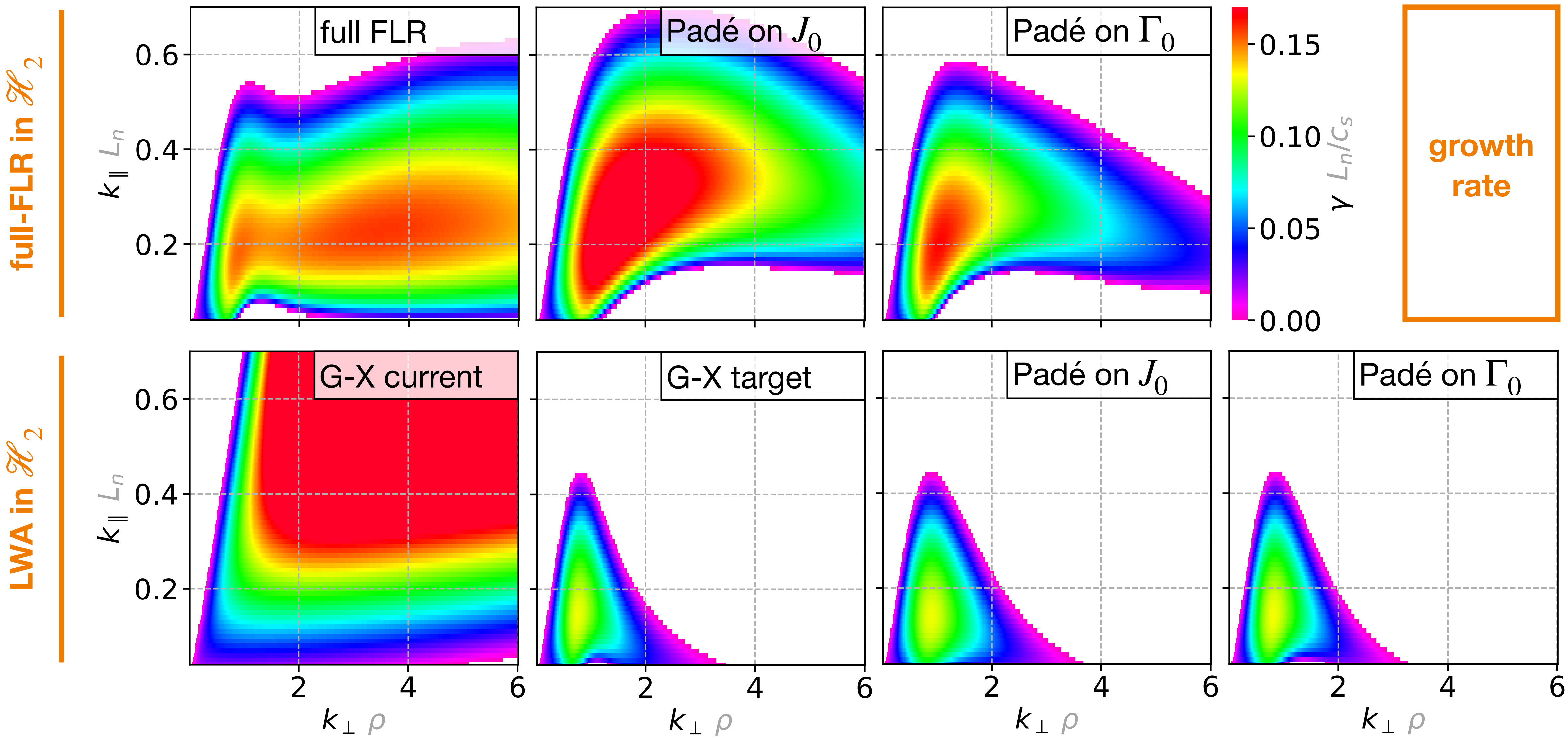
merging Vlasov eq. + quasi-neutrality eq.
$\Rightarrow$ J_0 respective operators combine to $\Gamma_0 \sim J_0^2$

Physics in Γ_0 !

Padé on J_0 from where we get Γ_0
or Padé on Γ_0 directly

FLR models

tested in the **electrostatic slab ion temperature gradient (ITG)** dispersion relation in Fourier space **with diffusion** in || and ⊥ dirs.



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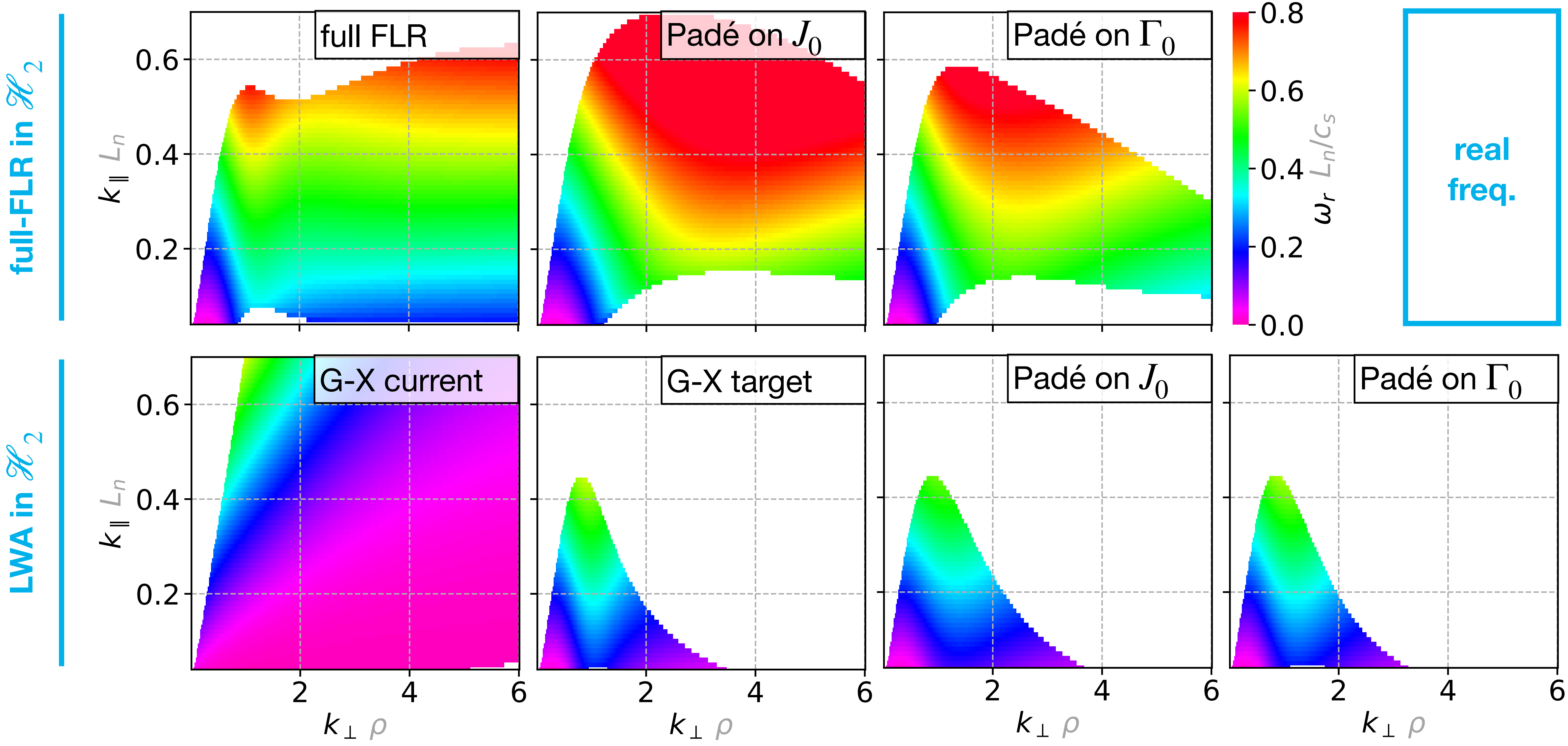
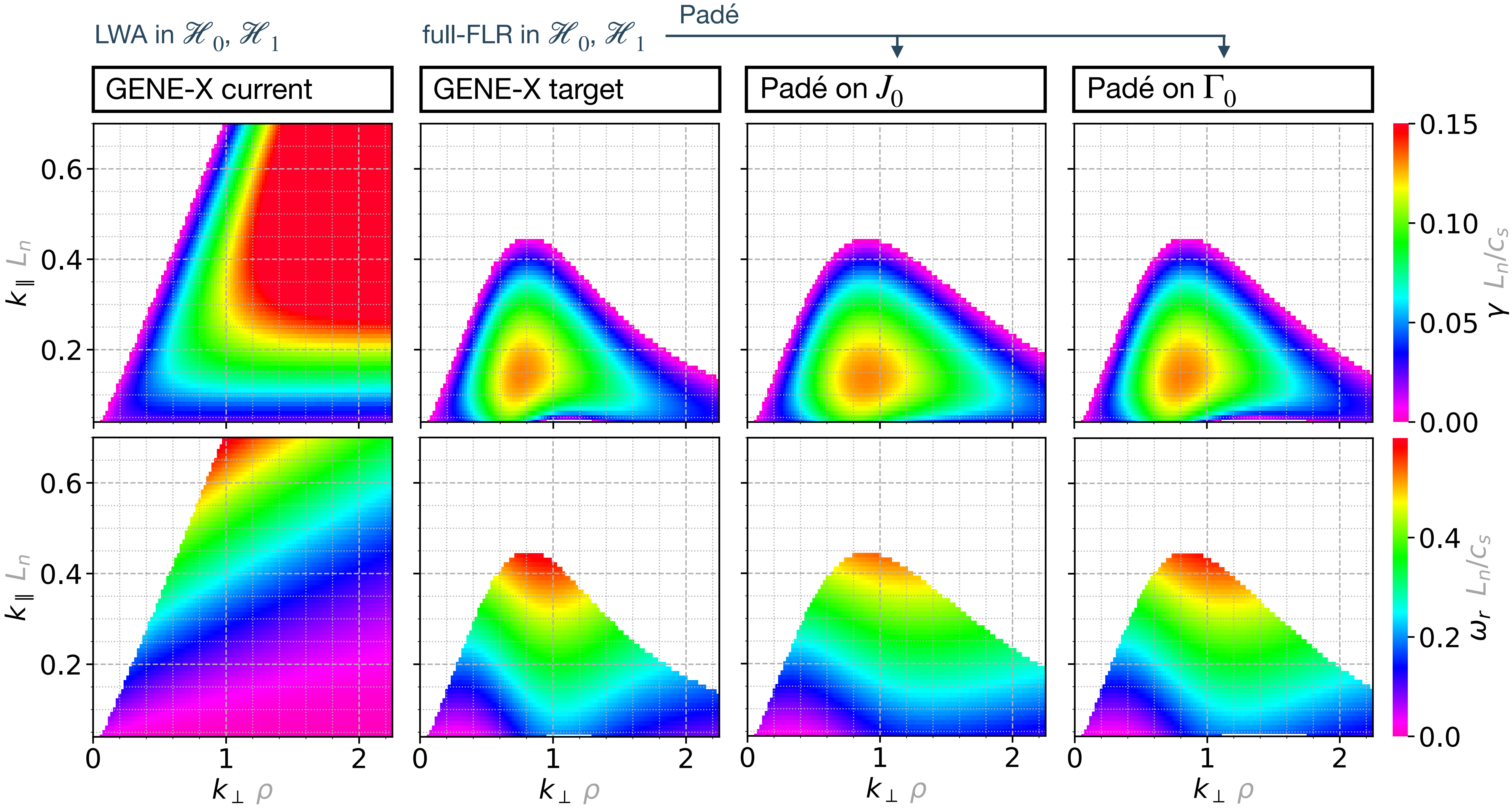
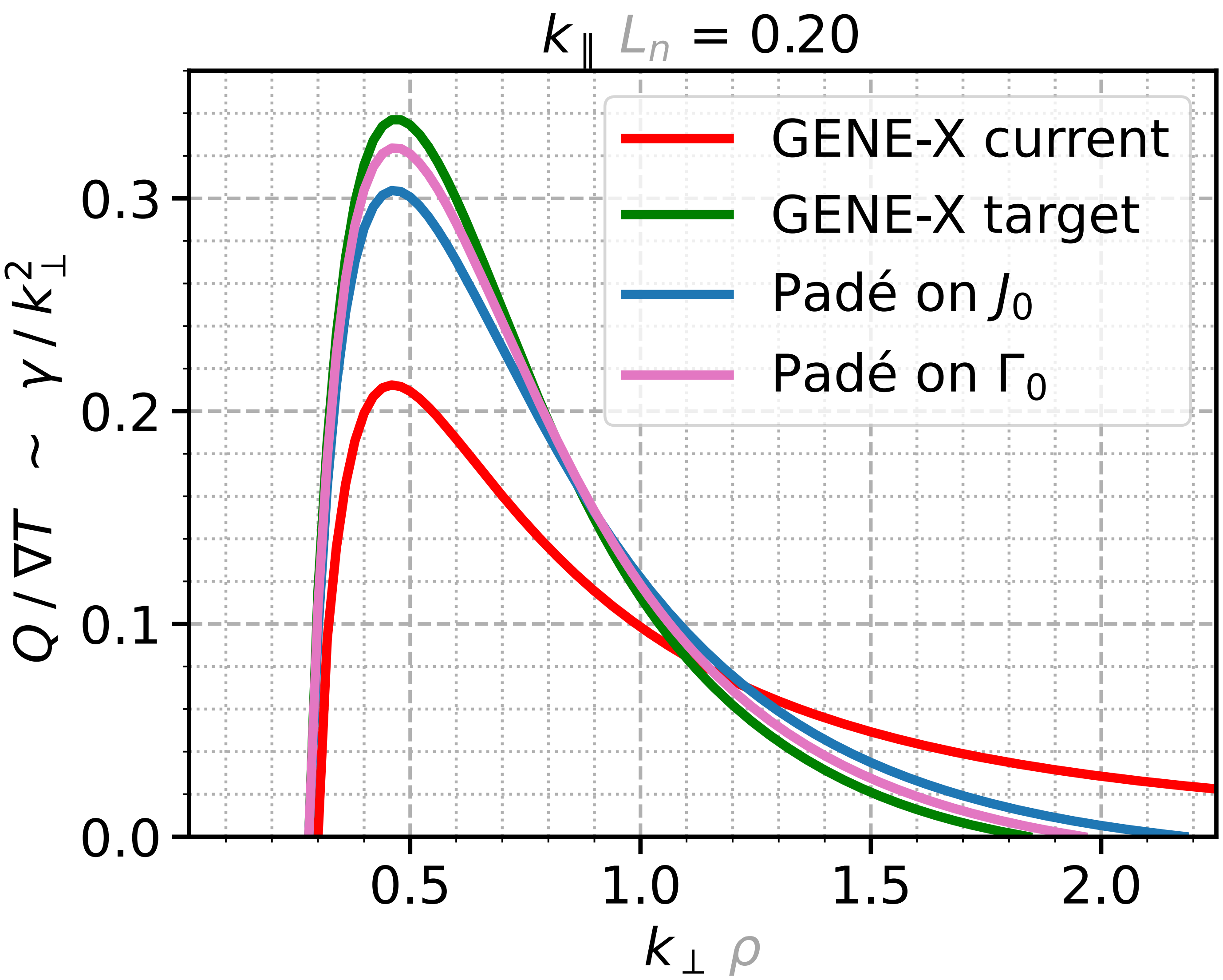




LWA in $\mathscr{H}_0, \mathscr{H}_1$
full-FLR in $\mathscr{H}_0, \mathscr{H}_1$
Padé
GENE-X current
GENE-X target
Padé on J_0
Padé on Γ_0
$k_\parallel L_n$
$k_\perp \rho$
$\gamma \; L_n/c_s$
$\omega_r \; L_n/c_s$
0.15
0.10
0.05
0.00
0.4
0.2
0.0
0.6
0
1
2

FLR models — ITG dispersion relation



analysis of quasi-linear flux [1]

- major Physics change from GENE-X **current** to **target**
- **Padé on Γ_0** more Physically sensible
- **Padé on J_0** not far-fetched and provides very convenient algorithm

$$\left(1 - \frac{1}{4}\,\rho_\alpha^2 \nabla_\perp^2\right) \langle \phi(\boldsymbol{r}) \rangle_\theta = \phi(\boldsymbol{R})$$

differential eq. for gyro-average

[1] A. Mariani *et al.*, Physics of Plasmas **25**, 012313 (2018).

MAX-PLANCK-INSTITUT
FÜR PLASMAPHYSIK

Study of finite Larmor radius (FLR) physics in tokamaks and stellarators with the gyrokinetic turbulence code GENE-X

José Capitán, Philipp Ulbl, Baptiste J. Frei, Frank Jenko

GENE-X

HEPP Introductory Talk

EUROfusion

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