MAX-PLANCK-INSTITUT
FÜR PLASMAPHYSIK

Turbulence in Molecular Clouds

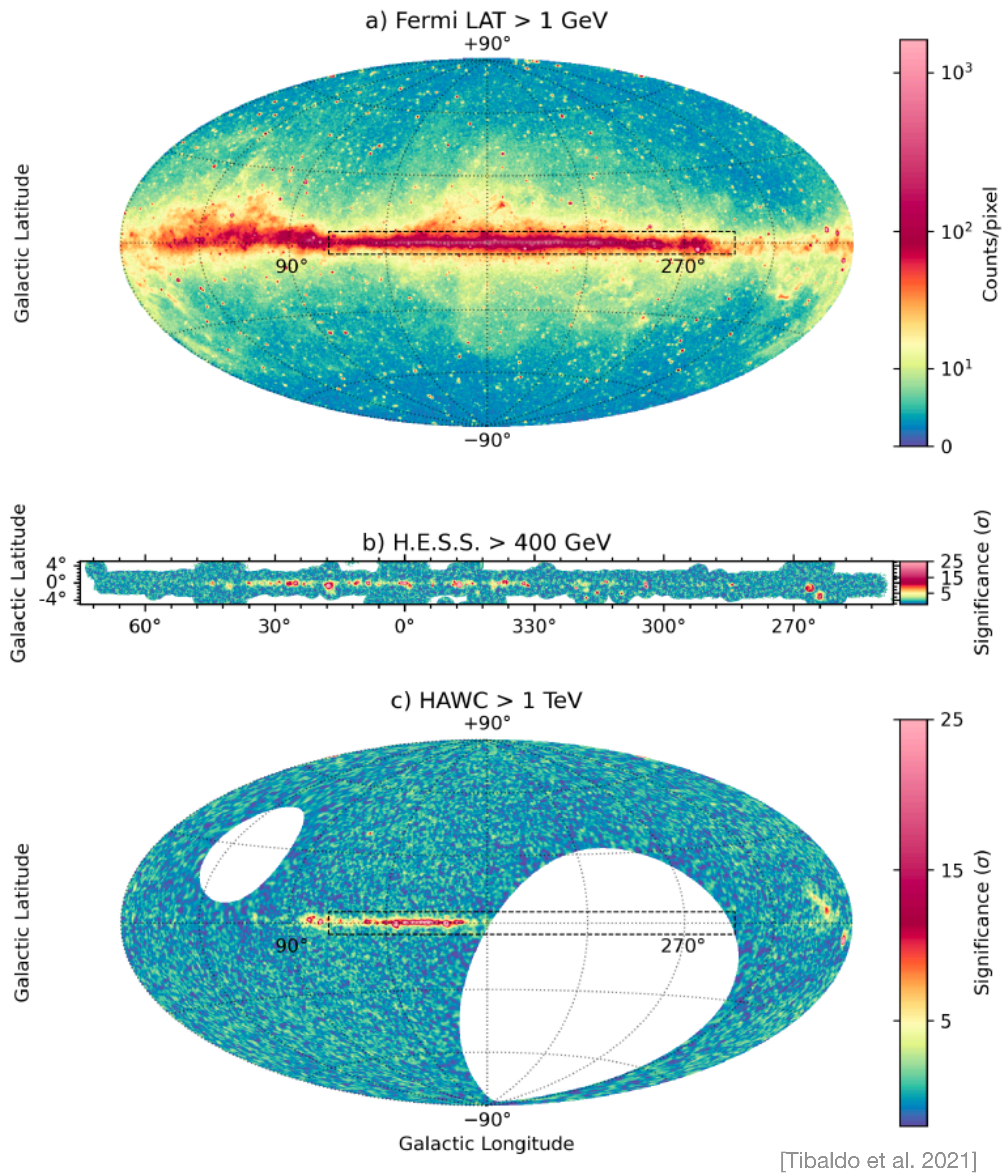
PLASMA
ASTRO

Pillars of Creation, JWST, https://stsci-opo.org/STScI-01GK2KMYS6HADS6ND8NRHG53RP.png

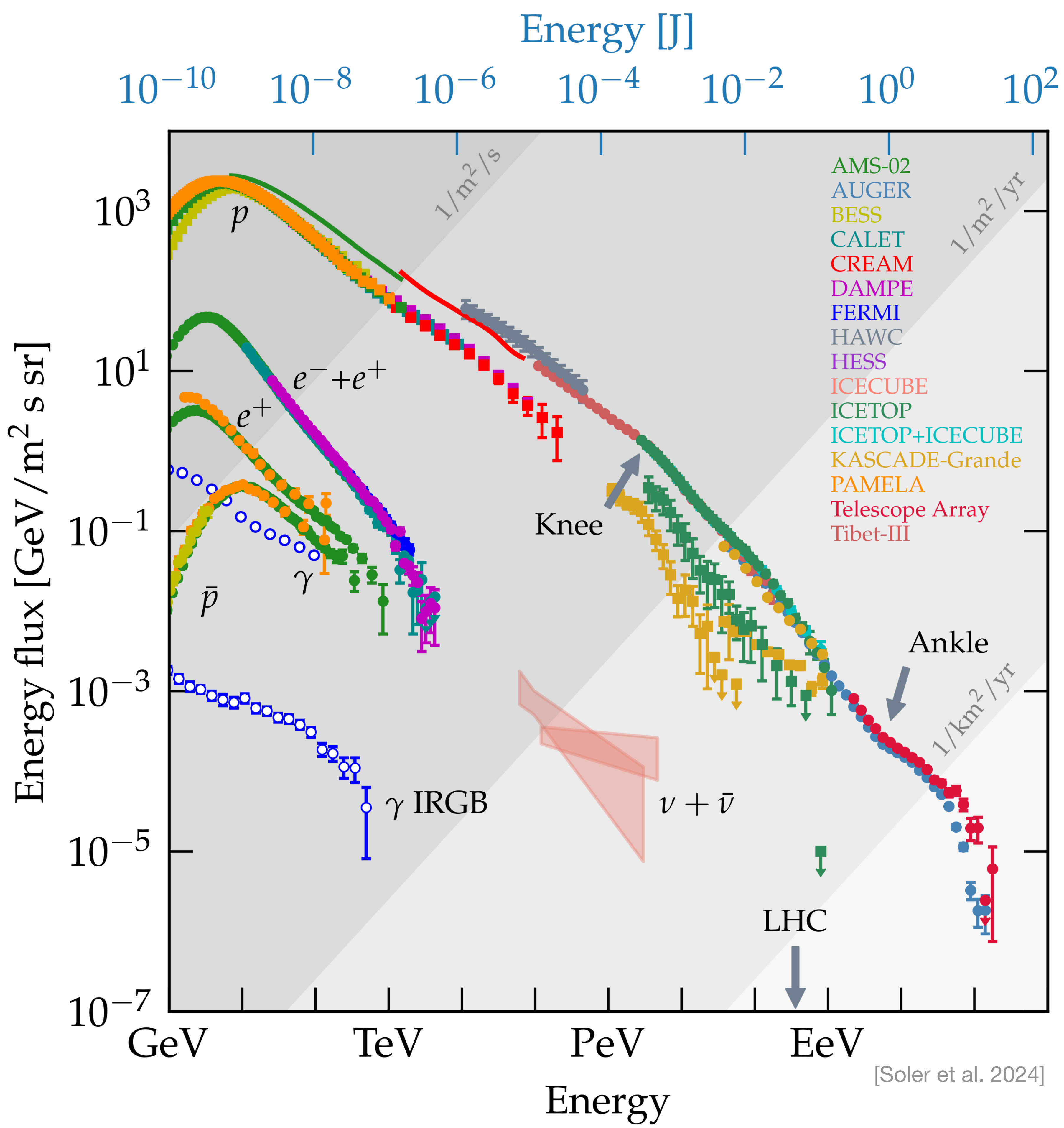
Christian Heppe

WHAT ARE COSMIC RAYS (CR)

„A dilute, non-thermal, high pressure relativistic gas“



[Tibaldo et al. 2021]



[Soler et al. 2024]

WHAT ARE COSMIC RAYS (CR)

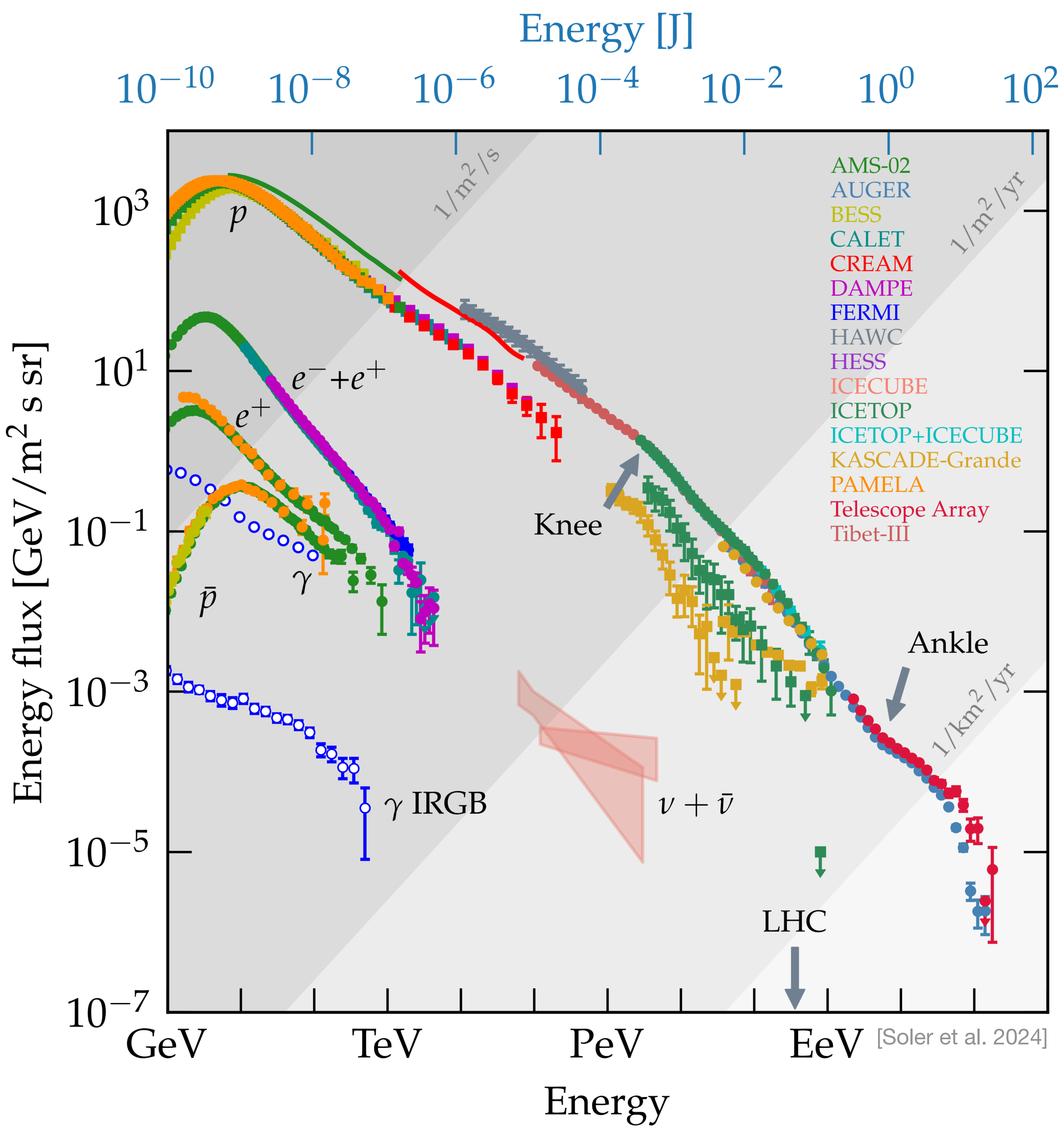
- Power law spectrum (GeV – ZeV)

$$dN(E) \propto E^{\alpha} dE$$

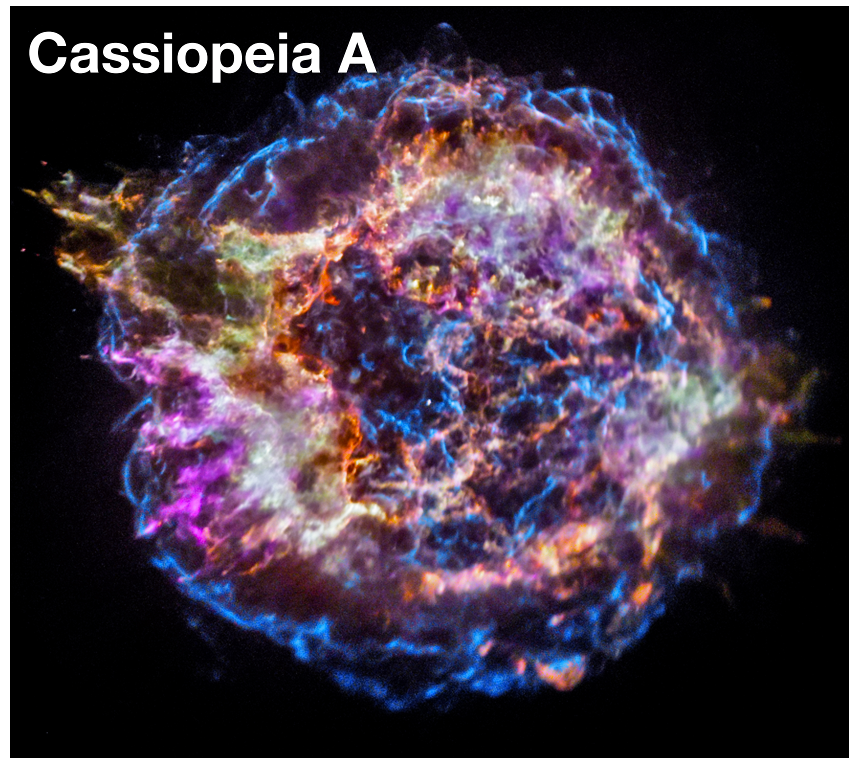
- 2nd-Order Fermi-Type Acceleration in shock environments

$$\langle \frac{\Delta E}{E} \rangle = \frac{4}{3}\gamma^2\beta^2 \simeq \frac{4}{3}\beta^2, \beta = V/c$$

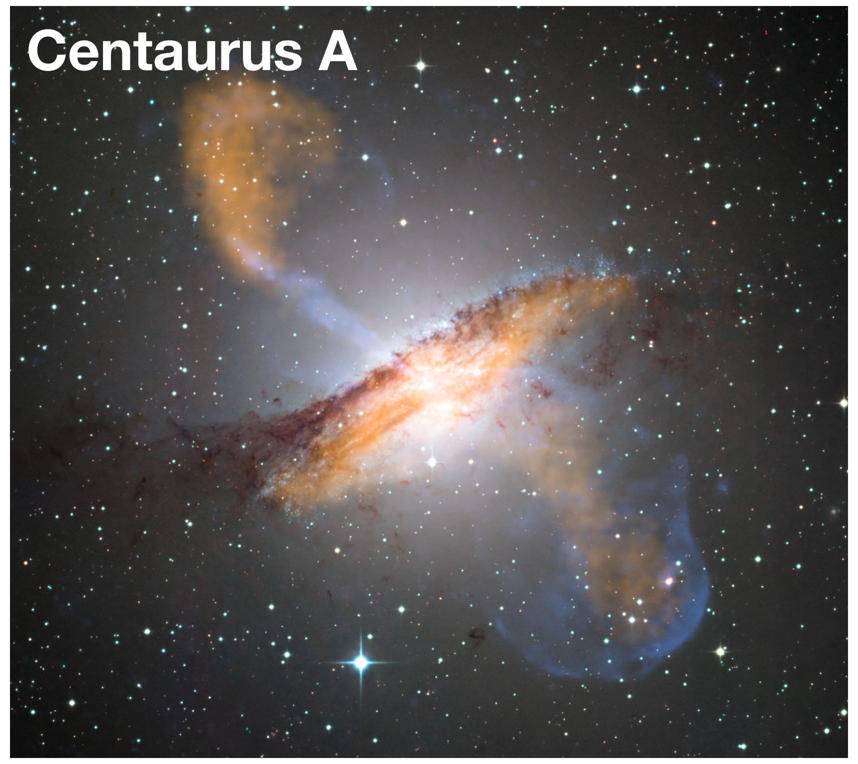
$$\langle \frac{dE}{dt} \rangle = \frac{E}{t_{acc}}, t_{acc} \propto \tau_s \simeq \lambda_{mfp}/c$$



[Soler et al. 2024]

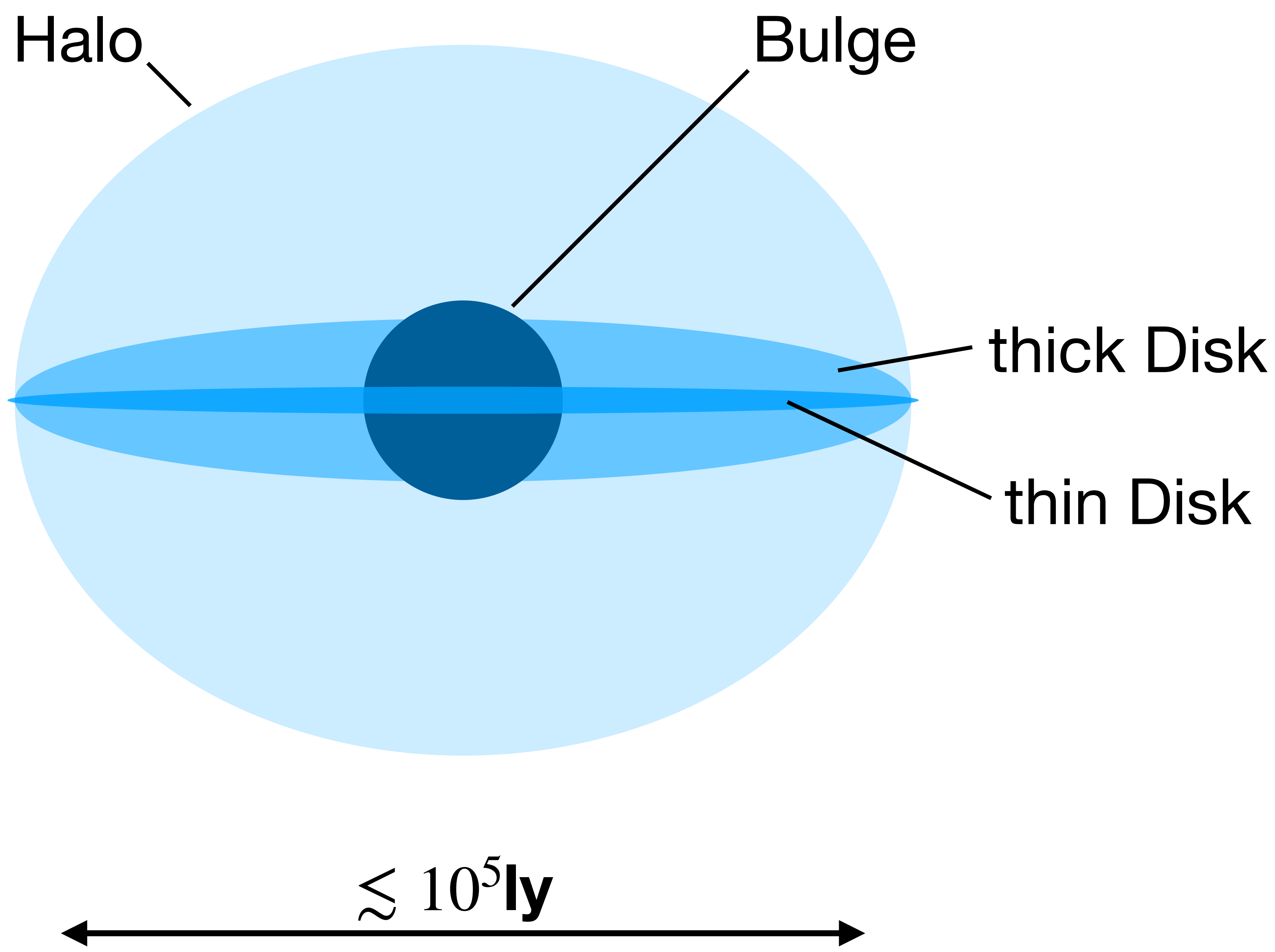


[Chandra NASA/CXC/SAO]



[ESO]

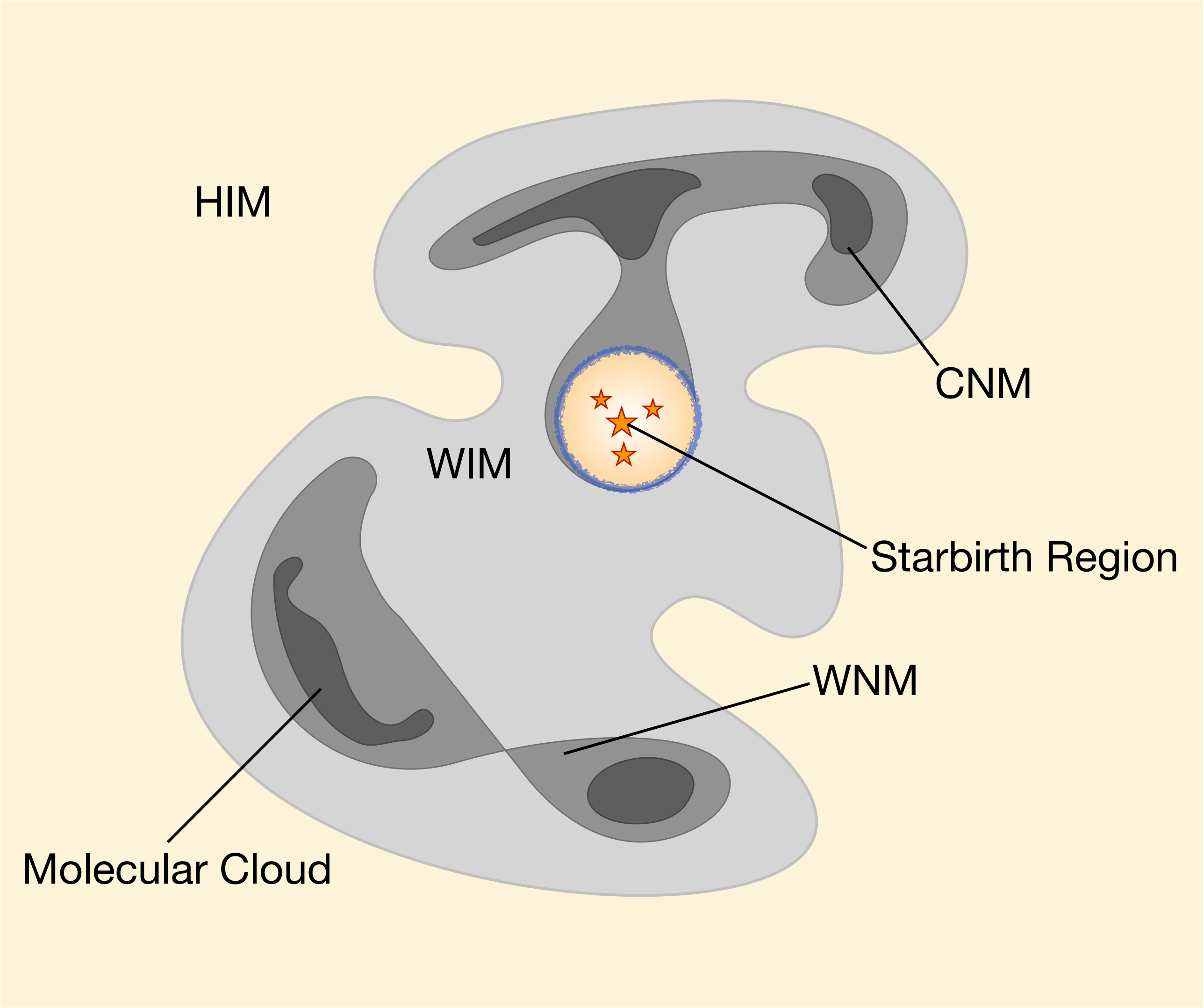
WHAT IS THE INTERSTELLAR MEDIUM (ISM)?

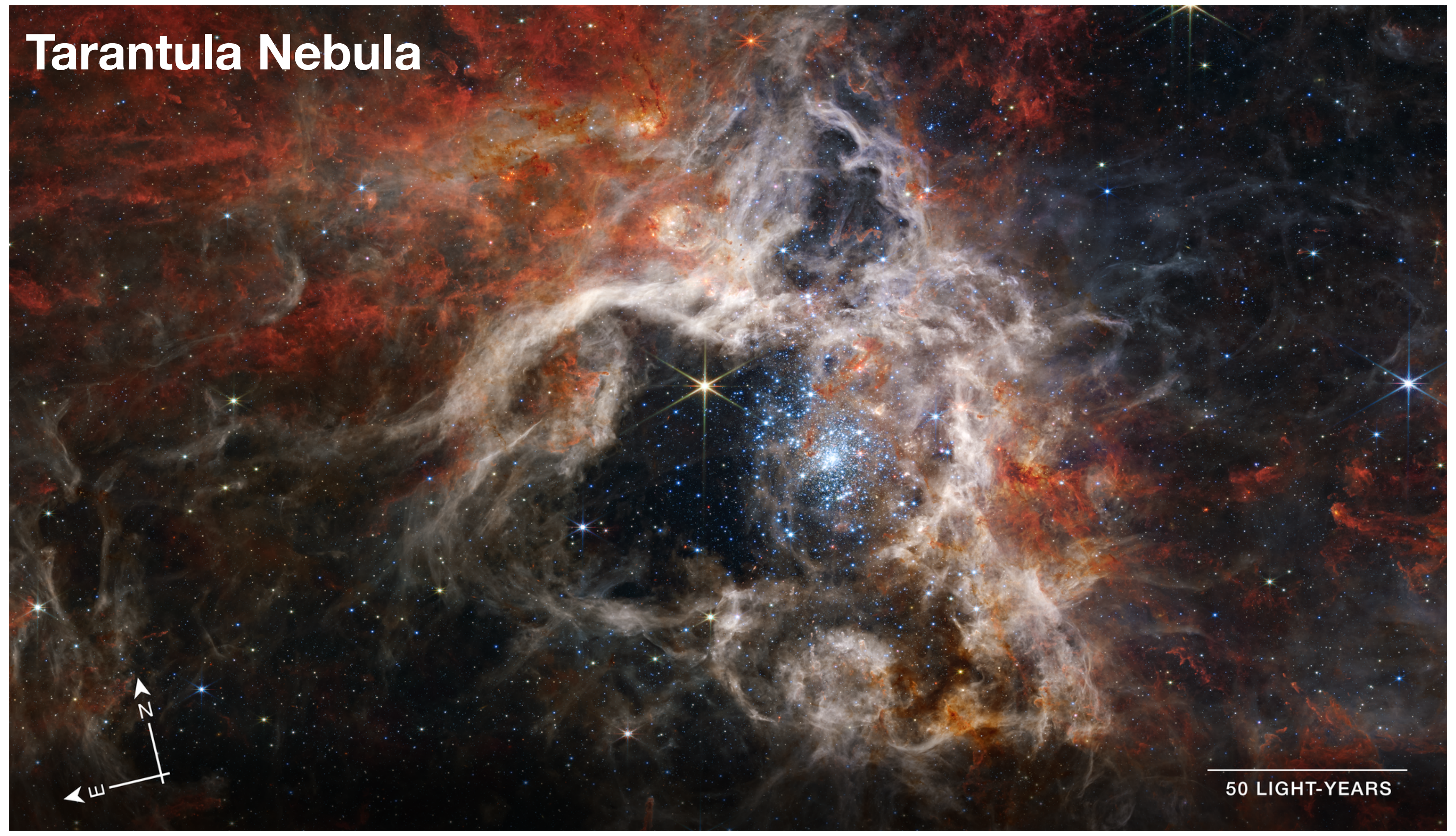


[Cartwheel Galaxy, Hubble]

WHAT IS THE ISM?

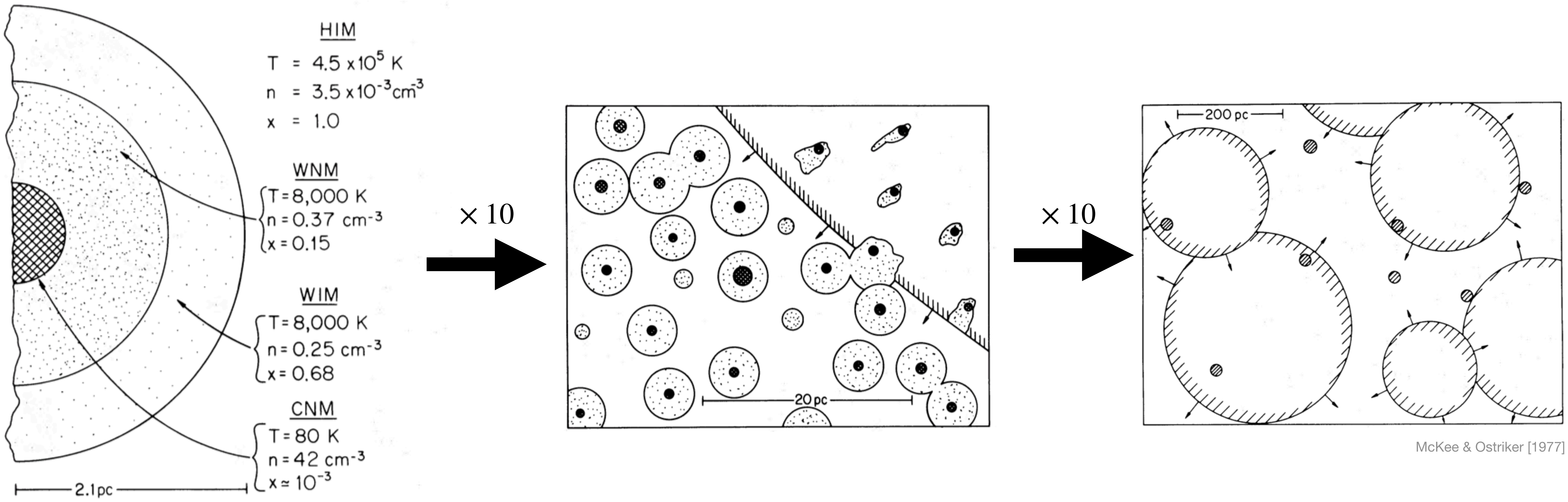
- **Hot Ionized Medium (HIM)**
  - Vol.: $30 - 60\,\%$
  - $T \gtrsim 10^{5.5}$K (Shock heated, adiab./X-ray cooling)
  - $\rho \sim 10^{-3}$ cm^{-3}
  - $\chi \sim 1$ (coll. Ionization)
- **Warm Ionized Medium (WIM, „HII“)**
  - Vol.: $\sim 0.1\,\%$
  - $T \sim 10^4$K (Photoelectron-heating, opt. & MIR line-emission cooling)
  - $\rho \sim 10^{-1}$ cm^{-3}
  - $\chi \sim 0.7$ (Photo-Ionized by UV)
- **Warm Neutral Medium (WNM, „warm HI“)**
  - Vol.: $\sim 40\,\%$
  - $T \sim 5000$K (Dust photoel.—heating, FIR line-emission cooling)
  - $\rho \sim 0.5$ cm^{-3}
  - $\chi \sim 10^{-1}$ (CRs & Starlight)
- **Cold Neutral Medium (CNM, „cold HI“ & „H_2-gas“)**
  - Vol.: $\sim 1\,\%$
  - $T \sim 10 - 100$K (Dust photoel.- & CR-heating, FIR line-emission)
  - $\rho \sim 30 - 10^3$ cm^{-3}
  - $\chi \lesssim 10^{-3}$ (CRs)





[JWST]

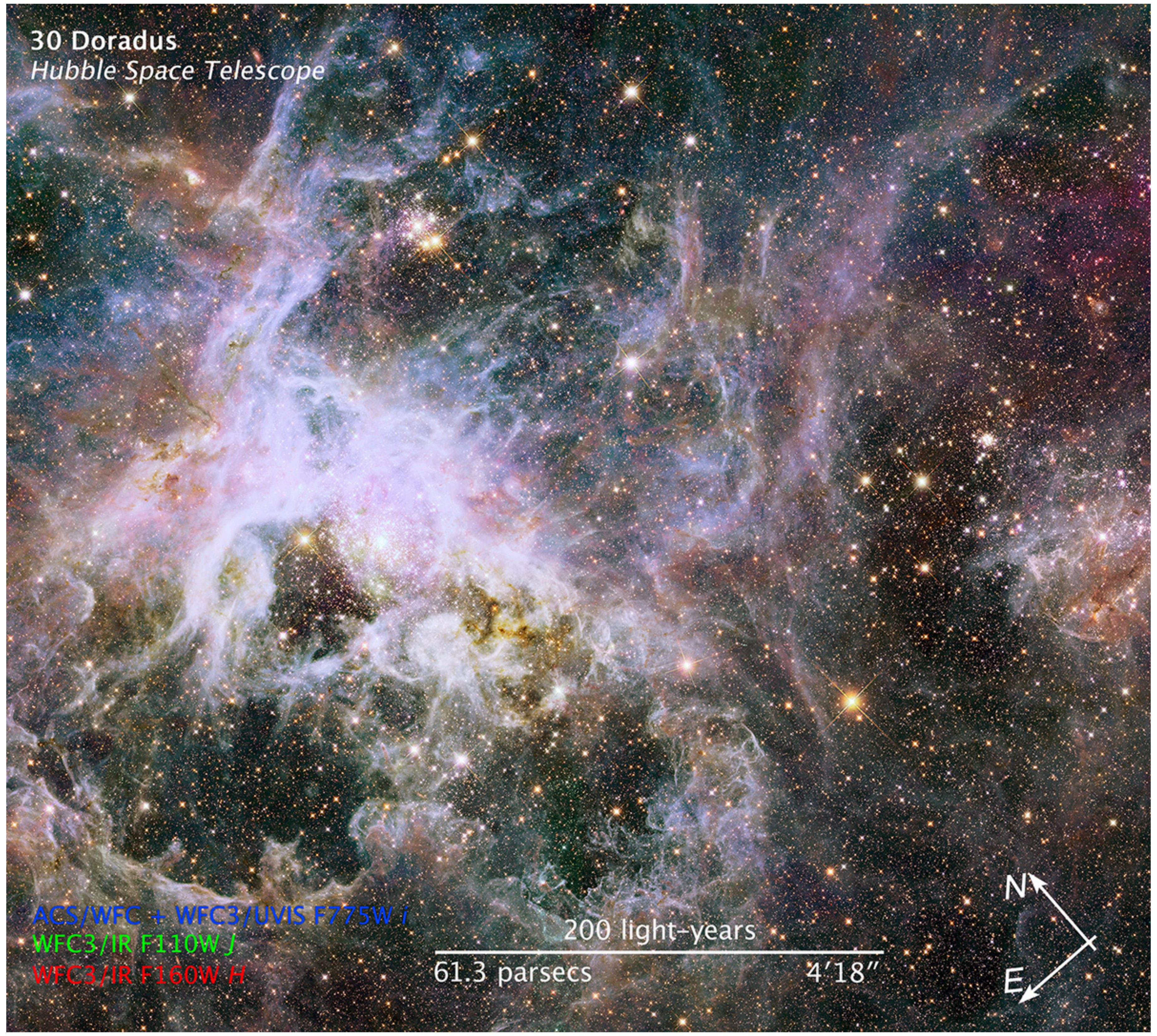
STRUCTURAL HIERARCHY OF THE ISM



McKee & Ostriker [1977]

WHAT IS THE ISM?

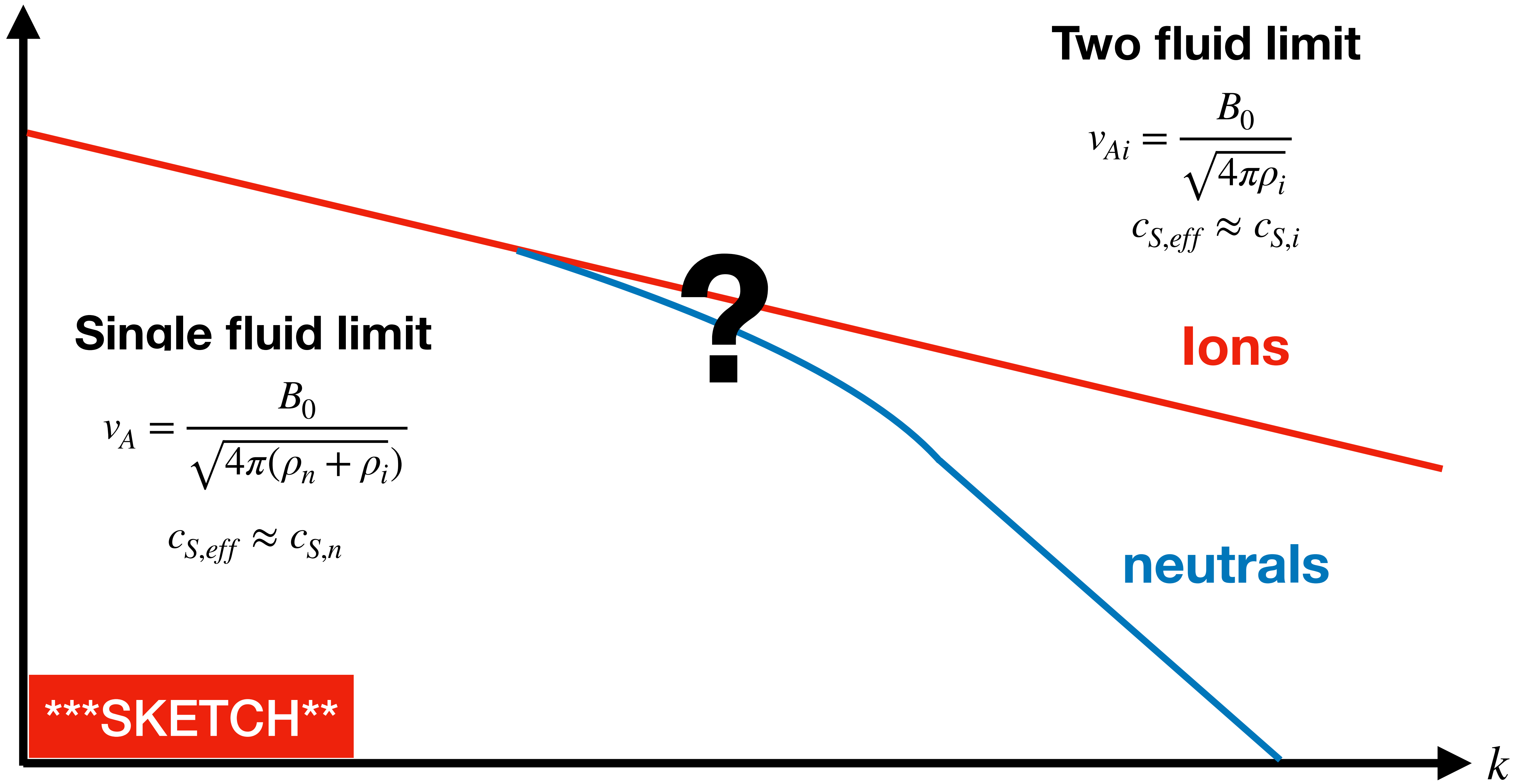
- Energy Budget:
  - i) $w_{turb} \approx 0.2$ ev/cm^{-3}
  - ii) $w_{CMB} \approx 0.265$ ev/cm^{-3}
  - iii) $w_{Dust} \approx 0.31$ ev/cm^{-3}
  - iv) $w_{Starlight} \approx 0.5$ ev/cm^{-3} (< 13.6 eV)
  - v) $w_{therm} \approx 0.5$ ev/cm^{-3} ($nT = 3800$ cm^{-3}K)
  - vi) $w_{mag} \approx 0.9$ ev/cm^{-3} ($B_{tot} = 6\mu$G)
  - vii) $w_{CR} \approx 1$ ev/cm^{-3}
- Large variety of conditions (4 Major Phases)
- Hierarchy of Scales & Structures

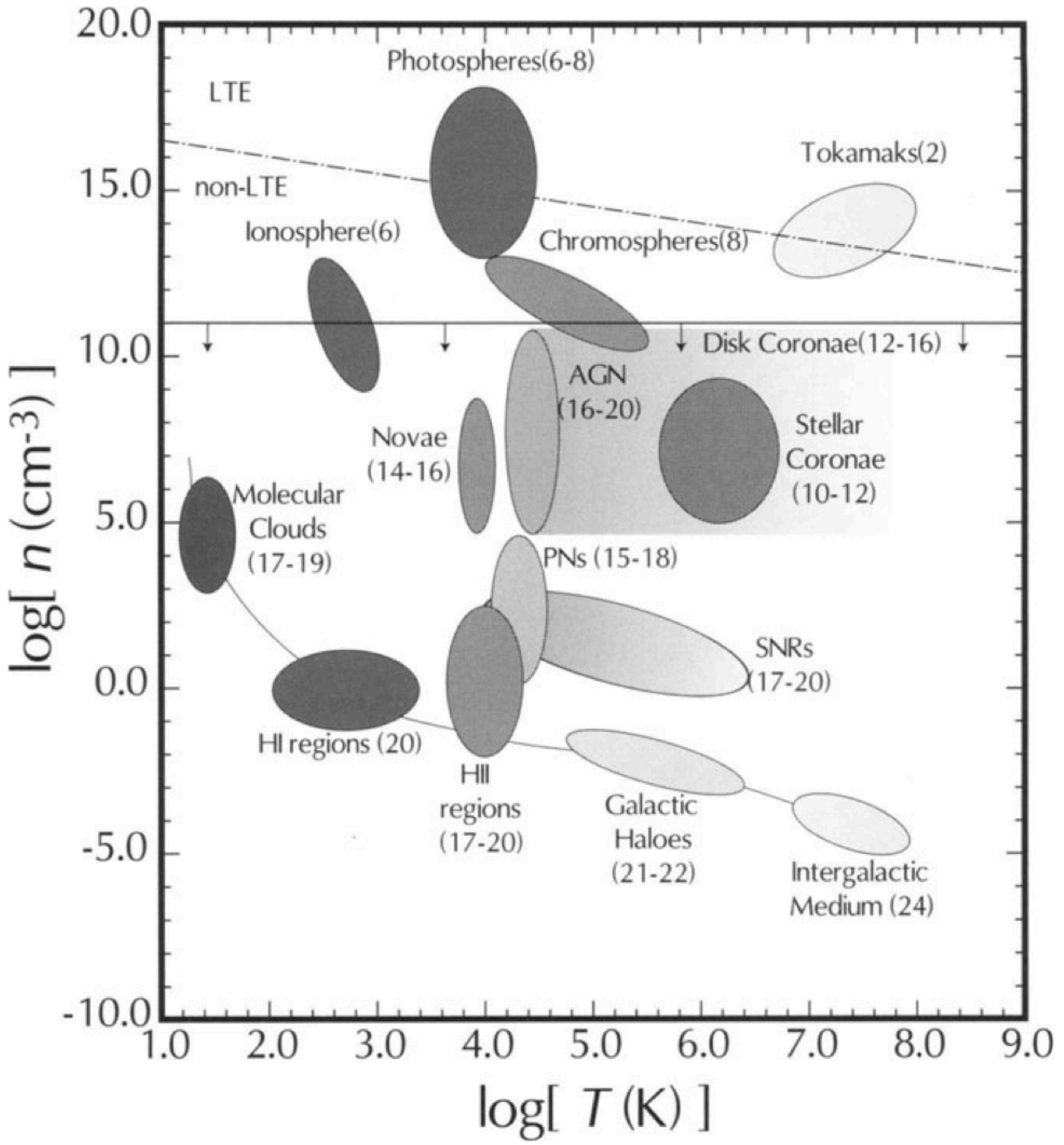


[Hubble]

TURBULENCE IN MOLECULAR CLOUDS

- Structure dictated by turbulence
- Turbulence in partially ionized media?
- Only now numerically feasible





Dopita & Sutherland [2005]

2FMHD EQUATIONS

Compressible 2FMHD EQs:

(1) $\frac{\partial \rho_i}{\partial t} + \nabla \cdot \left(\rho_i \mathbf{v}_i\right)$

(2) $\frac{\partial \rho_n}{\partial t} + \nabla \cdot \left(\rho_n \mathbf{v}_n\right)$

(3) $\frac{\partial \rho_i \mathbf{v}_i}{\partial t} + \nabla \left[\rho_i \mathbf{v}_i \mathbf{v}_i^T + \left(c_{S,i}^2 \rho_i + \frac{B^2}{8\pi}\right)\mathbf{I} - \frac{\mathbf{B}\mathbf{B}^T}{4\pi}\right] = \gamma_D \rho_i \rho_n \left(\mathbf{v}_n - \mathbf{v}_i\right) + f_i$

(4) $\frac{\partial \rho_n \mathbf{v}_n}{\partial t} + \nabla \left[\rho_i \mathbf{v}_n \mathbf{v}_n^T + c_{S,n}^2 \rho_n \mathbf{I}\right] = \gamma_D \rho_n \rho_i \left(\mathbf{v}_i - \mathbf{v}_n\right) + f_n$

(5) $\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v}_i \times \mathbf{B})$

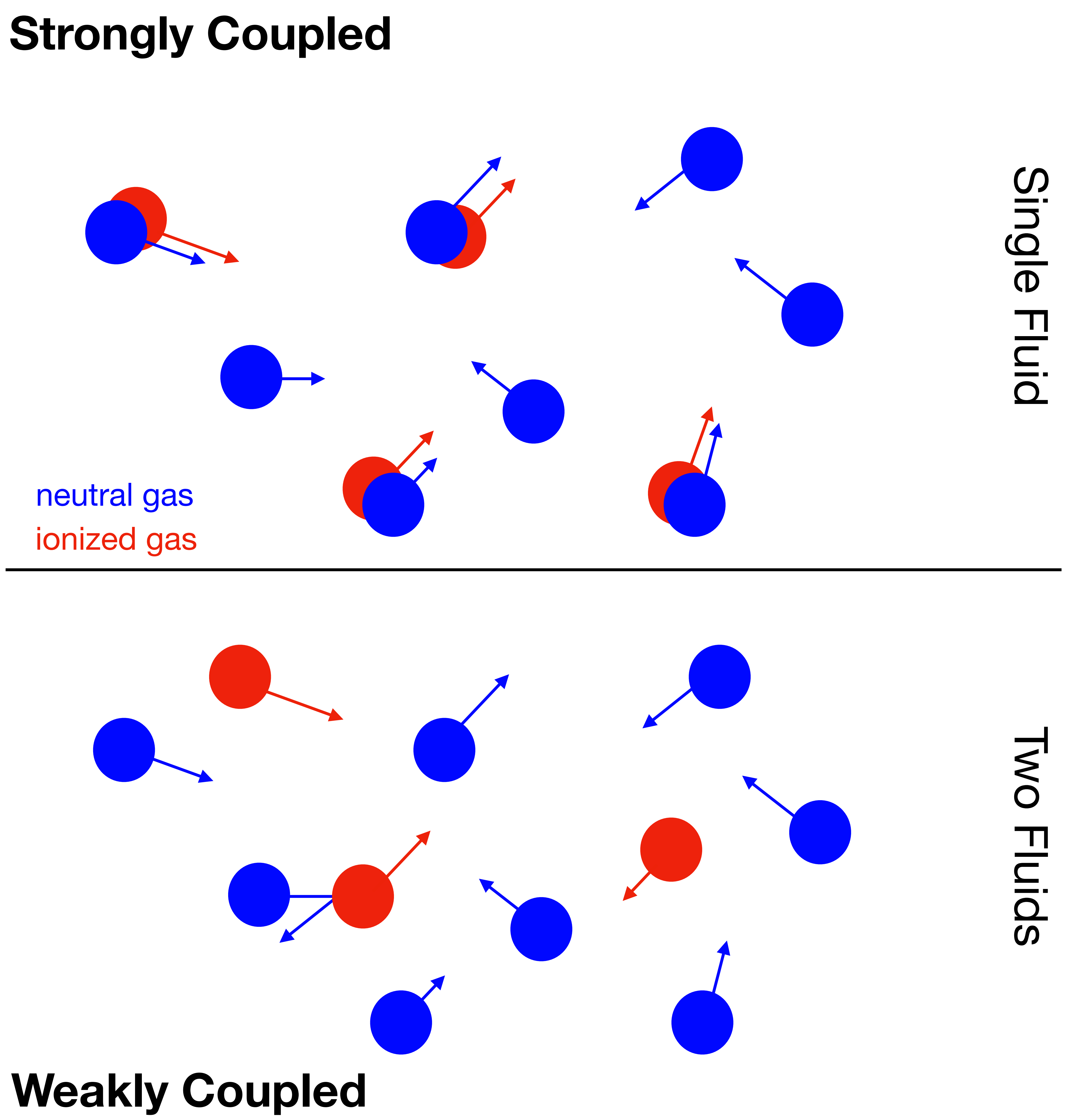
(6) $\nabla \cdot \mathbf{B} = 0$

With the current: $\mathbf{I} = -e\rho_i \mathbf{v}_i$

Collisional coupling:

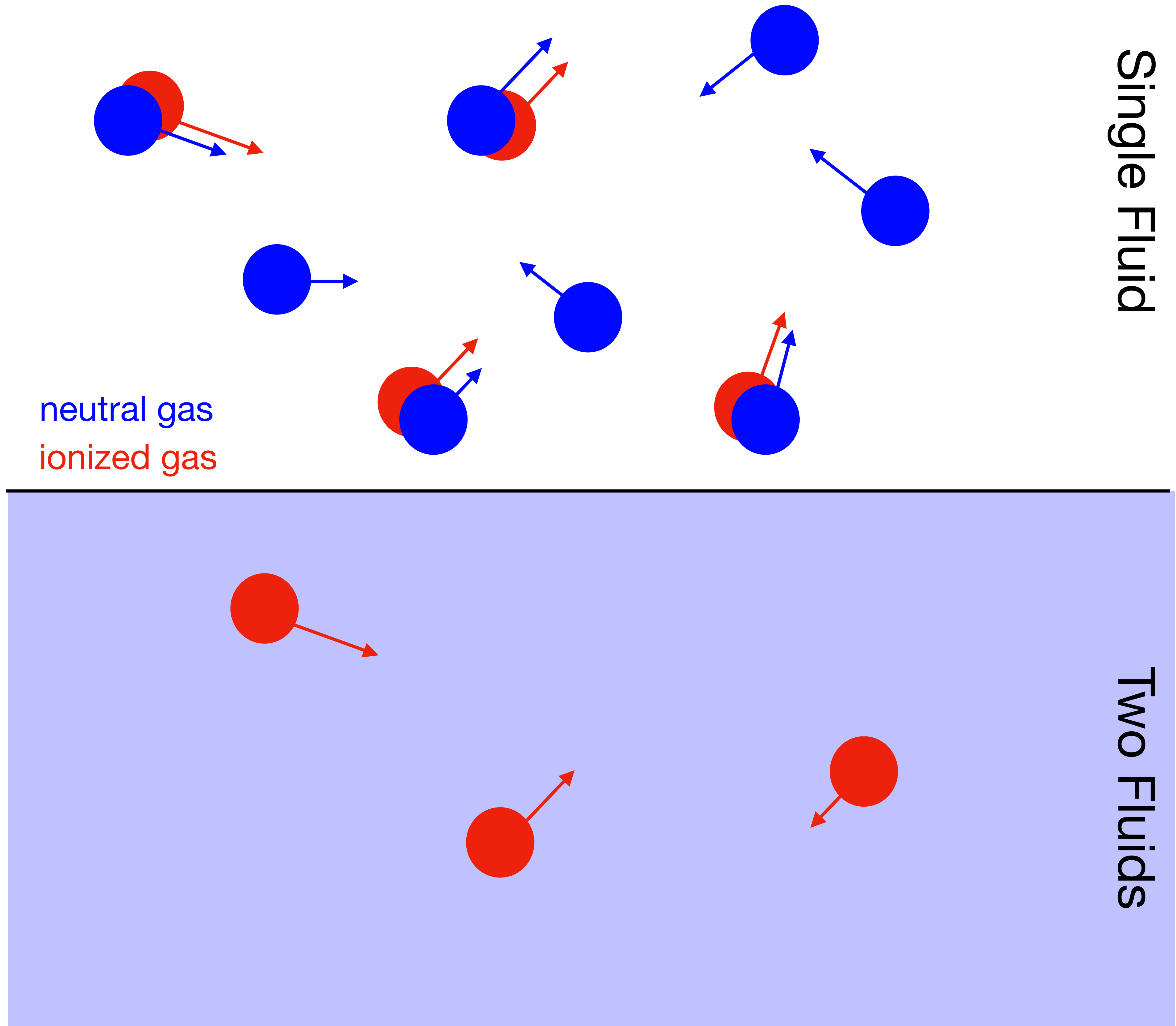
- Drag coefficient: $\gamma_D = \frac{1}{2m_n}\sqrt{\frac{16k_B T}{\pi m_i}}\sigma_{in}$
- Ion-neutral collisions: $\nu_{in} = \gamma_D \rho_n$
- Neutral-ion collisions: $\nu_{ni} = \gamma_D \rho_i$
- $\chi = \rho_n/\rho_i \implies \nu_{in} = \chi \nu_{ni}$

2FMHD - COUPLING LIMITS



2FMHD - SCALE LIMITS

Large Scales ($k \ll c_{ph}\nu_{coll}^{-1}$ & $\omega \ll \nu_{coll}$)



Small Scales ($k \gg c_{ph}\nu_{coll}^{-1}$ & $\omega \gg \nu_{coll}$)

LINEAR WAVES

Linearized compressible 2FMHD EQs:

(1) $\rho_i \frac{\partial \mathbf{v}_i}{\partial t} = -\nabla c_{S,i}^2 \rho_i + \frac{1}{\mu}(\nabla \times \mathbf{b}) \times \mathbf{B} - \gamma_D \rho_i \rho_n \left(\mathbf{v}_i - \mathbf{v}_n\right)$

(2) $\rho_n \frac{\partial \mathbf{v}_n}{\partial t} = -\nabla c_{S,n}^2 \rho_n - \gamma_D \rho_i \rho_n \left(\mathbf{v}_n - \mathbf{v}_i\right)$

(3) $\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{v}_i \times \mathbf{B})$

(4) $\frac{\partial p_i}{\partial t} = -c_{S,i}^2 \rho_i \nabla \cdot \mathbf{v}_i$

(5) $\frac{\partial p_n}{\partial t} = -c_{S,n}^2 \rho_n \nabla \cdot \mathbf{v}_n$

(6) $\nabla \cdot \mathbf{B} = 0$

Collisional coupling:

- Drag coefficient: $\gamma_D = \frac{1}{2m_n}\sqrt{\frac{16 k_B T}{\pi m_i}}\sigma_{in}$
- Ion-neutral collisions: $\nu_{in} = \gamma_D \rho_n$
- Neutral-ion collisions: $\nu_{ni} = \gamma_D \rho_i$
- $\chi = \rho_n/\rho_i \implies \nu_{in} = \chi \nu_{ni}$

LINEAR WAVES - ALFVÉN MODE

- **Helicity perturbations:**

$$\Gamma_i = (\nabla \times \mathbf{v}_i) \cdot \mathbf{e}_z = ik_x v_{i,y} - ik_y v_{i,x}$$

$$\Gamma_n = (\nabla \times \mathbf{v}_n) \cdot \mathbf{e}_z = ik_x v_{n,y} - ik_y v_{n,x}$$

- **Rewrite 2FMHD-eq's in terms of Γ_i & Γ_n**

$$\frac{\partial^2 \Gamma_i}{\partial t^2} + \rho_n \gamma_D \frac{\partial \Gamma_i}{\partial t} + k^2 \cos^2 \theta c_{Ai}^2 \Gamma_i = \rho_n \gamma_D \frac{\partial \Gamma_n}{\partial t}$$

$$\frac{1}{\rho_i} \frac{\partial \Gamma_n}{\partial t} + \gamma_D \Gamma_n = \Gamma_i$$

- **Dispersion via normal mode analysis:**

$$\omega^3 + i(1+\chi)\nu_{ni}\omega^2 - k_z^2 c_{Ai}^2 \omega - i\nu_{ni} k_z^2 c_{Ai}^2 = 0$$

$$\iff \left(\frac{k_z c_A}{\omega}\right)^2 = \frac{\omega + i(1+\chi)\nu_{ni}}{\omega + i\nu_{ni}}$$

Alfvén velocity:

- Ion-Alfvén velocity: $c_{Ai} = \dfrac{B^2}{\sqrt{4\pi\rho_i}}$
- Loaded-Alfvén velocity: $c_{Ai} = \dfrac{B^2}{\sqrt{4\pi(\rho_i + \rho_n)}}$

Decoupling

- Decoupling approximation:

$$k_{dec}^- v_A \sim \nu_{ni} \quad \& \quad k_{dec}^+ v_{Ai} \sim \nu_{in}$$

- Exact solution:

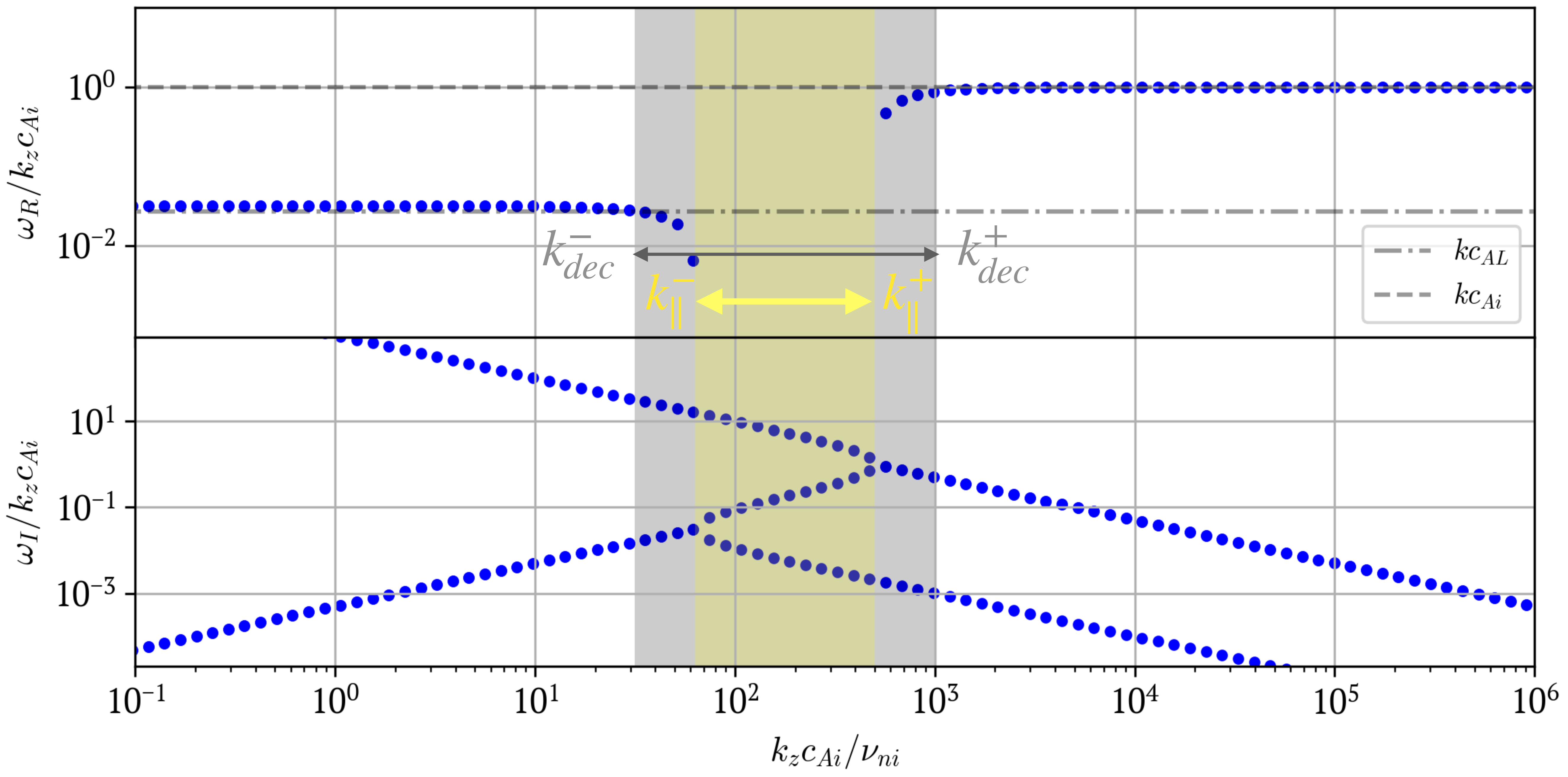
$$k_\parallel^\pm = \frac{\nu_{ni}}{c_{Ai}} \left[\frac{\chi^2 + 20\chi - 8}{8(1+\chi)^3} \pm \frac{\chi^{1/2}(\chi - 8)^{3/2}}{8(1+\chi)^3} \right]$$

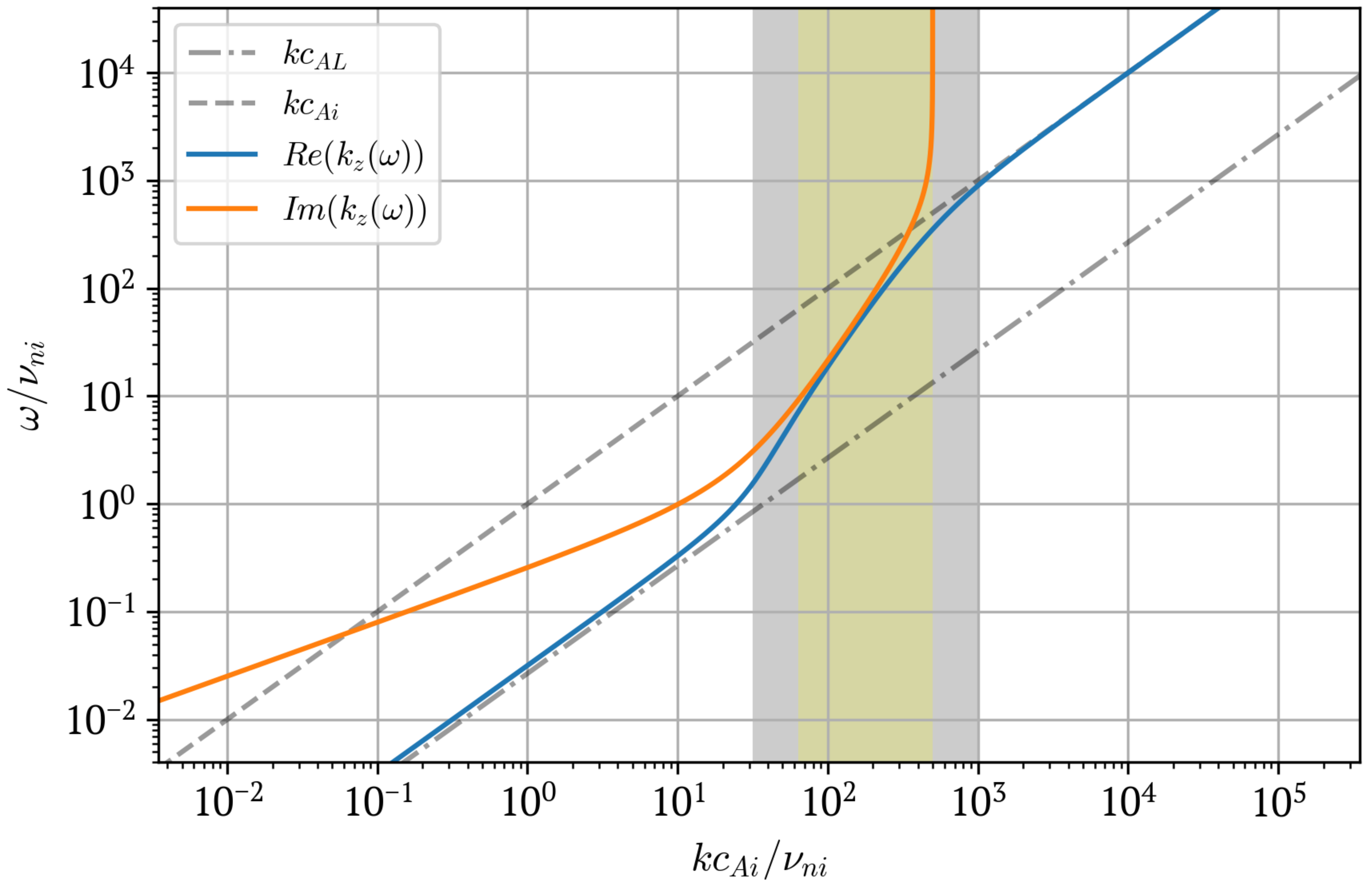
Solved for $\omega_R = 0$ with $\overrightarrow{k} = k_\parallel \hat{e}_B$

LINEAR WAVES - ALFVÉN MODE

Parameter:

- $\rho_i = 0.001,\ \rho_n = 0.999$
- $c_{Sn} = 2,\ c_{Si} = 1 \times \mathrm{km/s}$
- $\beta = 0.1$ & $\gamma_D = 25$





LINEAR WAVES - MAGNETOSONIC MODE

- **Compressibilitiy perturbations:**

$$\Delta_i = \nabla \cdot \mathbf{v}_i = ik_x v_{i,x} + ik_y v_{i,y} + ik_z v_{i,z}$$

$$\Delta_n = \nabla \cdot \mathbf{v}_n = ik_x v_{n,x} + ik_y v_{n,y} + ik_z v_{n,z}$$

- **Rewrite 2FMHD-eq's in terms of Δ_i & Δ_n & Normal mode analysis:**

$$D(\omega)\Delta_i = 0$$

$$i\nu_{ni}\omega\frac{D(\omega)}{D_n(\omega)}\Delta_n = 0$$

$$D(\omega) = D_i(\omega)D_n(\omega) + D_c^2(\omega)$$

$$D_i(\omega) = \omega^3(\omega + i\nu_{in}) - \omega^2 k^2(c_{Ai}^2 + c_{S,i}^2) + \frac{\omega + i\nu_{ni}}{\omega + i(\nu_{in} + \nu_{ni})}k^4 c_{Ai}^2 c_{S,n}^2 \cos^2\theta$$

$$D_n(\omega) = \omega(\omega + i\nu_{ni}) - k^2 c_{S,n}^2$$

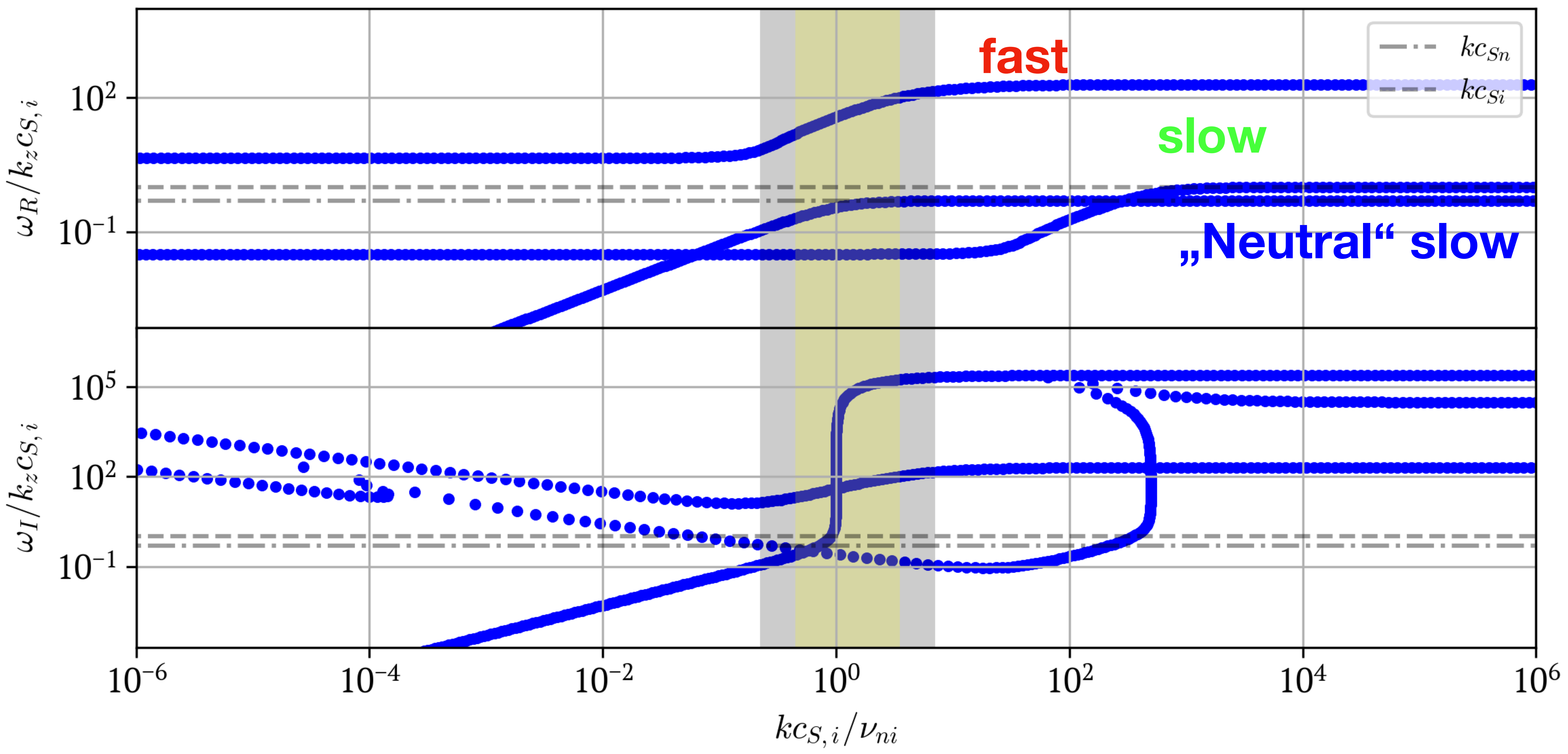
$$D_c^2(\omega) = \frac{\omega\nu_{ni}\nu_{in}}{\omega + i(\nu_{in} + \nu_{ni})}\left[\omega^3(\omega + i(\nu_{in} + \nu_{ni})) - k^4 c_{Ai}^2 c_{S,n}^2 \cos^2\theta\right]$$

Effective sound velocity:

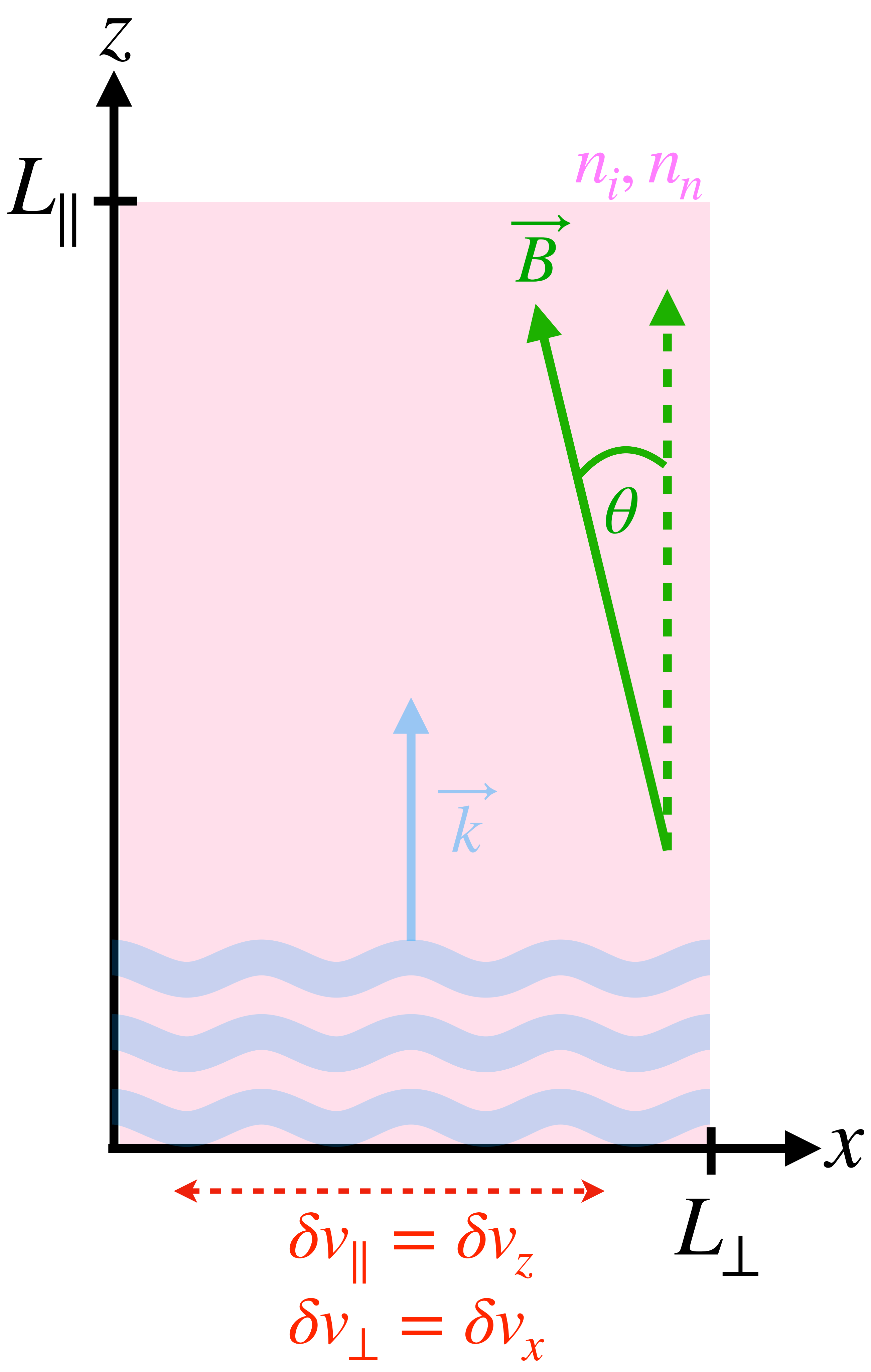
$$c_{S,eff}^2 \approx \frac{c_{S,i}^2 + \chi c_{S,n}^2}{1 + \chi}$$

LINEAR WAVES - MAGNETOSONIC MODE

Parameter:
- $\rho_i = 0.001,\ \rho_n = 0.999$
- $c_{Sn} = 2,\ c_{Si} = 1 \times \mathrm{km/s}$
- $\beta = 0.1$ & $\gamma_D = 25$



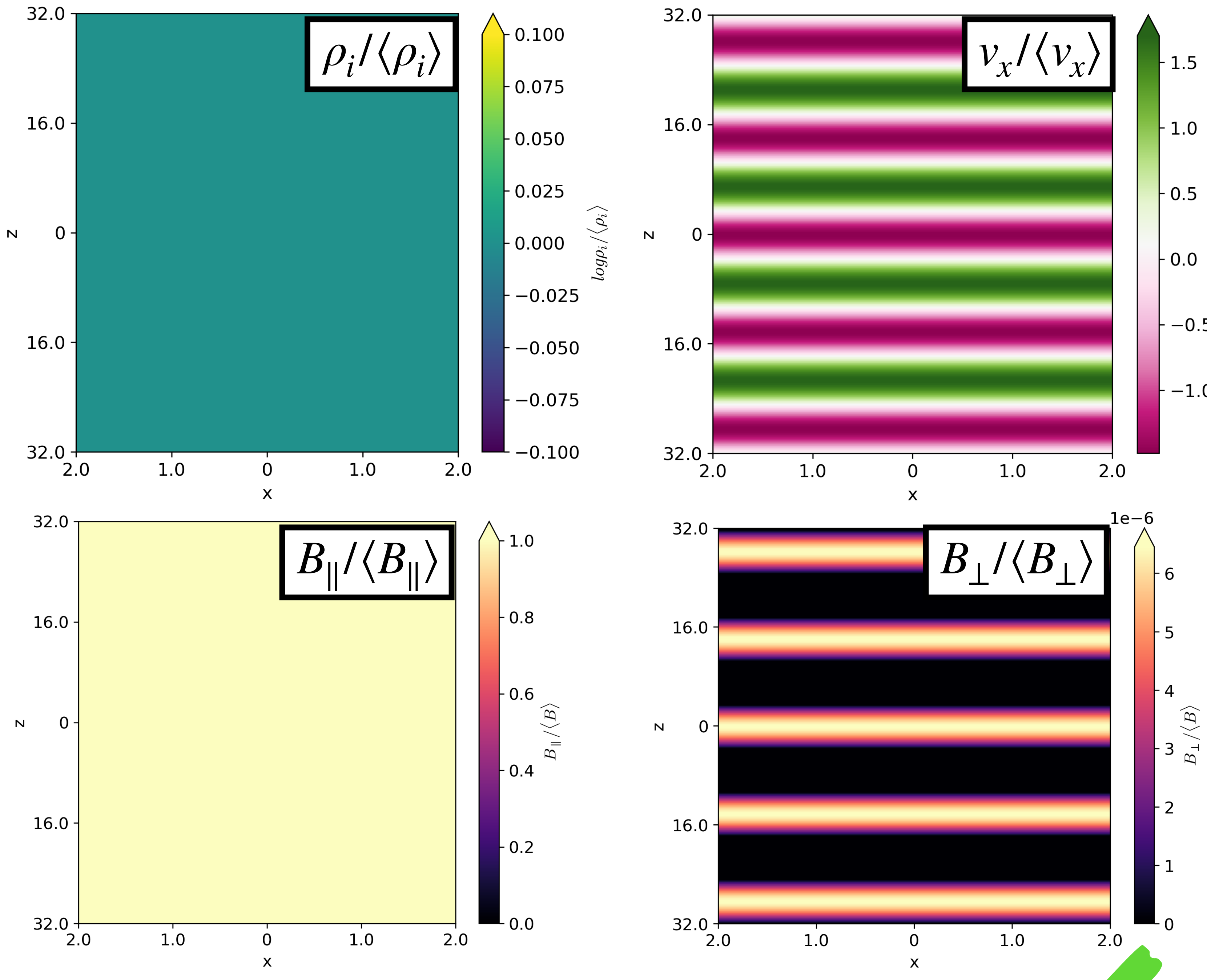
BASIC SETUP FOR LIN. PERTURBATION-SIM'S



- AthenaK
- Trying to probe „frequency response“ of the (twofluid) system
- Quasi 1D: $L_{\|} = 128 \times L_{\perp}$
  - ➡ Boundary-Conditions:
    - Transverse: Periodic
    - Longitudinal: Outflow
  - ➡ MB: 1x1x256 MBs à 32^3 px appears to be most efficient on full 4x A100-Node
- Drive any time dependent perturbation, at xy-face of box at $z = z_{min}$
  - ➡ Primarily xz-polarization

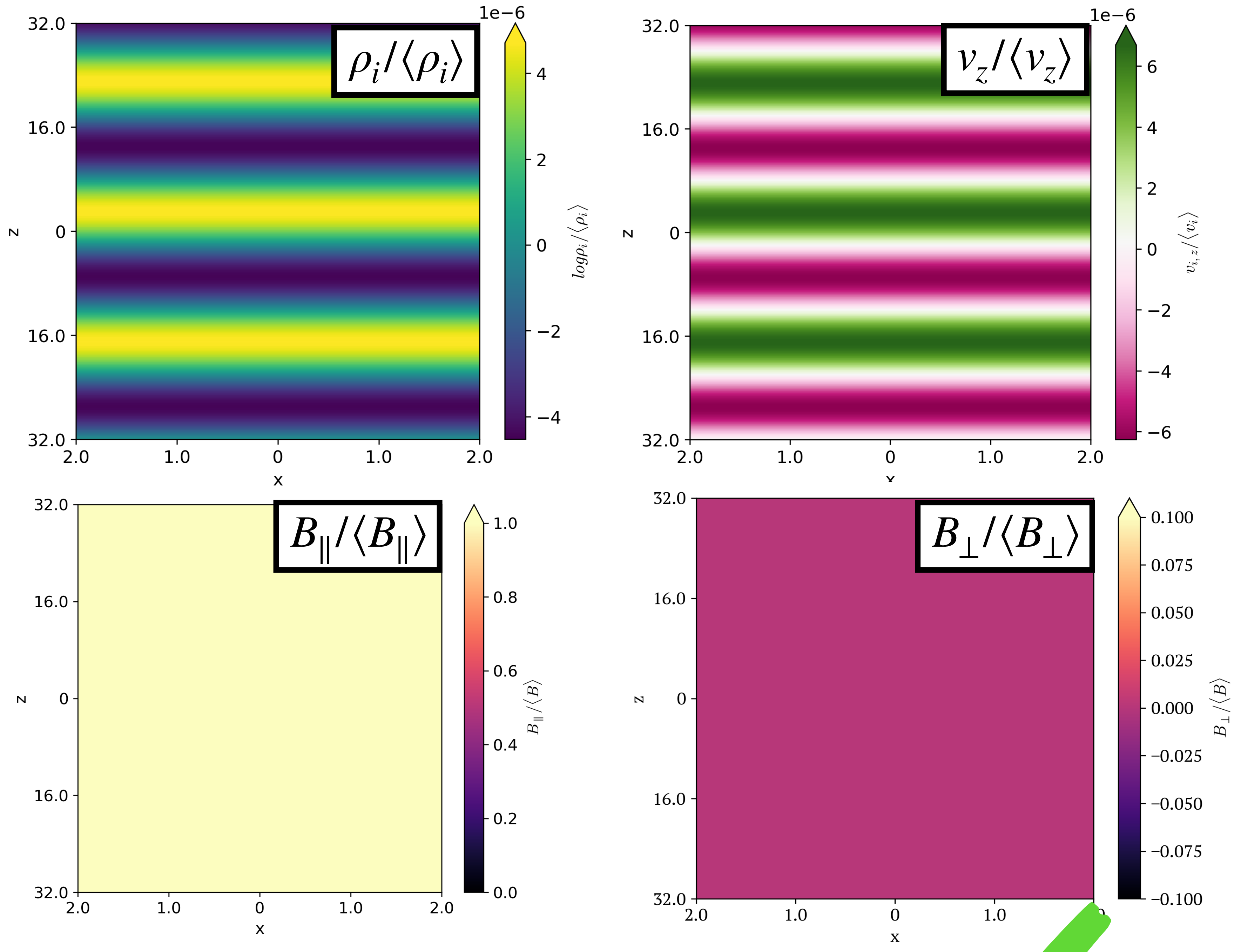
LINEAR TESTING (FULLY IONIZED/MHD)

Transverse Perturb.



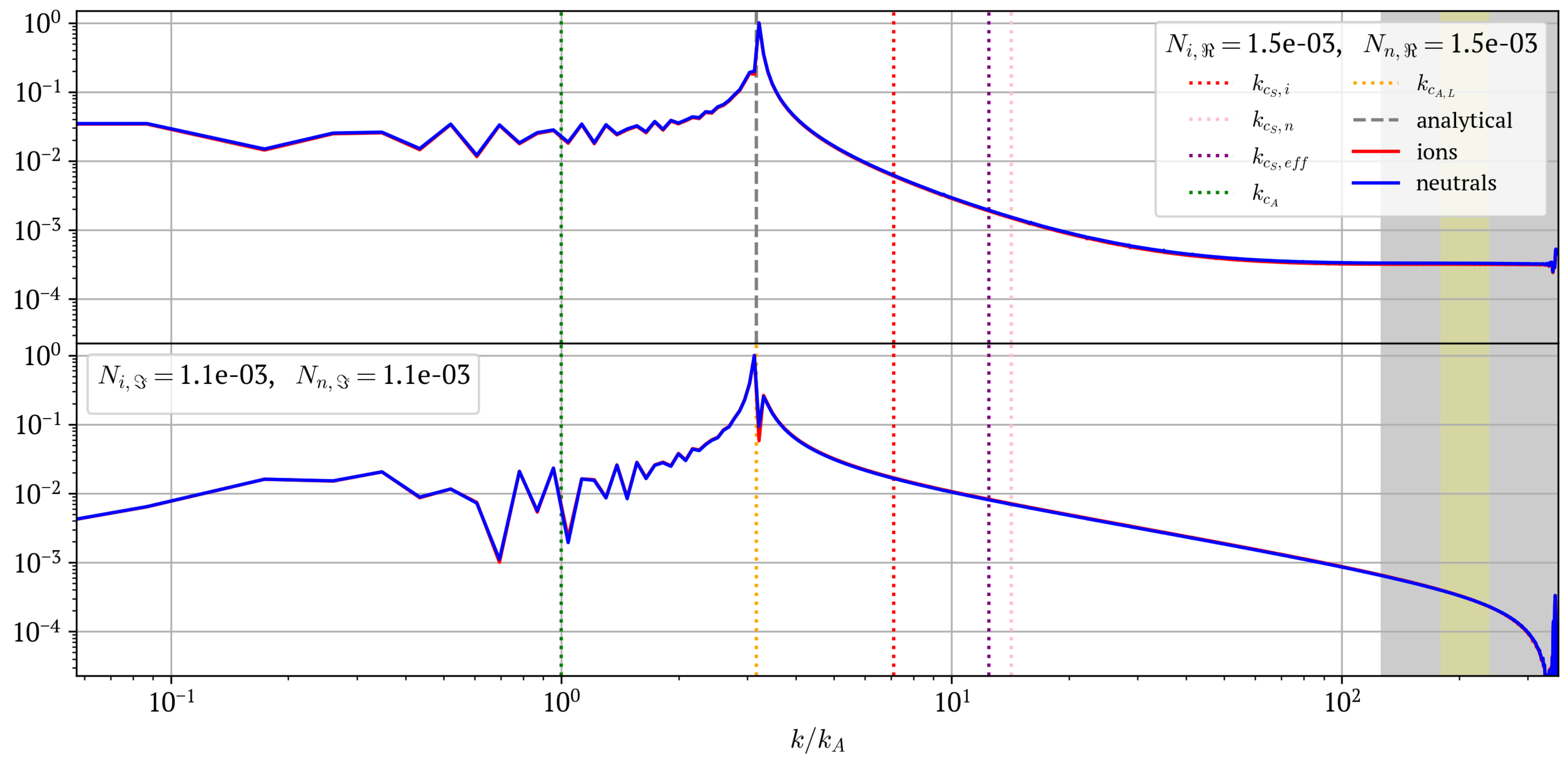
$$\delta\rho = \delta B_\parallel = 0 \,\&\, \delta B_\perp \neq 0$$

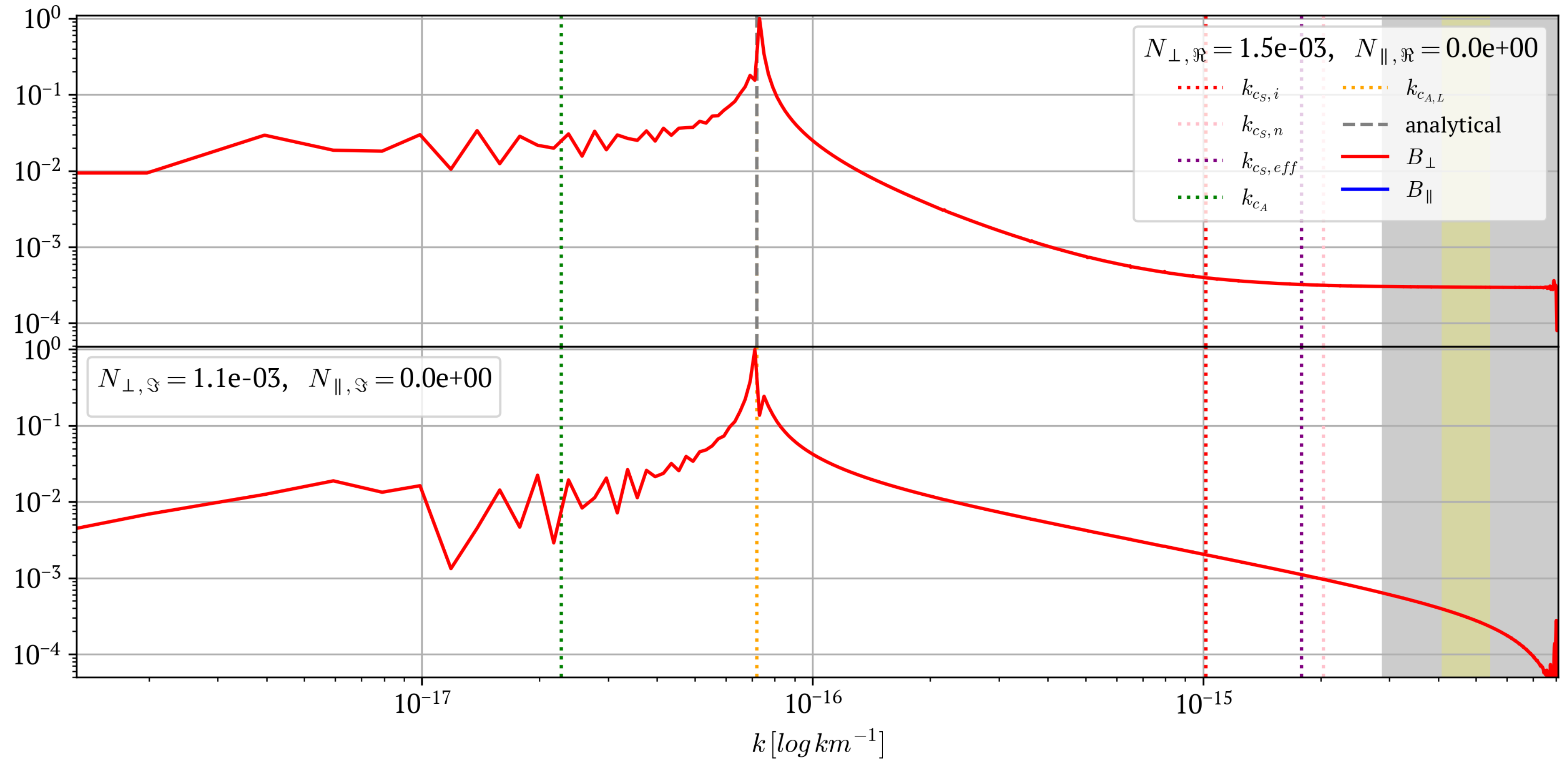
Longitudinal Perturb.

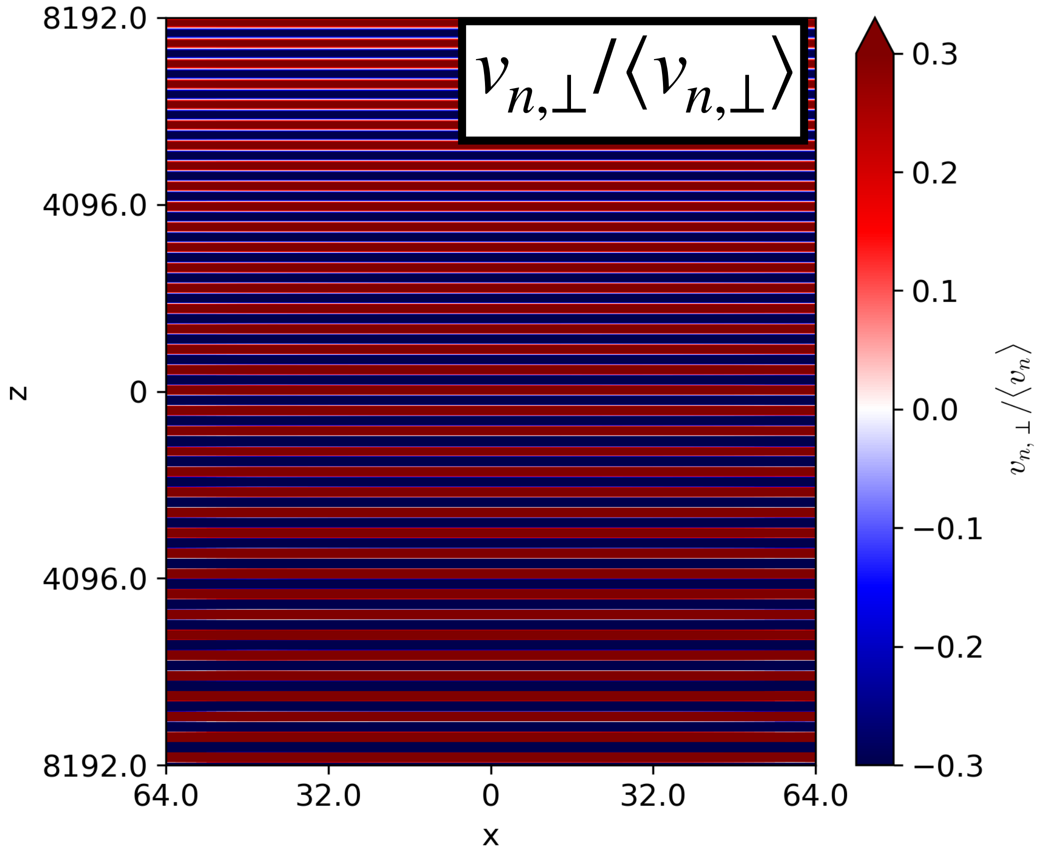


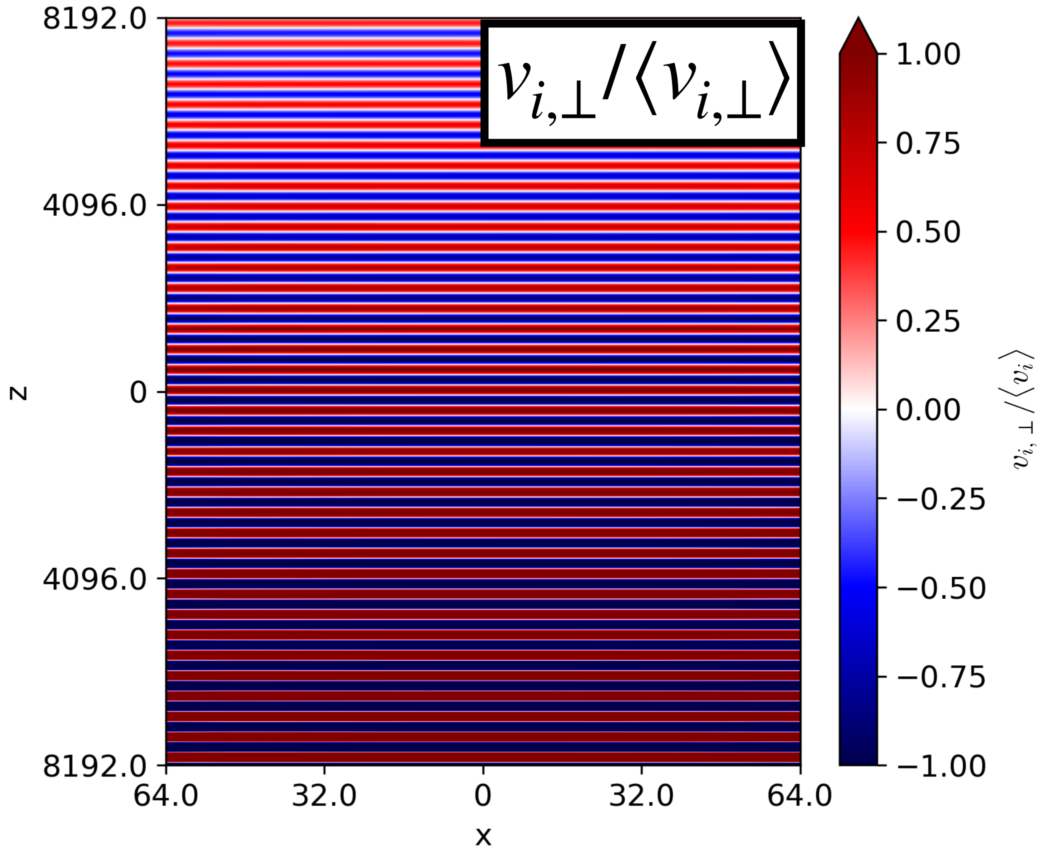
$$\delta\rho \neq 0 \,\&\, \delta B_\perp = \delta B_\parallel = 0$$

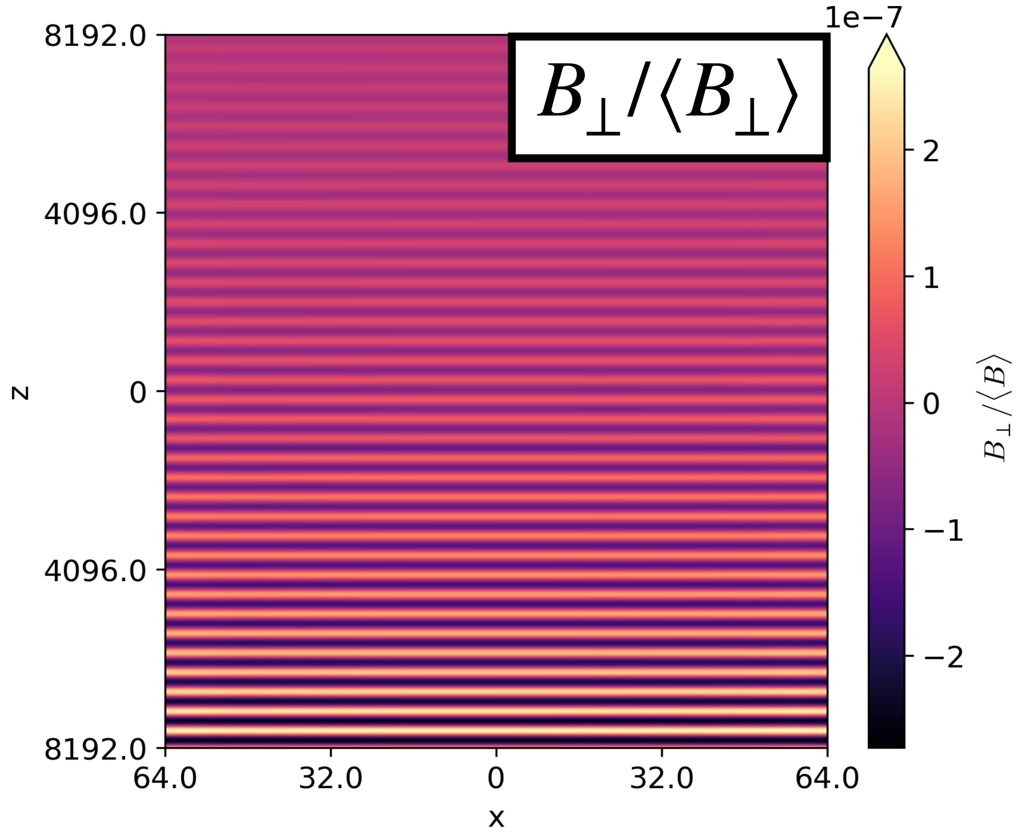
LINEAR TESTING (2FMHD)











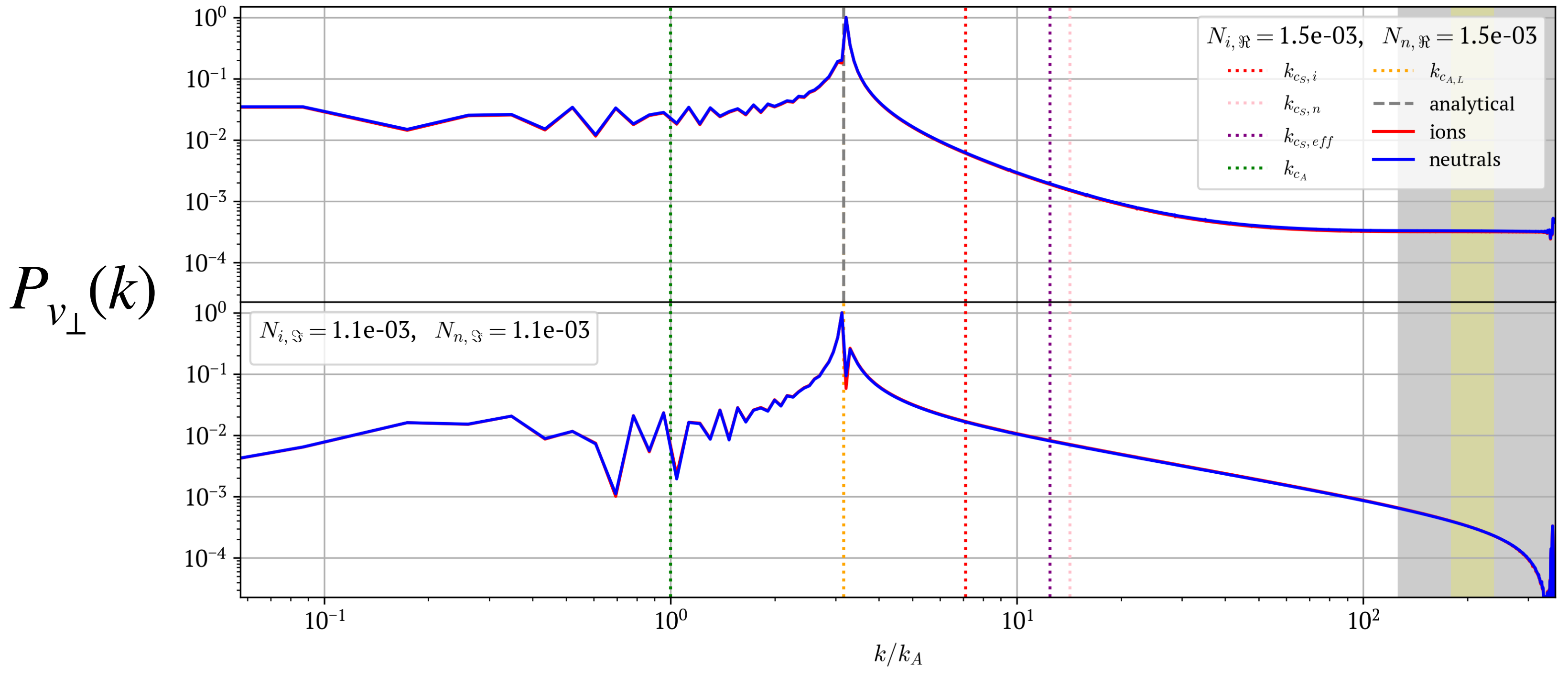
ASYMPTOTIC TESTING - TRANSVERSE $\delta v_\perp$

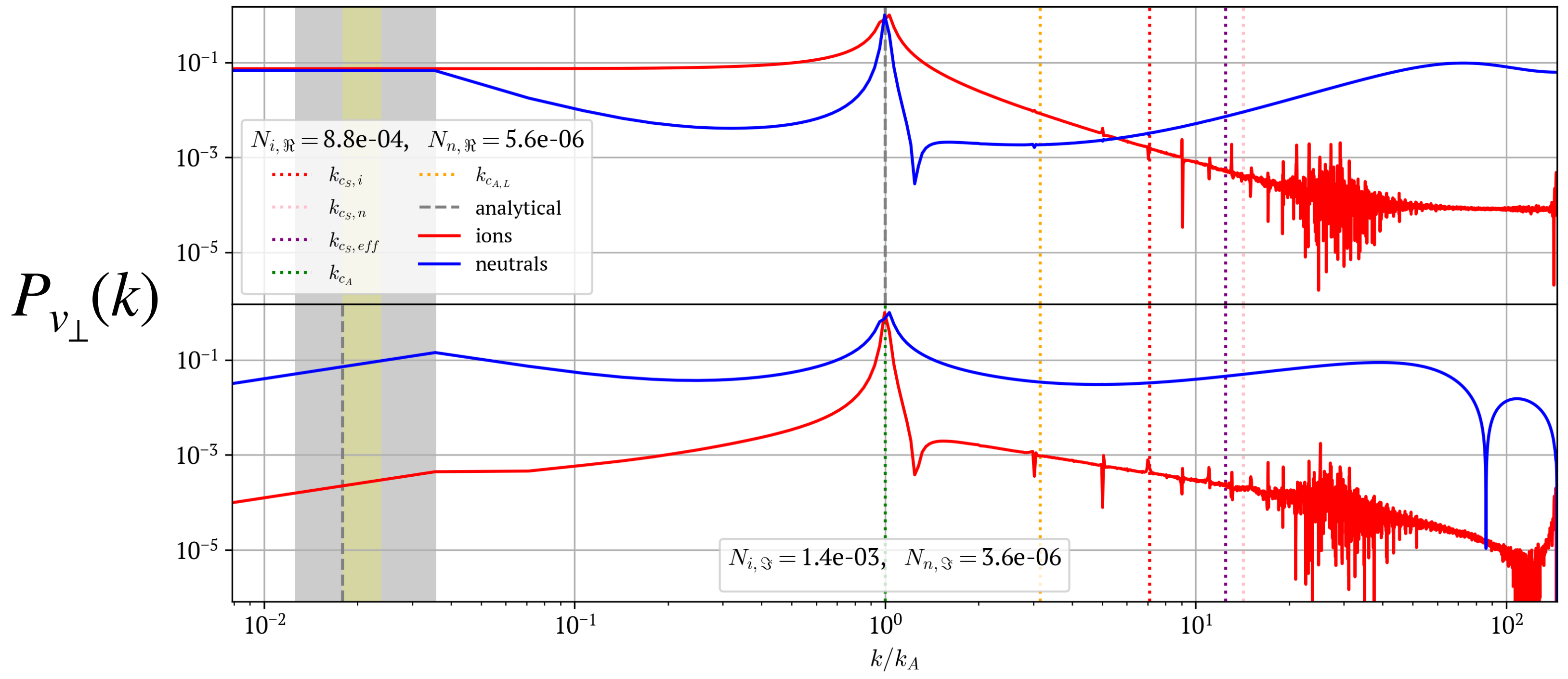
Parameter:
- $\rho_i = 0.1,\ \rho_n = 0.9$
- $c_{Sn} = 2,\ c_{Si} = 1 \times \mathrm{km/s}$
- $\beta = 0.1$ & $\gamma_D = 25$

Low-Freq. Limit ($\omega = 0.01$)

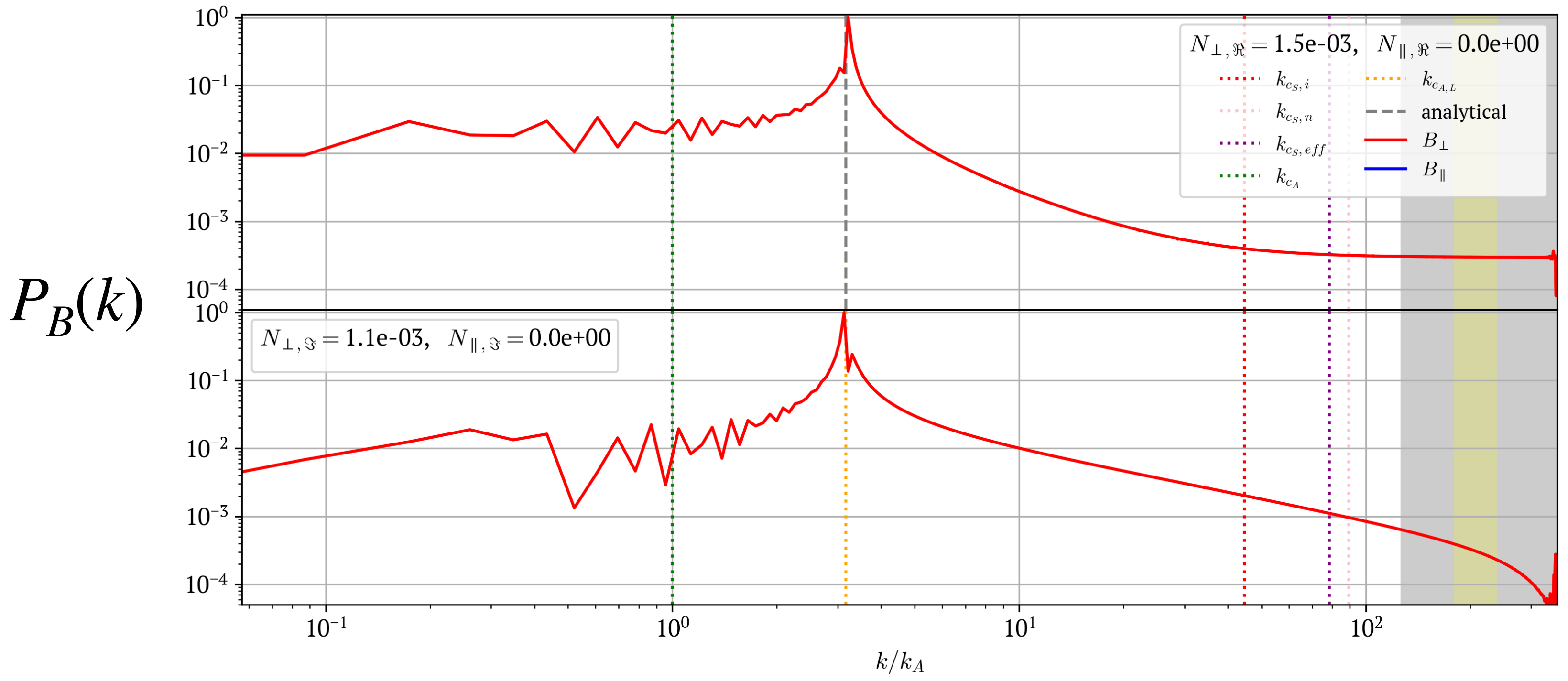
High-Freq. Limit ($\omega = 100$)

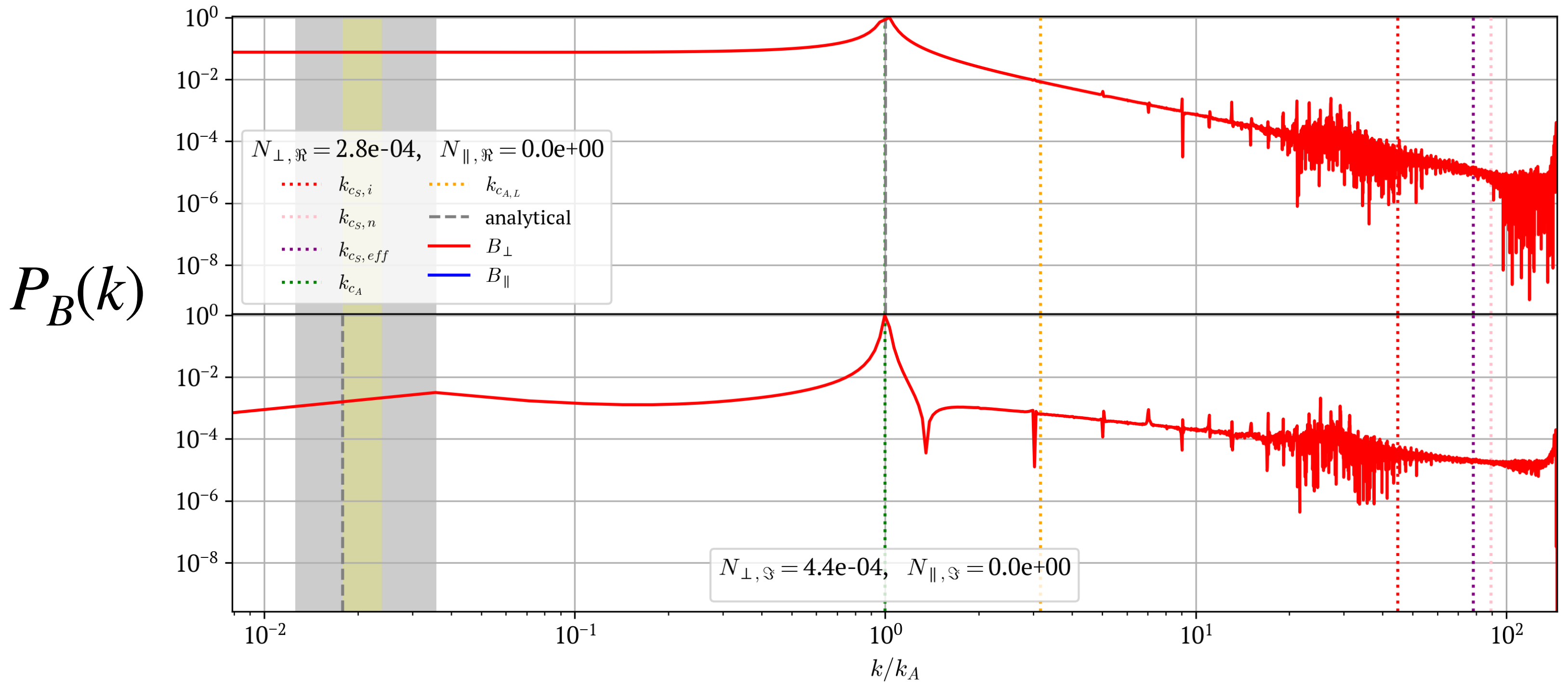
Velocity $v_\perp$





B-Field B

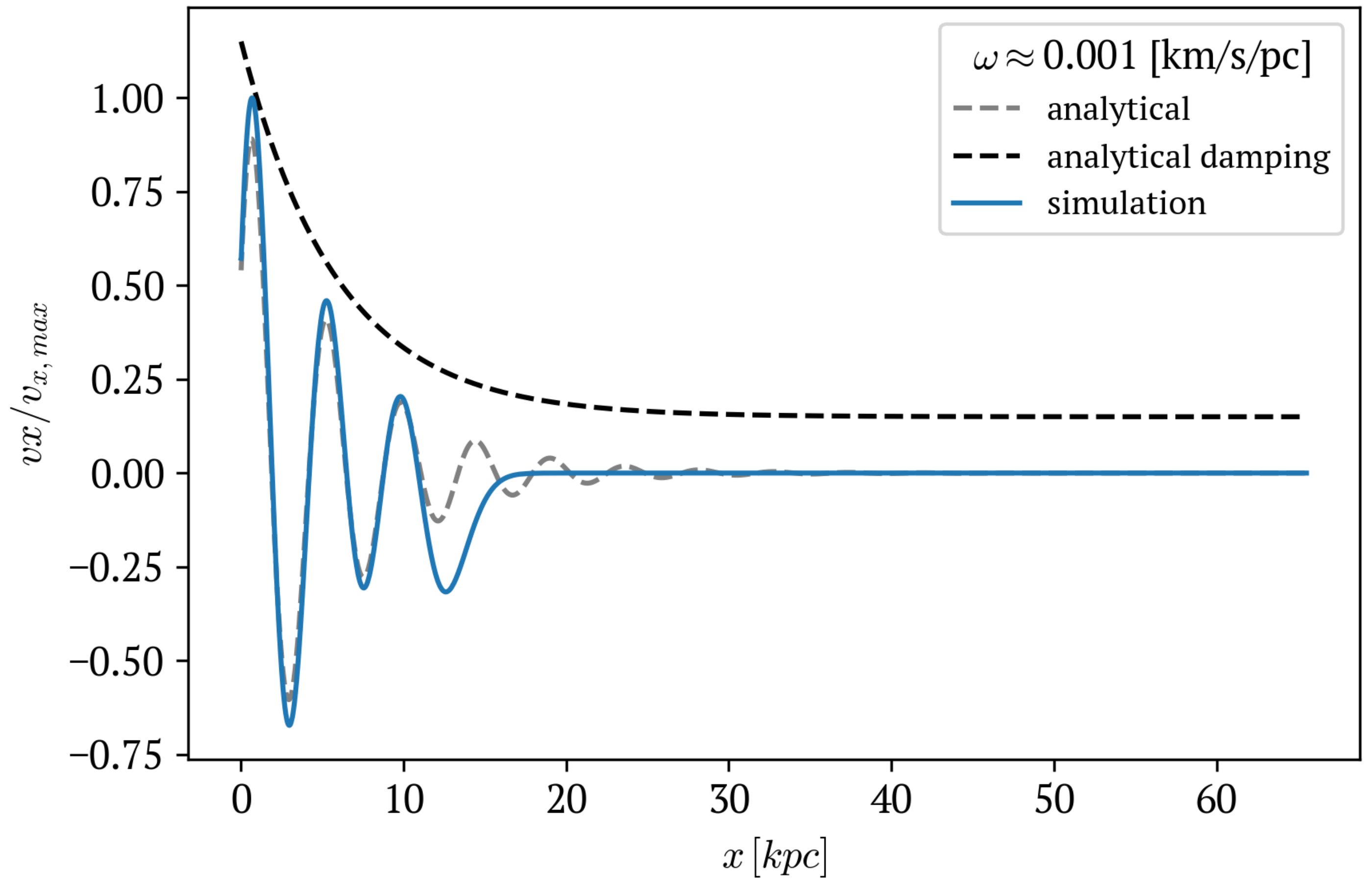


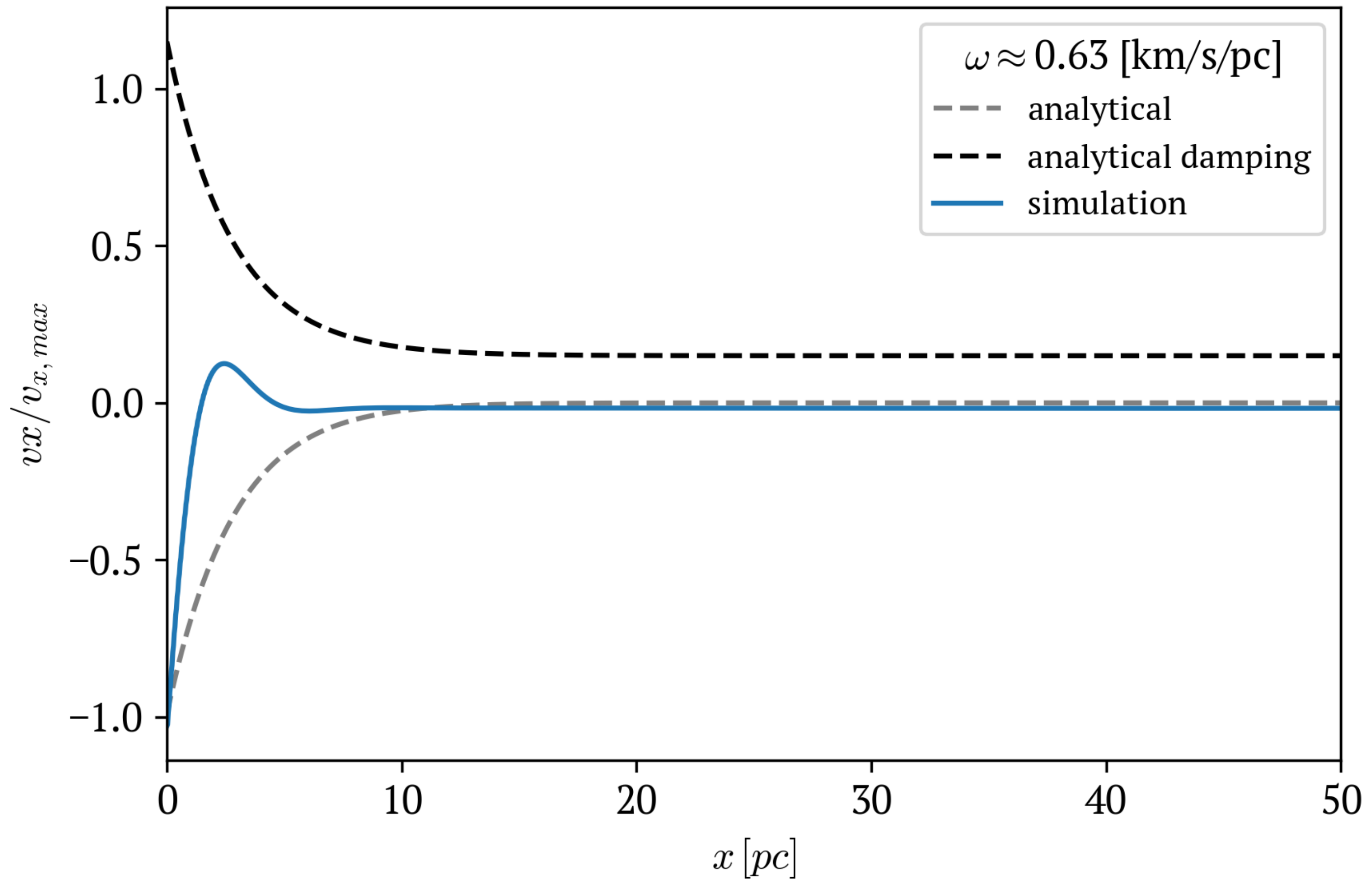


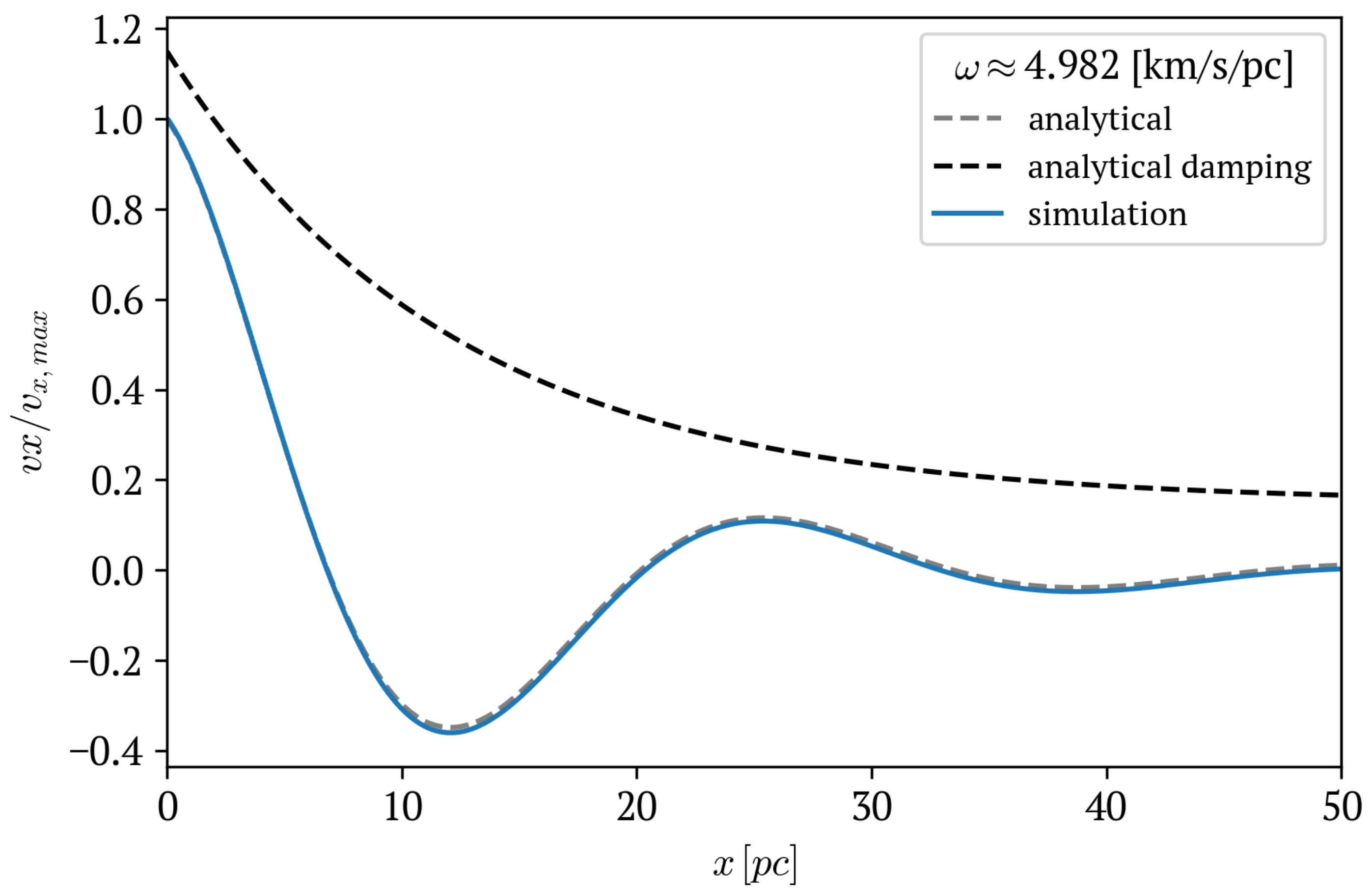
COMPARING WITH ANALYTICS - TRANSVERSE $\delta v_{\perp}$

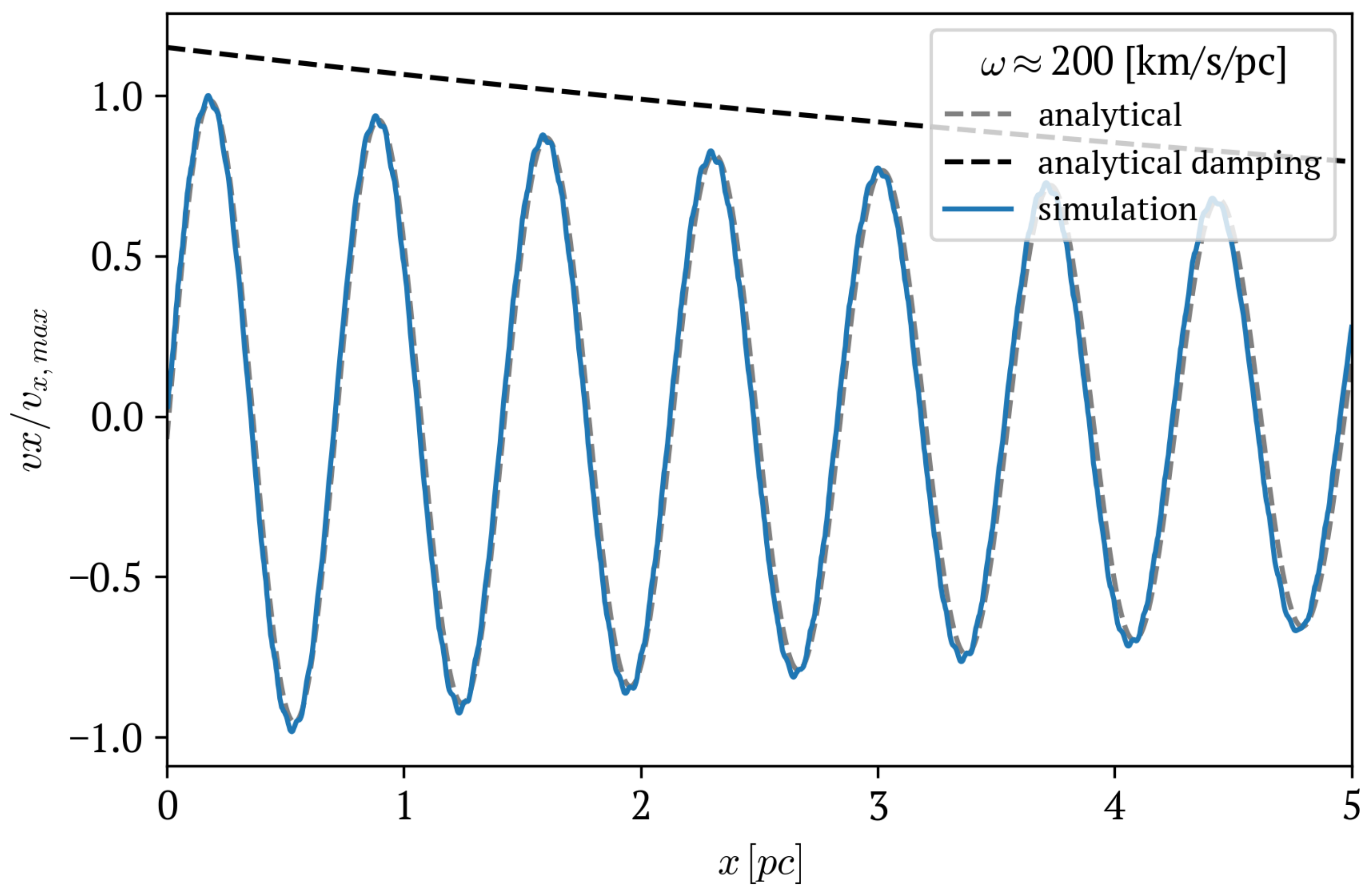
Parameter:

- $\rho_i = 0.001,\ \rho_n = 0.999$
- $c_{Sn} = 2,\ c_{Si} = 1 \times \mathrm{km/s}$
- $\beta = 0.1$ & $\gamma_D = 25$
- $\theta = 0°$



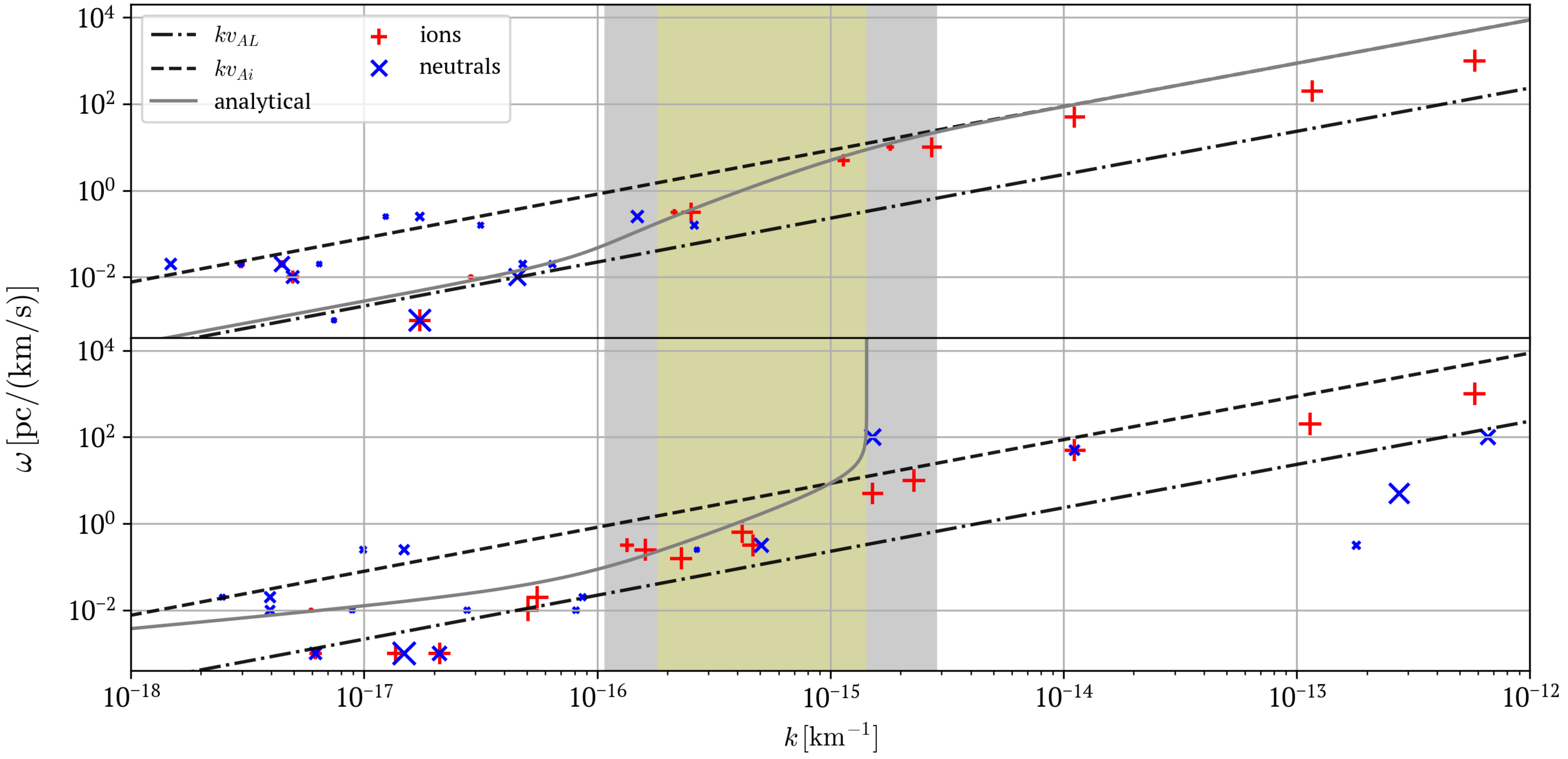






GAP TESTING - „DISPERSION RELATION“

Parameter:
- $\rho_i = 0.001,\ \rho_n = 0.999$
- $c_{Sn} = 2,\ c_{Si} = 1 \times \mathrm{km/s}$
- $\beta = 0.1$ & $\gamma_D = 25$



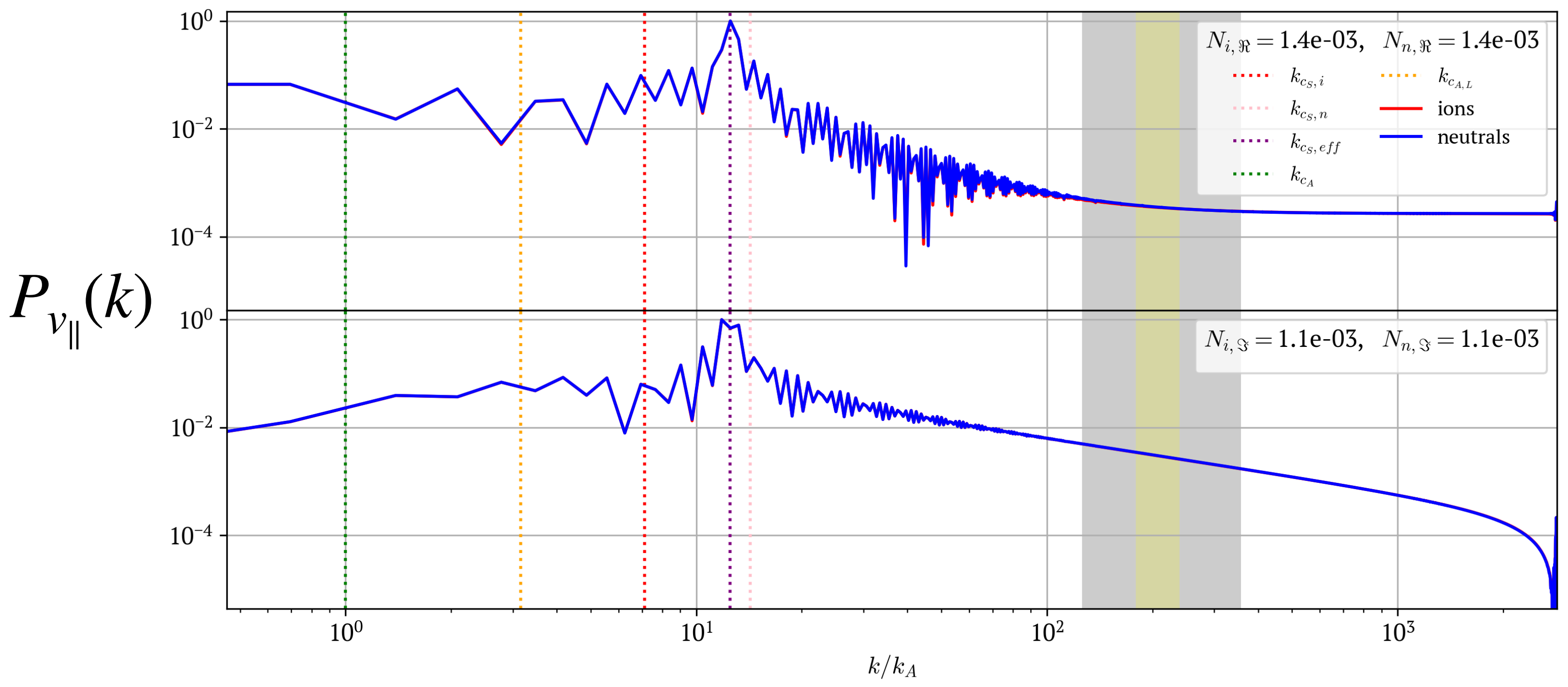
ASYMPTOTIC TESTING - LONGITUDINAL $\delta v_{\|}$ (2FMHD)

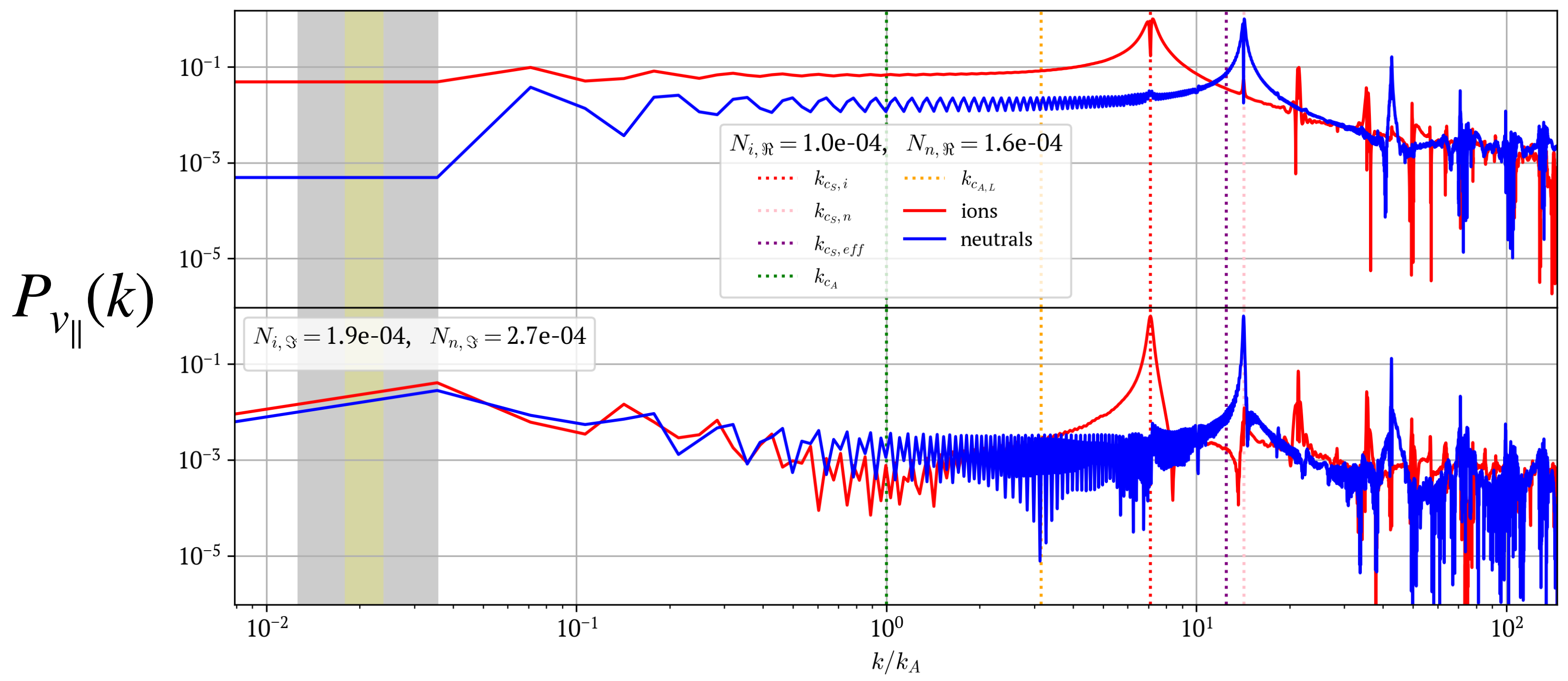
Parameter:
- $\rho_i = 0.1,\ \rho_n = 0.9$
- $c_{Sn} = 2,\ c_{Si} = 1 \times \mathrm{km/s}$
- $\beta = 0.1$ & $\gamma_D = 25$

Low-Freq. Limit ($\omega = 0.01$)

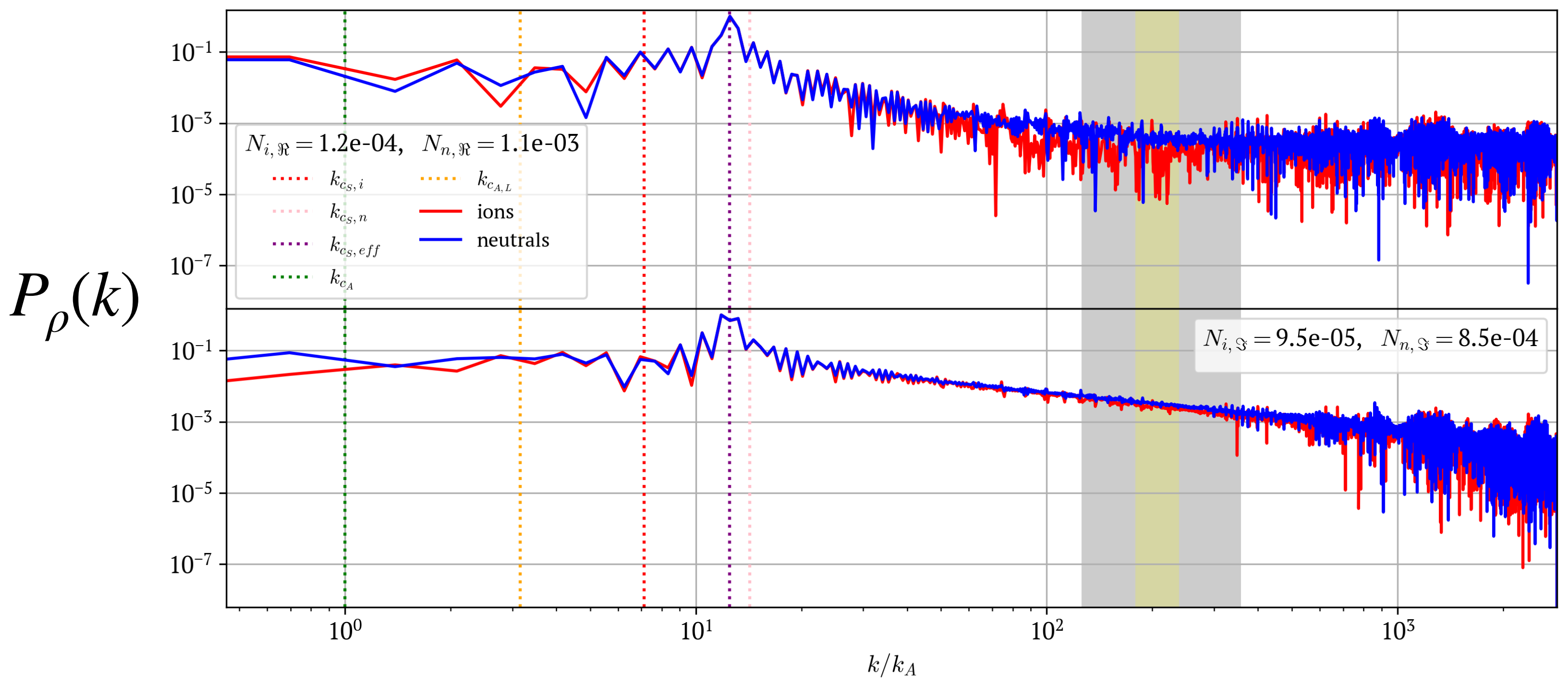
High-Freq. Limit ($\omega = 100$)

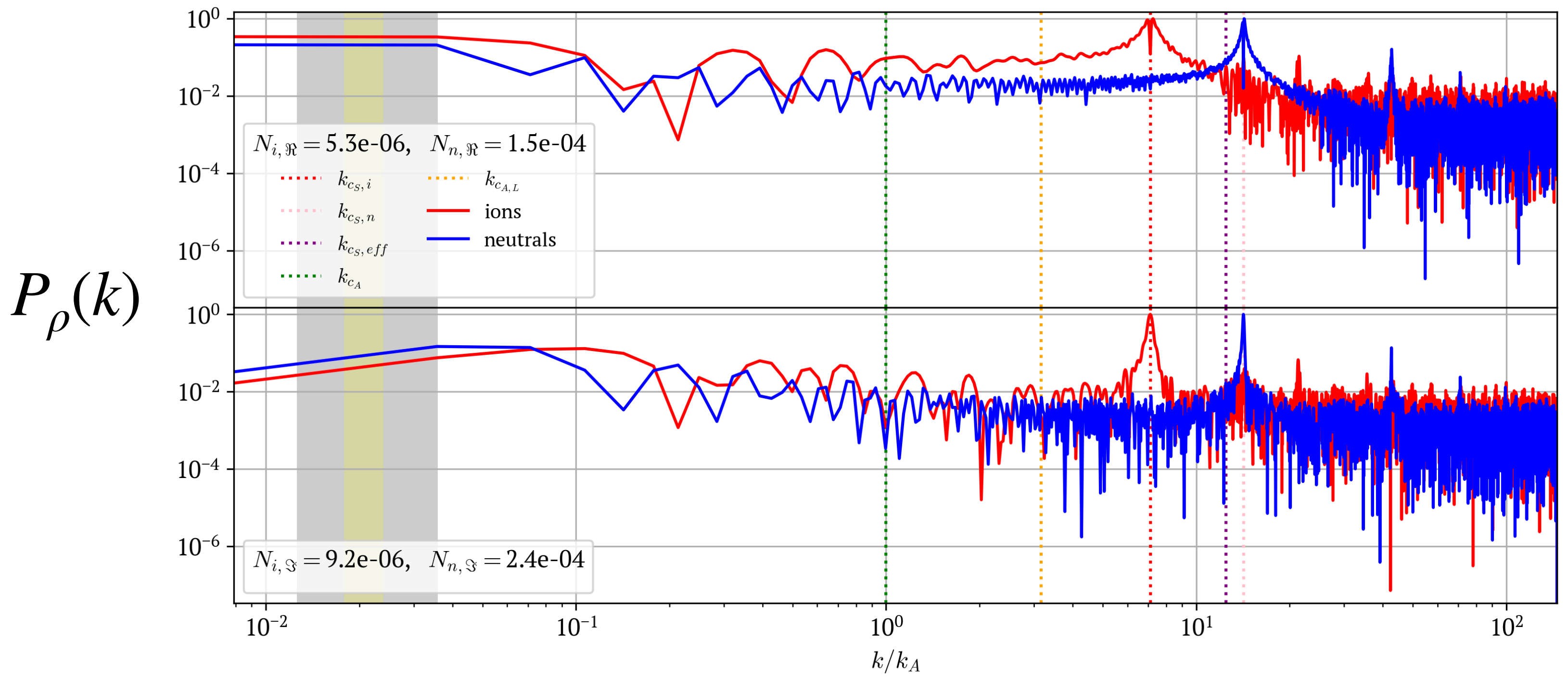
Velocity $v_{\|}$



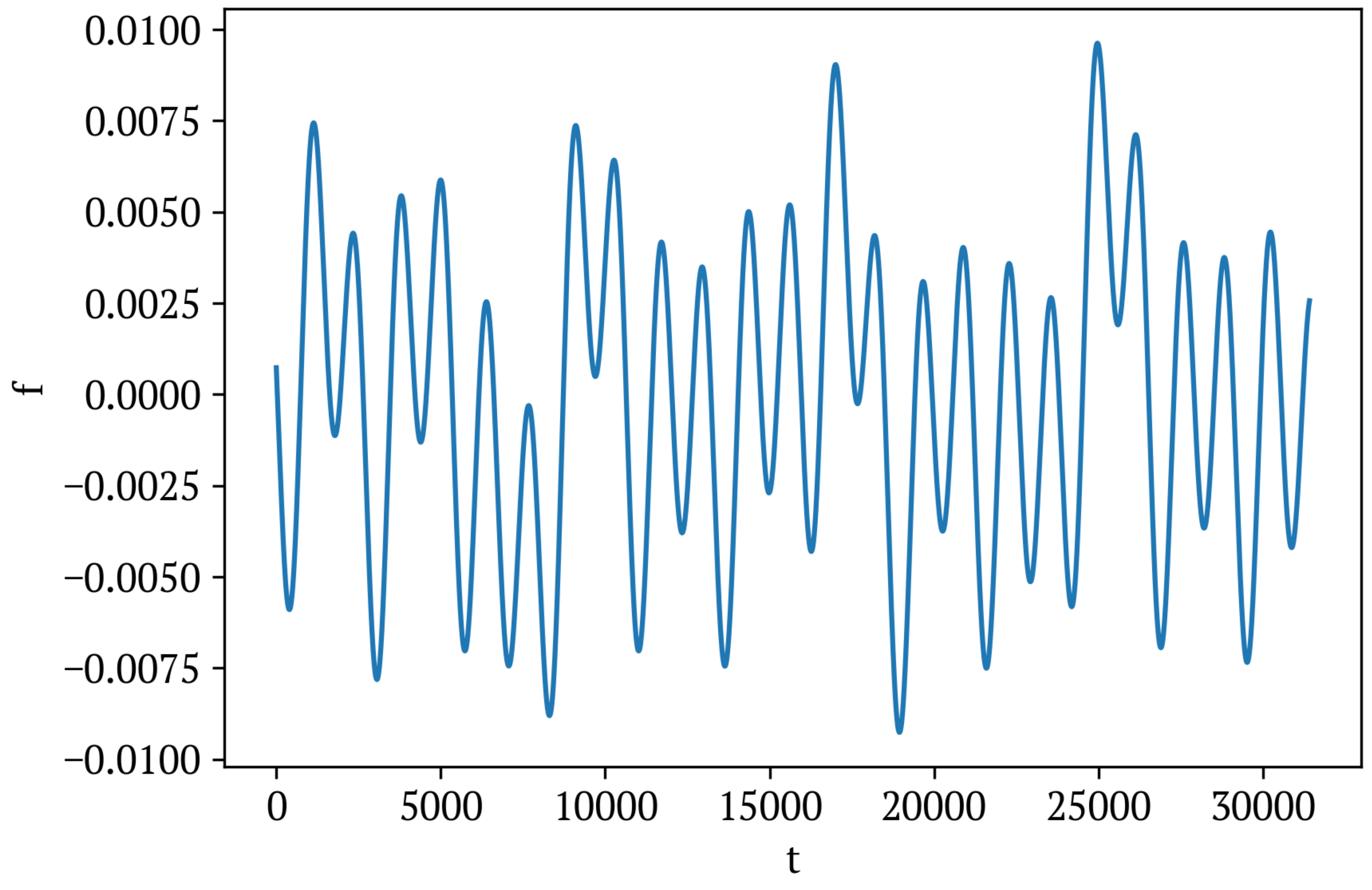


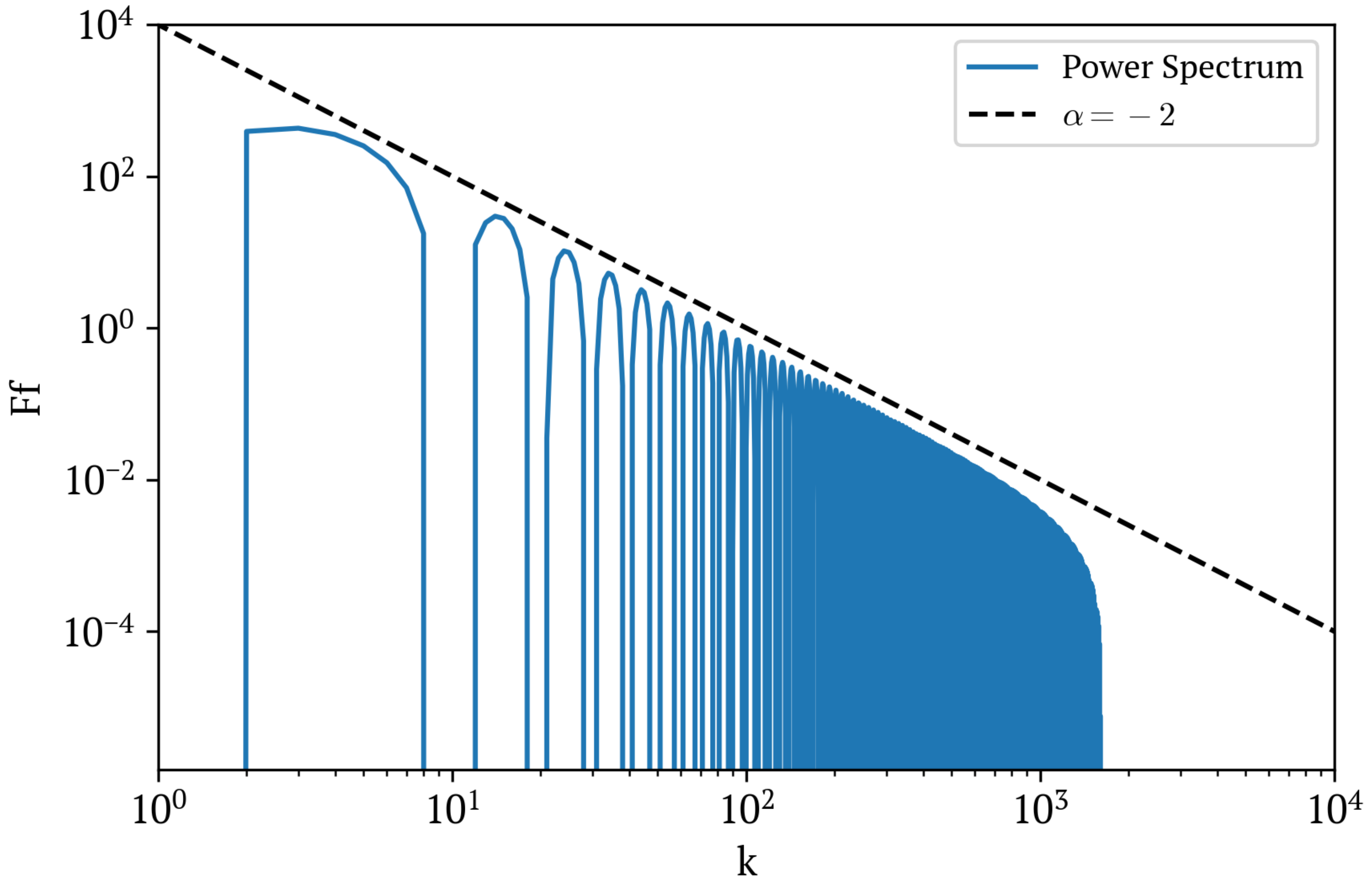
Density ρ





OUTLOOK - DISPERSION





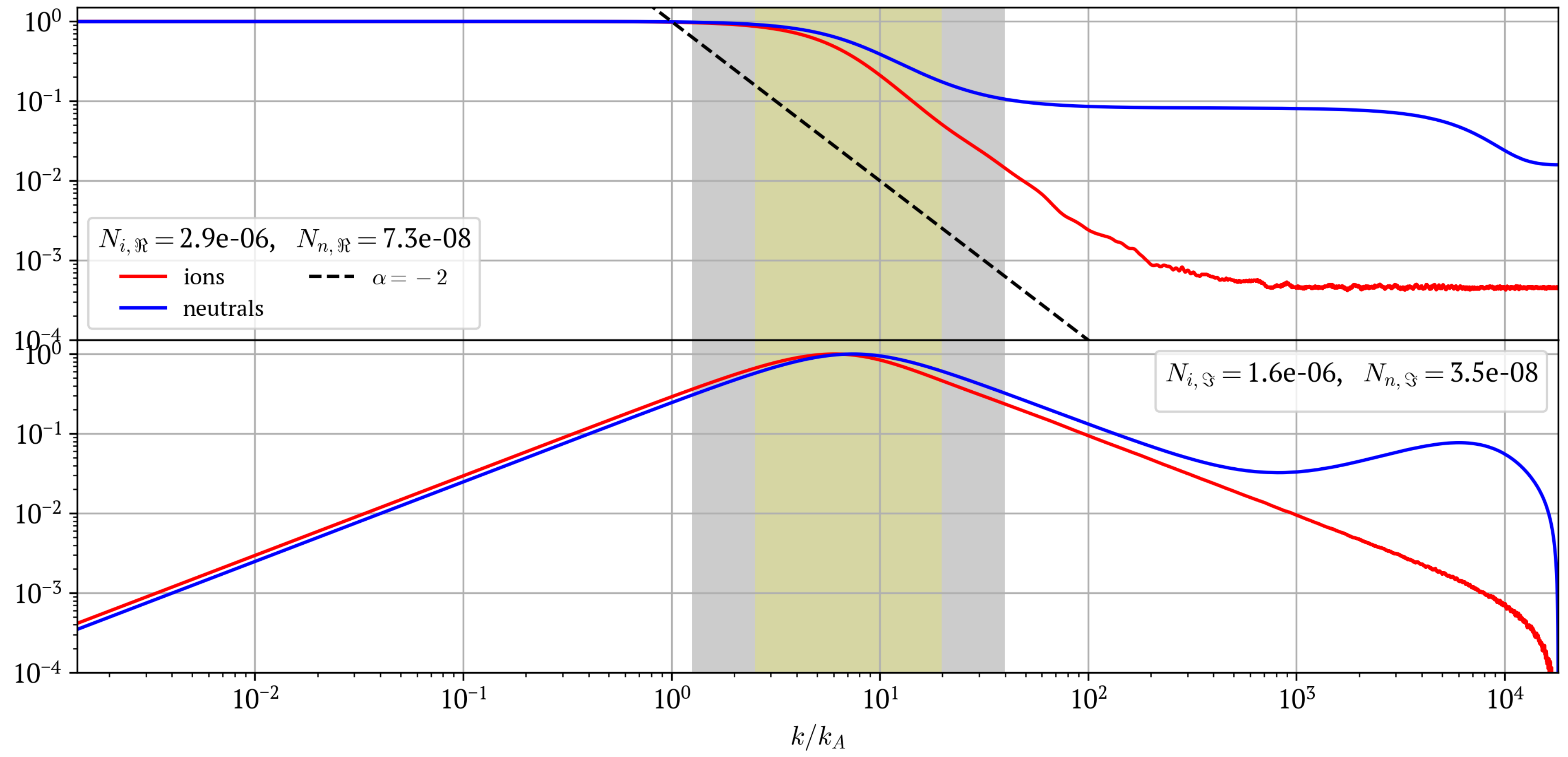
Parameter:
- $\rho_i = 0.001,\ \rho_n = 0.999$
- $c_{Sn} = 2,\ c_{Si} = 1 \times \text{km/s}$
- $\beta = 0.1$ & $\gamma_D = 25$

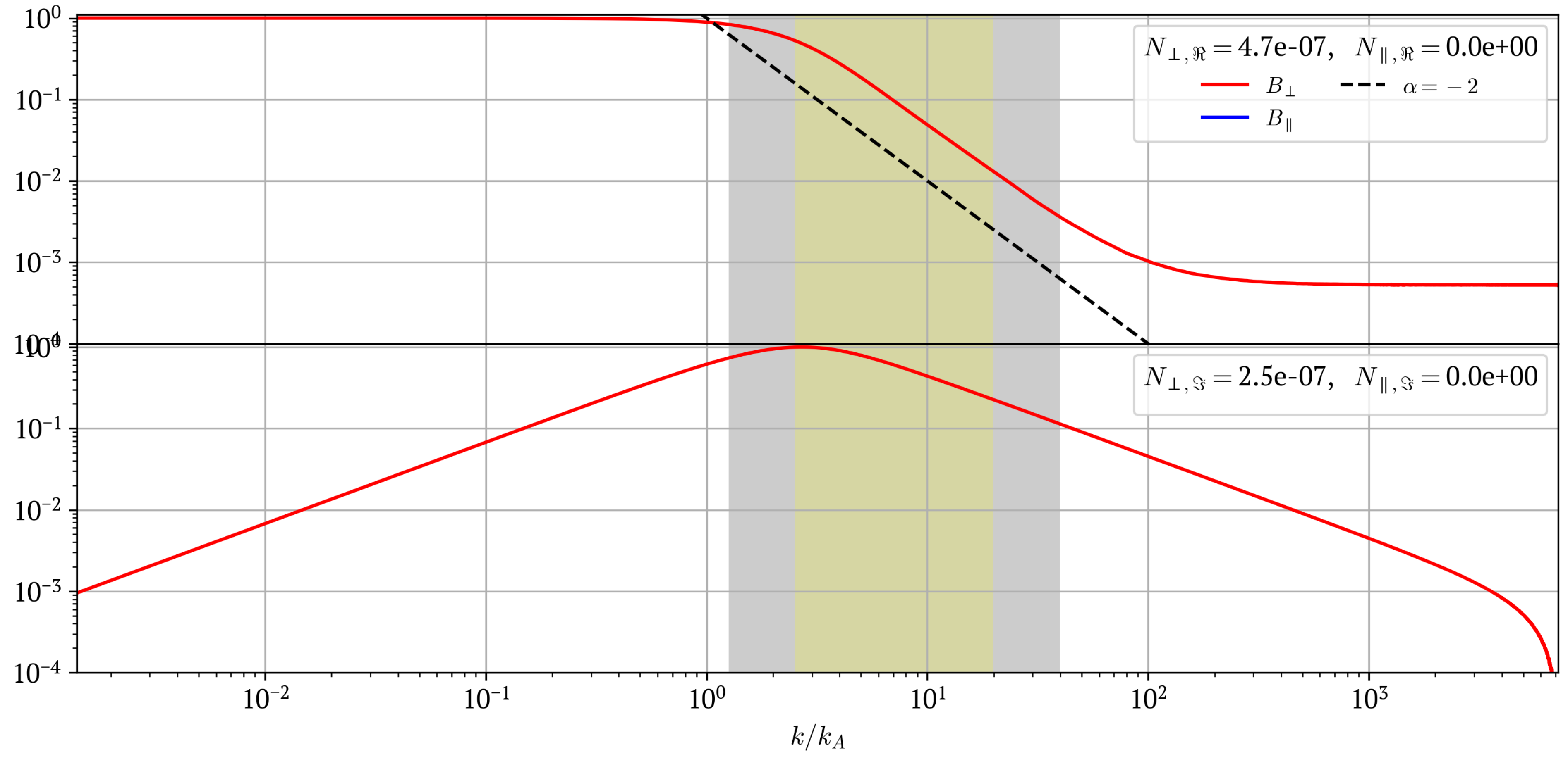
Excite sum of waves with randomized phases

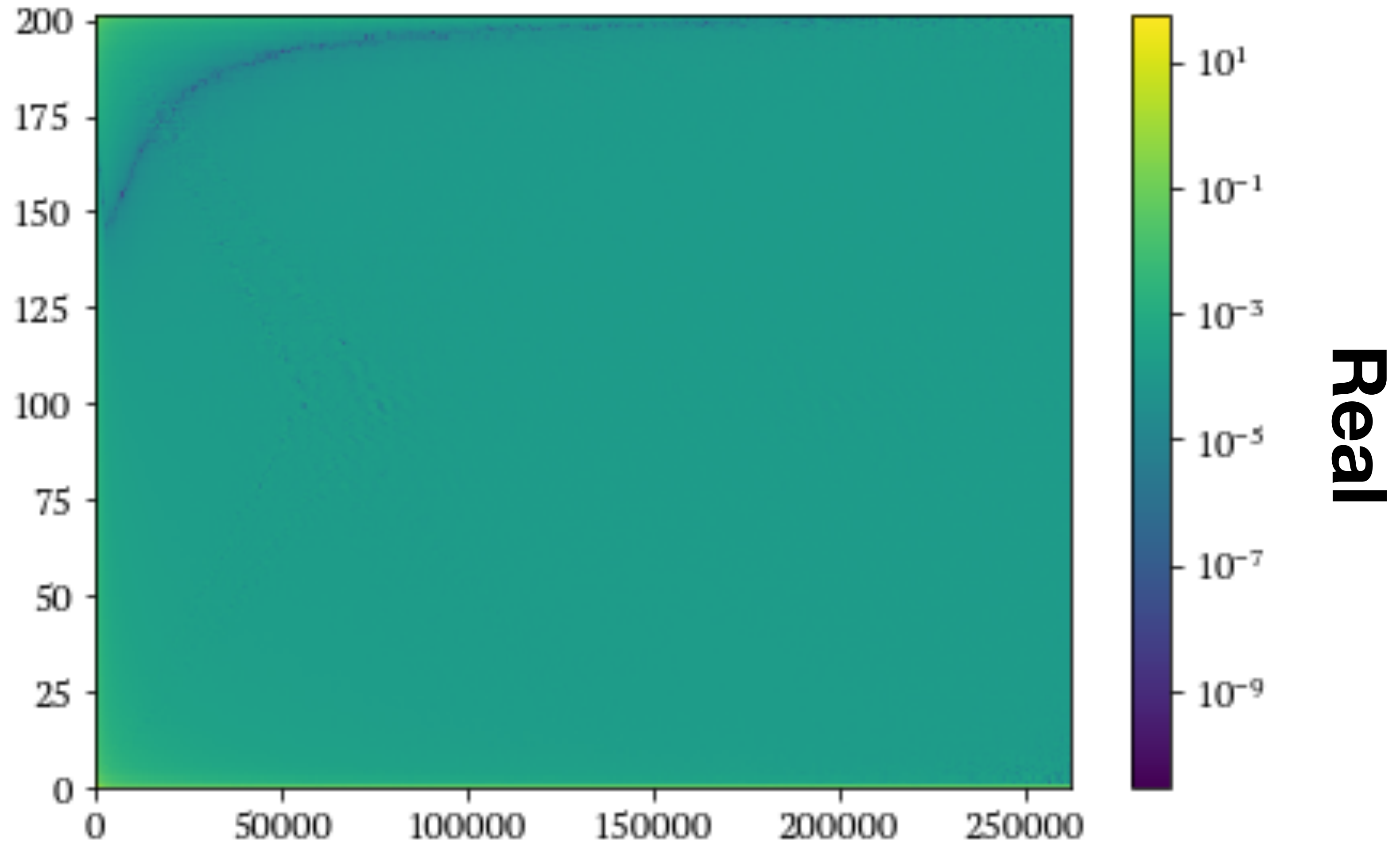
$$F(t) = \sum_{k=1}^{N} \omega_k \cos\left(\omega_k t + \phi_k\right),\ \omega_k = \frac{2\pi v_A}{L(k+1)},\ N = \frac{L}{2\pi v_A} 10^3$$

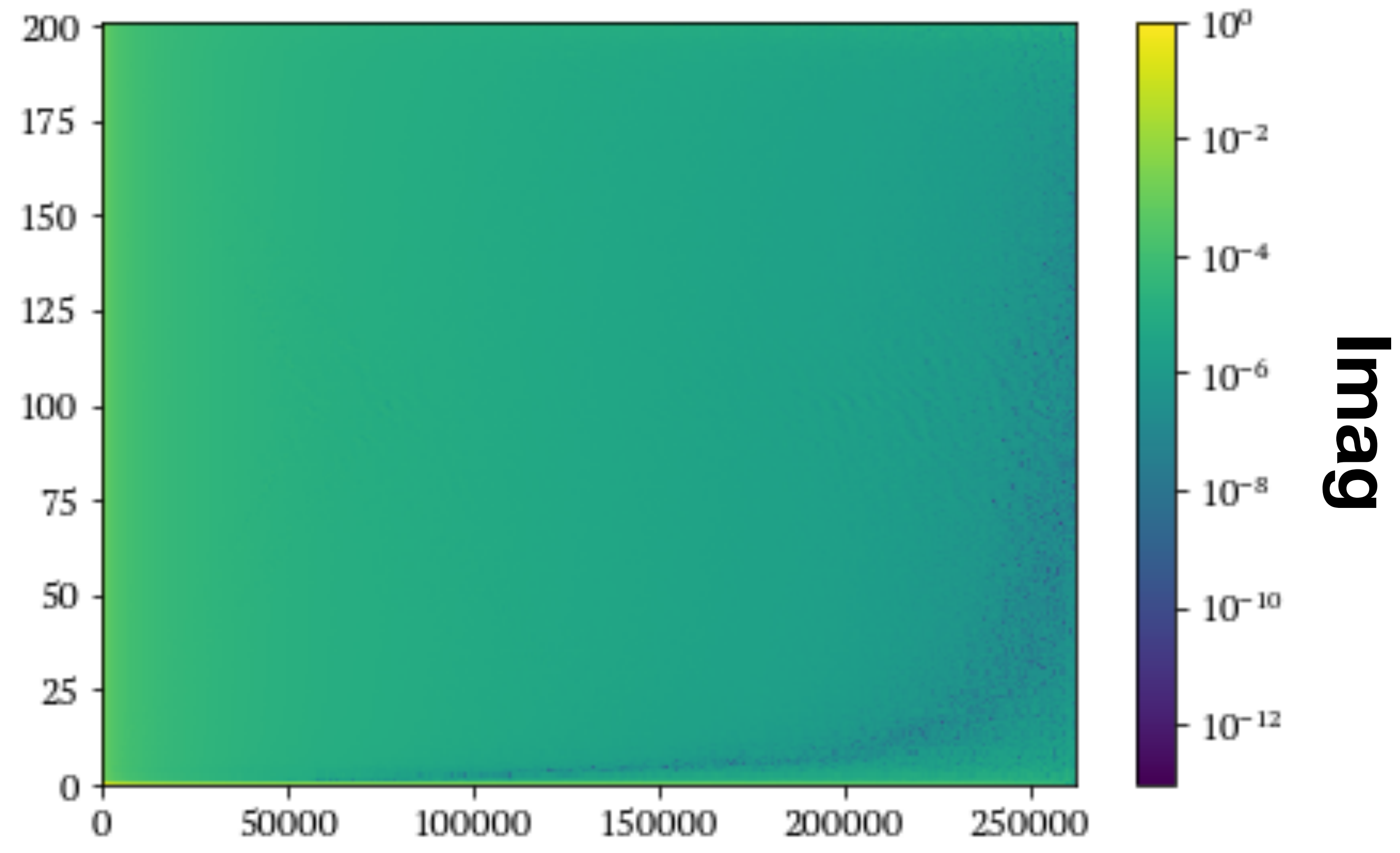
$$\implies \omega_0 > \frac{2\pi v_A}{L},\ \omega_N \gtrsim \frac{1}{\Delta t}$$

OUTLOOK - DISPERSION







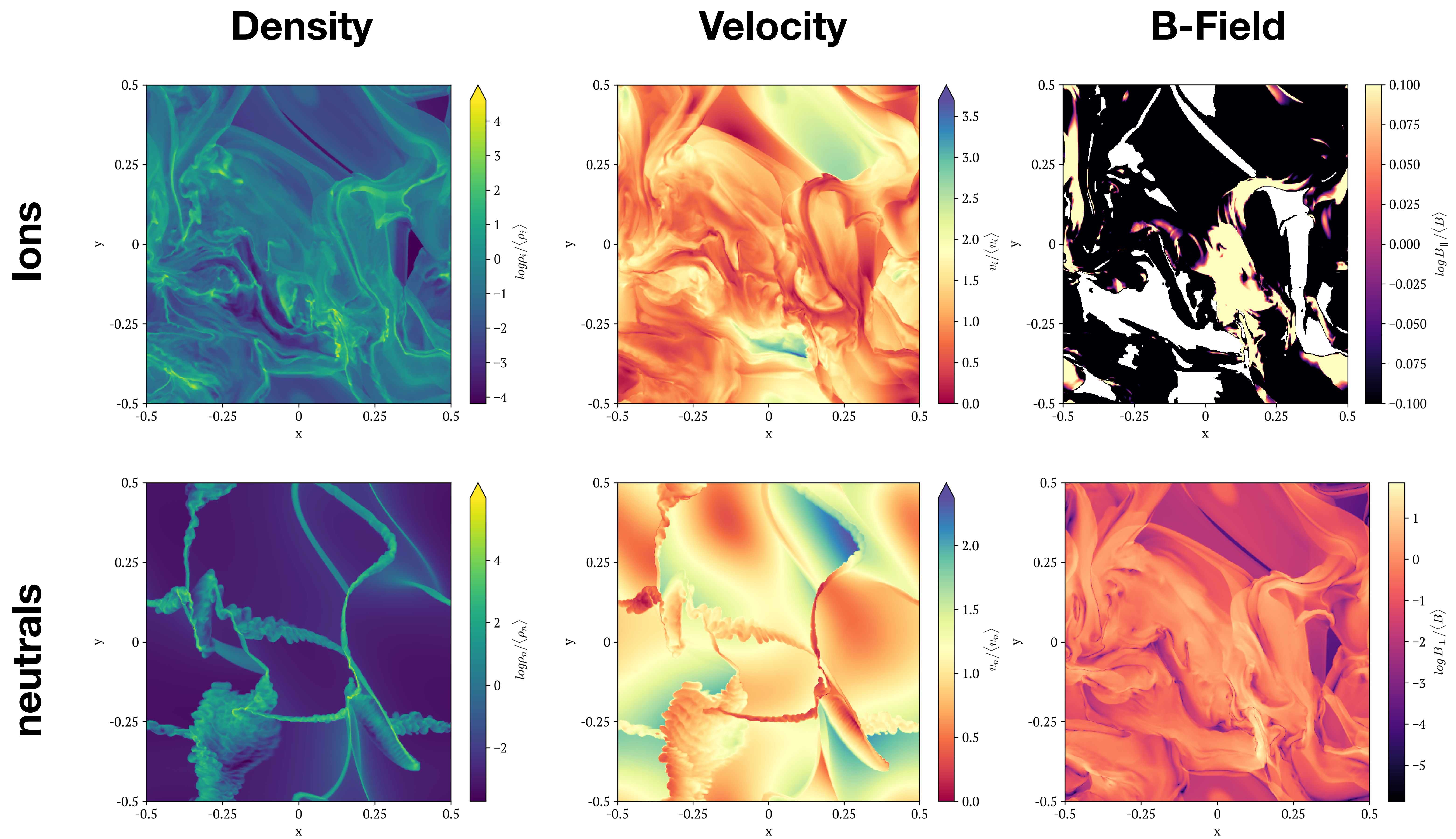


Parameter:

- $\rho_i = 0.001, \ \rho_n = 0.999$
- $c_{Sn} = 2, \ c_{Si} = 1 \times \text{km/s}$
- $\beta = 0.1$ & $\gamma_D = 25$

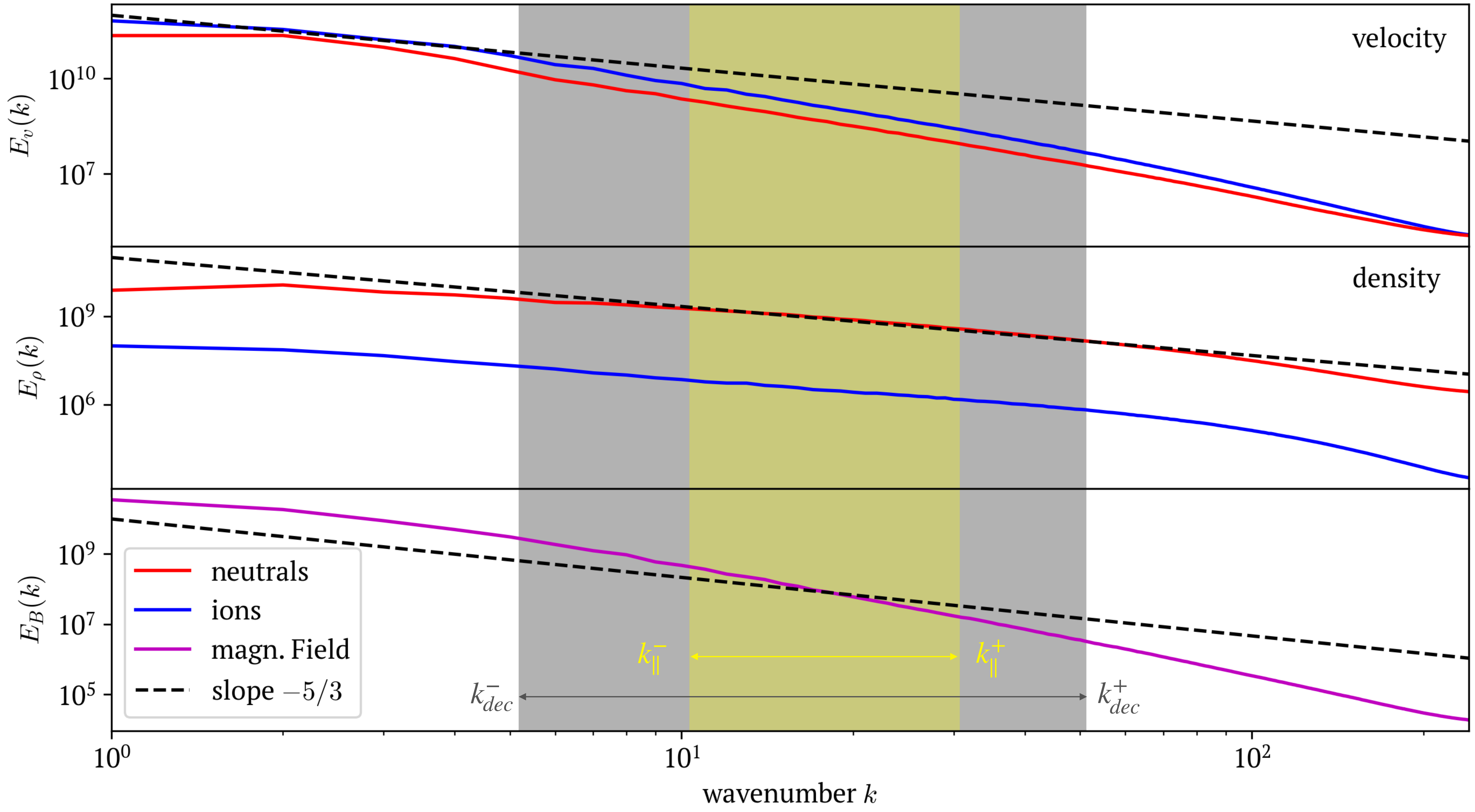
OUTLOOK - TURBULENT SIMULATIONS

Parameter:
- $\rho_i = 0.001,\ \rho_n = 0.999$
- $c_{Sn} = 2,\ c_{Si} = 1 \times \mathrm{km/s}$
- $\beta = 0.1$ & $\gamma_D = 25$



OUTLOOK - TURBULENT SIMULATIONS

Parameter:
- $\rho_i = 0.001,\ \rho_n = 0.999$
- $c_{Sn} = 2,\ c_{Si} = 1 \times \mathrm{km/s}$
- $\beta = 0.1$ & $\gamma_D = 25$



THANK YOU FOR YOUR ATTENTION!